CE 201 STATICS (Sections 4&6)
First Semester 081
H.W. #11 Solution

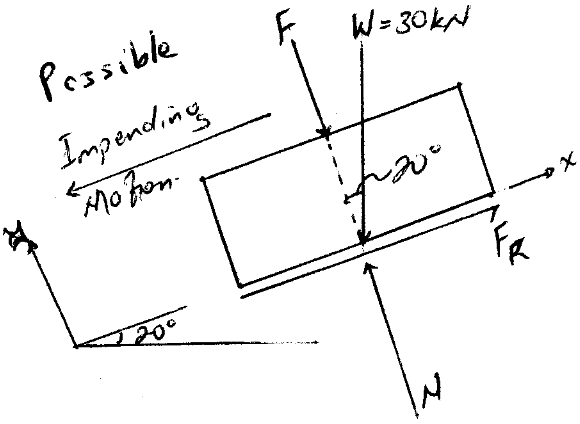
Problem 1.

Given: -figure P1 as shown in question sheet
- $W = 30\,kN$ and $\mu_s = 0.2$

Required: -(a) If $F = 30\,kN$, what is the magnitude of the friction force exerted on the box?
(b) If $F = 10\,kN$, show that the box cannot remain on the inclined surface.

Solution.

(a) When $F = 30\,kN$



$\nearrow^{+} \sum F_x = 0 \Rightarrow F_R - 30\sin 20^\circ = 0$

$\Rightarrow F_R = 10.261\,kN$

Now check, $F_R \overset{?}{\leq} F_{max}$

$+\nwarrow \sum F_y = 0 \Rightarrow -30 - 30\cos 20^\circ + N = 0$

$\Rightarrow N = 58.191\,kN$

$F_{max} = \mu_s N$

$= 0.2 \times 58.191$

$\boxed{F_{max} = 11.683\,kN > F_R = 10.261\,kN, \text{ Thus ok.}}$

(b) When $F = 10\,kN$.

$+\nwarrow \sum F_y = 0 \Rightarrow -10 - 30\cos 20^\circ + N = 0$

$\Rightarrow N = 38.191\,kN.$

$F_{max} = \mu_s N \Rightarrow 0.2 \times 38.191 \Rightarrow F_{max} = 7.638\,kN$

$\nearrow^{+} \sum F_x = 0 \Rightarrow F_R - 30\sin 20^\circ = 0 \Rightarrow F_R = 10.261\,kN$

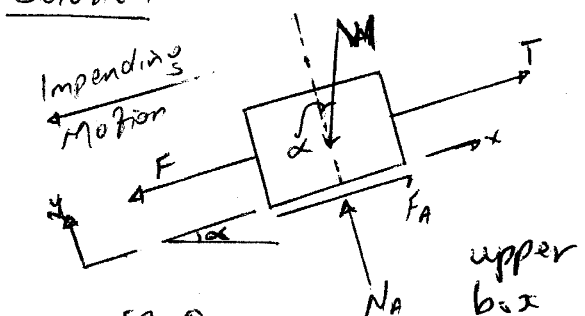
$\therefore \boxed{F_R = 10.261\,kN > F_{max} = 7.638\,kN, \text{ Thus box will move.}}$

Problem 2

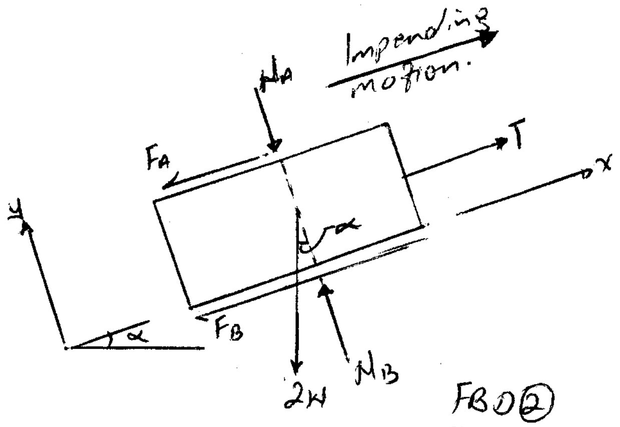
Given: The two boxes in Figure P2 as shown in the question sheet.

Required: The largest force, F that will not cause the boxes to slip.

Solution.



FBD①



FBD②

lower box

x along the incline.

For the upper box - FBD ①

$$\overset{+}{\nearrow} \sum F_x = 0 \Rightarrow T - F - W\sin\alpha + F_A = 0$$

$$+\nwarrow \sum F_y = 0 \Rightarrow N_A - W\cos\alpha = 0$$
$$\Rightarrow N_A = W\cos\alpha$$

Since we have impending motion, $F_A = \mu_A N_A$

$$\Rightarrow T - F - W\sin\alpha + \mu_s W\cos\alpha = 0$$
$$\Rightarrow T = F + W\sin\alpha - \mu_s W\cos\alpha = 0 \quad -①$$

For the lower box - FBD ②

$$+\nwarrow \sum F_y = 0 \Rightarrow N_B = W\cos\alpha + 2W\cos\alpha \Rightarrow N_B = 3W\cos\alpha$$
$$\overset{+}{\nearrow} \sum F_x = 0 \Rightarrow T - 2W\sin\alpha - F_B - F_A = 0$$

Since we have impending motion, $F_B = \mu_s N_B$

$$T - 2W\sin\alpha - \mu_s(3W\cos\alpha) - \mu_s(W\cos\alpha) = 0$$
$$\Rightarrow T = 2W\sin\alpha + 4\mu_s W\cos\alpha = 0 \quad -②$$

Equating ① to ② $\Rightarrow F + W\sin\alpha - \mu_s W\cos\alpha = 2W\sin\alpha + 4\mu_s W\cos\alpha$

$$\Rightarrow \boxed{F = W(\sin\alpha + 5\mu_s\cos\alpha)}$$

