CE 201 STATICS (Sections 4 & 6)
First Semester (081)
H.W # 7 Solutions.

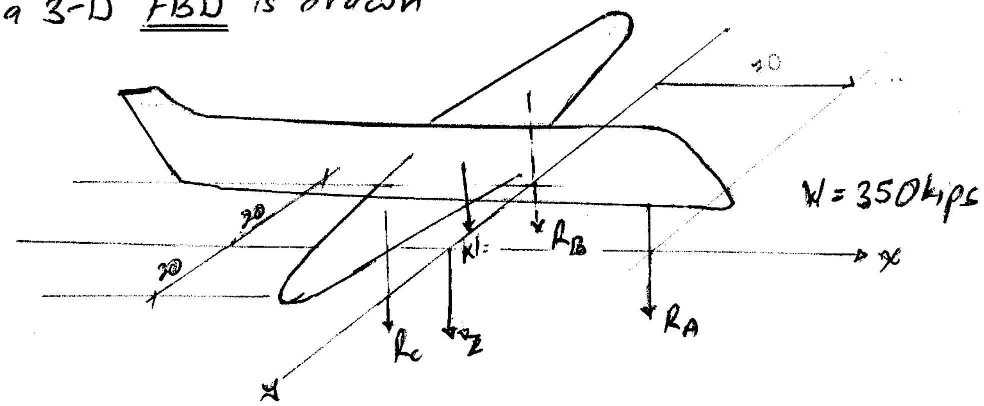
Problem 1

Given:- Given Figure P1 in the question paper
- Coordinates (10, 1.5, -15) ft of point G where the 350-kip weight of the plane acts.

Required: The magnitudes of the normal reactions exerted by the ground on the plane's landing gear.

Solution

First, a 3-D FBD is drawn



$$\Sigma F_z = 0 \Rightarrow R_A + R_B + R_C + 350 = 0 \quad -①$$

Taking Moments of the forces about the axes

$$\Sigma M_y = 0, \circlearrowleft^{+} \Rightarrow$$

$$(-350 \times 10) - (70 \times R_A) = 0 \quad - ②$$

$$R_A = -50 \text{ kips}$$

$$\Rightarrow \boxed{R_A = 50 \text{ kips, upward } (-z)}$$

$\Sigma M_x = 0, \circlearrowleft^{+} \Rightarrow$

$$(R_c \times 20) - (R_B \times 20) + (350 \times 1.5) = 0 \quad - ③$$

$$20R_c - 20R_B = -525 \ (ft \cdot kips)$$

Substituting R_A into Equation ①

$$R_c + R_B = -300 \quad - *$$

From equation ③

$$R_c = -26.25 + R_B \quad - **$$

Substituting ** into *

$$2R_B - 26.25 = -300$$

$$R_B = -136.9 \text{ kips.}$$

$\Rightarrow$ $\boxed{R_B = 136.9 \text{ kips, upwards } (-z)}$

Substituting R_B into **

$$R_c = -163.1 \text{ kips.}$$

$\Rightarrow$ $\boxed{R_c = 163.1 \text{ kips, upwards } (-z)}$

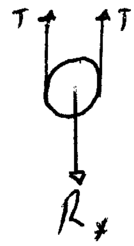
Problem 2

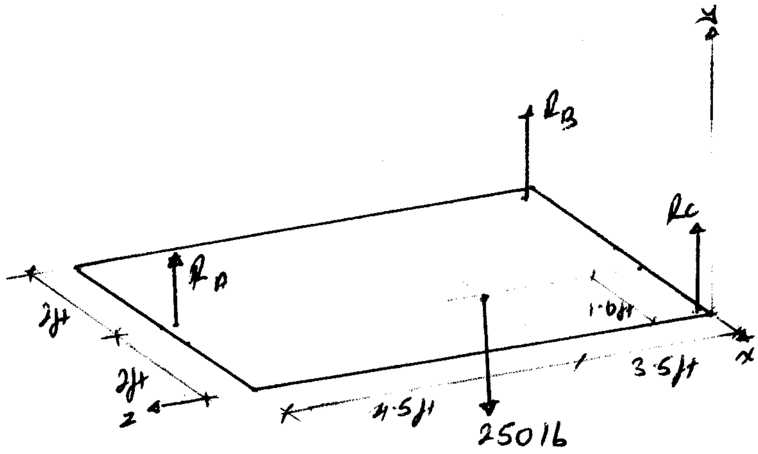
Given: figures P2 in question sheet

Required: Tensions in the 3 ropes when the system is in equillibrium.

Solution

Drawing the FBD, first.





where $*$ is A, B or C

$$2T = R_*$$

$$T = \frac{R_*}{2} \quad \text{— (1)}$$

$$\Sigma F_y = 0 \Rightarrow$$

$$R_A + R_B + R_C - 250 = 0$$

$$R_A + R_B + R_C = 250\,lb \quad \text{— (2)}$$

Taking Moments of the forces about the axes

$$\circlearrowleft + \Sigma M_x = 0; \Rightarrow$$

$$(-R_A \times 8) + (250 \times 3.5) = 0$$

$$R_A = 109.375\,lb \quad \text{— (3)}$$

Substituting Equation (3) into (1)

$$\Rightarrow \boxed{T_A = \frac{R_A}{2} = 54.69\,lb}$$

$\circlearrowleft + \sum M_z = 0,$

$$(-R_A \times 2) + (-R_B \times 4) + (250 \times 1.6) = 0$$

$$R_B = 45.3125 \text{ lb} \quad — \quad (4)$$

$$\Rightarrow \boxed{T_B = \frac{R_B}{2} = 22.66 \text{ lb}}$$

Substituting R_A and R_B into Equation (2)

$$R_c = 95.3125 \text{ lb}$$

$$\Rightarrow \boxed{T_c = \frac{R_c}{2} = 47.66 \text{ lb}}$$

* Note that you may take the moment about a point instead of an axis. Try it!

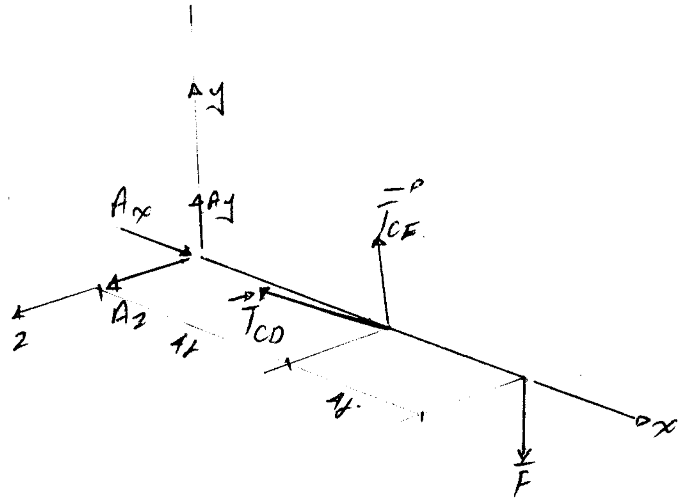
Problem 3

Given: A boom as shown in figure P3 of question sheet

Required: Maximum force F which can be applied if the maximum allowable tension in any cable is 5000 lb.

Solution

First, the FBD is drawn.



A(0, 0, 0)

B(8, 0, 0)

C(4, 0, 0)

D(0, 2, 4)

E(0, 4, -3)

$$\vec{r}_{CD} = -4\vec{i} + 2\vec{j} + 4\vec{k}$$

$$CD = \left[(-4)^2 + (2)^2 + (4)^2\right]^{1/2}$$

$$= 6 ft.$$

$$\vec{u}_{CA} = \frac{\vec{r}_{CA}}{CA} = -\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}$$

$$\vec{T}_{CD} = T_{CD}\left(-\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}\right)$$

$$\vec{u}_{CE} = \frac{\vec{r}_{CE}}{CE} = \frac{-4\vec{i} + 4\vec{j} - 3\vec{k}}{\left[(4)^2 + (4)^2 + (3)^2\right]^{1/2}}$$

$$= \frac{-4\vec{i}}{\sqrt{41}} + \frac{4\vec{j}}{\sqrt{41}} - \frac{3\vec{k}}{\sqrt{41}}$$

$$\vec{T}_{CE} = T_{CE}\left(-\frac{4}{\sqrt{41}}\vec{i} + \frac{4}{\sqrt{41}}\vec{j} - \frac{3}{\sqrt{41}}\vec{k}\right)$$

$$\vec{r}_{AB} = 8\vec{j}$$

$$\vec{r}_{AC} = \frac{1}{2}\vec{r}_{AB} = 4\vec{i}$$

$$\vec{F} = -F\vec{j}$$

Summing Moments about point A, yields,

$$\sum \vec{M}_A = 0; \Rightarrow$$

$$\Rightarrow \vec{r}_{AC} \times (\vec{T}_{CD} + \vec{T}_{CE}) + \vec{r}_{AB} \times \vec{F} = 0$$

$$\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 0 \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{vmatrix} T_{CD} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 0 \\ \frac{-4}{\sqrt{41}} & \frac{4}{\sqrt{41}} & \frac{-3}{\sqrt{41}} \end{vmatrix} T_{CE} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & 0 & 0 \\ 0 & -F & 0 \end{vmatrix} = 0$$

$$\Rightarrow \left[-\frac{8}{3} T_{CD} + \frac{12}{\sqrt{41}} T_{CE} \right] \vec{j} + \left[\frac{4}{3} T_{CD} + \frac{16}{\sqrt{41}} T_{CE} - 8F \right] \vec{k} = 0 \quad — (1)$$

from (1) above;

$\sum M_x = 0;$ « trivial ! ⇒ ?! »

$$\sum M_y = 0; \Rightarrow \frac{8}{3} T_{CD} = \frac{12}{\sqrt{41}} T_{CE}$$

$$T_{CD} = \frac{9}{2\sqrt{41}} T_{CE} \quad — * \qquad \text{or} \qquad T_{CE} = \frac{2\sqrt{41}}{9} T_{CD} \quad — **$$

$$\sum M_z = 0; \Rightarrow 8F = \frac{4}{3} T_{CD} + \frac{16}{\sqrt{41}} T_{CE} \quad — ***$$

Substituting * into *** yields.

$$8F = \frac{4}{3}\left(\frac{9}{2\sqrt{41}} T_{CE}\right) + \frac{16}{\sqrt{41}} T_{CE}$$

$$F = \frac{3T_{CE}}{4\sqrt{41}} + \frac{2T_{CE}}{\sqrt{41}} \Rightarrow F = 0.42948\, T_{CE} \quad — (1)$$

** info ***

$$\Rightarrow 8F = \frac{4}{3} T_{CD} + \frac{16}{\sqrt{41}} \left(\frac{2\sqrt{41}\, T_{CD}}{9} \right)$$

$$F = \frac{T_{CD}}{6} + \frac{8\, T_{CD}}{9}$$

$$F = \frac{11}{18} T_{CD} = 0.61111\, T_{CD} \quad \text{— (2)}$$

From equations (1) and (2), if the maximum allowable tension in any of the cables T_{CE} and T_{CD} is 5000 lb.

$F = 2147$ lb or $F = 3055$ lb

F_{min} controls "to get max. all. force" (Why?!)

From the above,

$$\boxed{T_{CE} \text{ controls and } F = 2147 \text{ lb}}$$

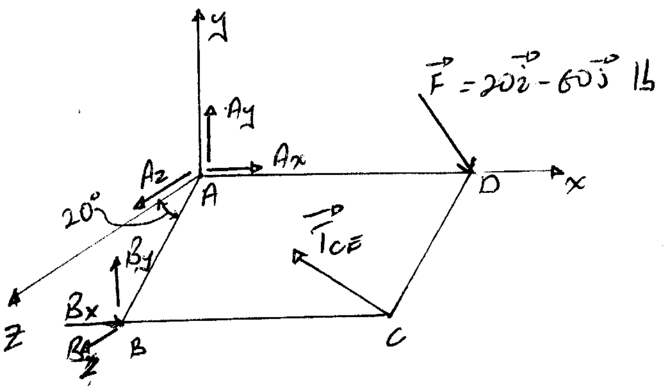
Problem 4

Given: The plate in figure P4 as shown in question sheet.

Required: All forces.

Solution:

The FBD is drawn first



A(0, 0, 0)

B(0, −4Sin 20, 4Cos 20)

C(4, −4Sin 20, 4Cos 20)

D(4, 0, 0)

E(0, 2, 6)

Takings moments about point A:

$$\sum \vec{M}_A = 0; \Rightarrow \vec{r}_{AC} \times T_{CE} + \vec{r}_{AB} \times \vec{B} + \vec{r}_{AD} \times F \quad - (*)$$

$$\vec{r}_{AC} = 4\vec{i} - (4\sin 20)\vec{j} + (4\cos 20)\vec{k}$$
$$= 4\vec{i} - 1.3681\vec{j} + 3.7588\vec{k}$$

$$\vec{r}_{CE} = -4\vec{i} + 3.3681\vec{j} + 2.2412\vec{k}$$

$$CE = \left[(-4)^2 + (3.3681)^2 + (2.2412)^2\right]^{1/2}$$

$$CE = 5.6892 \text{ ft}$$

$$\vec{U}_{CE} = \frac{\vec{r}_{CE}}{CE} = \frac{-4\vec{i} + 3.3681\vec{j} + 2.2412\vec{k}}{5.6892}$$

$$\vec{T}_{CE} = T_{CE}\vec{U}_{CE}$$
$$= T_{CE}(-0.7031\vec{i} + 0.5920\vec{j} + 0.3939\vec{k})$$

$\vec{r}_{AB} = 0\vec{i} - 1.3681\vec{j} + 3.7588\vec{k}$

$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$

$\vec{r}_{AD} = 4\vec{i}$

$\vec{F} = 20\vec{i} - 60\vec{j}$

Substituting the above vectors into equation ⊛

$$\sum \vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1.3681 & 3.7588 \\ -0.7031 & 0.5920 & 0.3939 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1.3681 & 3.7588 \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 0 \\ 20 & -60 & 0 \end{vmatrix} = 0$$

$\Rightarrow$

$\sum M_x = 0 \Rightarrow -2.7641\, T_{CE} - 3.759\, B_y - 1.368\, B_z = 0 \quad — ①$

$\sum M_y = 0; \Rightarrow -4.2184\, T_{CE} + 3.759\, B_x = 0 \quad — ②$

$\sum M_z = 0; \Rightarrow 1.4061\, T_{CE} + 1.3681\, B_x = 240 \quad — ③$

Solving (2) and (3)

$$\boxed{T_{CE} = 81.596 \text{ lb}} \cdots (4)$$

$$\boxed{B_x = 91.564 \text{ lb}}$$

Since the hinge at B exerts no force parallel to the hinge axis, then the hinge has two reactions; B_{Normal} and B_x.

$$\Rightarrow B_y = B_N \cos 20 \cdots (5)$$

$$B_z = B_N \sin 20 \cdots (6)$$

Replacing (4), (5) and (6) into (1) gives

$$\boxed{B_N = -56.4 \text{ lb}}$$

From the above,

$$\boxed{B_y = -53 \text{ lb}} \quad \text{and} \quad \boxed{B_z = 19.3 \text{ lb}}$$

Now taking $\sum \vec{F} = 0 \Rightarrow$

$$\sum F_x = 0 = A_x + B_x - 0.7031 T_{CE} + 20 = 0$$

$$\Rightarrow \boxed{A_x = -54.2 \text{ lb}}$$

$\Sigma F_y = 0 = A_y + B_y + 0.592\, T_{CE} - 60 = 0$

$\Rightarrow \boxed{A_y = 64.7 \text{ lb}}$

$\Sigma F_z = 0 \Rightarrow A_z + B_z + 0.3939\, T_{CE} = 0$

$\Rightarrow \boxed{A_z = 12.9 \text{ lb}}$

Note that we can find the magnitude of $\vec{A}$ and $\vec{B}$

$\vec{A} = \sqrt{A_x^2 + A_y^2 + A_z^2} = 85.4 \text{ lb}$

$\vec{B} = \sqrt{B_x^2 + B_y^2 + B_z^2}$

$= \sqrt{B_x^2 + B_H^2} = 107.5 \text{ lb}$

Problem 5.

Given: Figure P5 as shown in the question sheet.

Required: Tension in cable BD.

Solution.

First, the FBD is drawn.

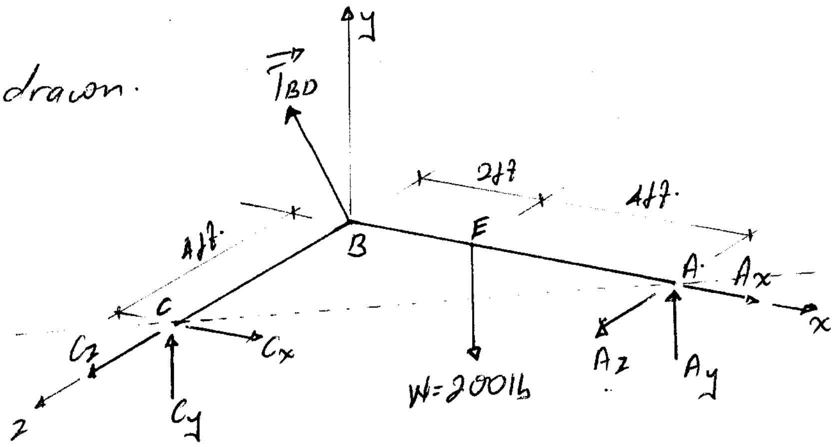
A (6, 0, 0)

B (0, 0, 0)

C (0, 0, 4)

D (-2, 2, -1)

E (2, 0, 0)



Take moments about axis CA. (Why??!!)

$$\sum M_{axis} = \vec{u}_{axis} \sum(\vec{r} \times \vec{F}) = 0$$

$$= \vec{u}_{CA} \left(\vec{r}_{AB} \times \vec{T}_{BD} + \vec{r}_{AE} \times \vec{W} \right) = 0 \quad \text{—— (1)}$$

$$\vec{u}_{CA} = \frac{\vec{r}_{CA}}{CA} = \frac{6\vec{i} - 4\vec{k}}{\sqrt{6^2+4^2}}$$

$$\Rightarrow \vec{u}_{CA} = \frac{6\vec{i}}{\sqrt{52}} - \frac{4\vec{k}}{\sqrt{52}}$$

$\vec{r}_{AB} = -6\vec{i}$

$\vec{r}_{AE} = -4\vec{i}$

$\vec{W} = -200\vec{j}$

$\vec{r}_{BD} = (D) - (B)$
$= -2\vec{i} + 2\vec{j} - \vec{k}$

$r_{BD} = \sqrt{9} = 3\,ft$

$\vec{T}_{BD} = T_{BD}\,\vec{u}_{BD}$
$= T_{BD}\left(-\frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k} \right)$

Substitutings into equation ① yields.

$$\Rightarrow \quad \sum \vec{M}_{CA} = \begin{vmatrix} \frac{6}{\sqrt{52}} & 0 & \frac{-4}{\sqrt{52}} \\ -6 & 0 & 0 \\ -\frac{2}{3}T_{BD} & \frac{2}{3}T_{BD} & \frac{1}{3}T_{BD} \end{vmatrix} + \begin{vmatrix} \frac{6}{\sqrt{52}} & 0 & \frac{-4}{\sqrt{52}} \\ -4 & 0 & 0 \\ 0 & -200 & 0 \end{vmatrix}$$

$$\Rightarrow \quad \frac{-4}{\sqrt{52}}(-6)\left(\frac{2}{3}T_{BD}\right) + \frac{-4}{\sqrt{52}}(-4)(-200) = 0$$

$$\Rightarrow \quad \boxed{T_{BD} = 200 \text{ lb}}$$

Note that $\vec{r}_{CD}$, or $\vec{r}_{AD}$, can be taken instead of $\vec{r}_{AB}$. Which is easier?!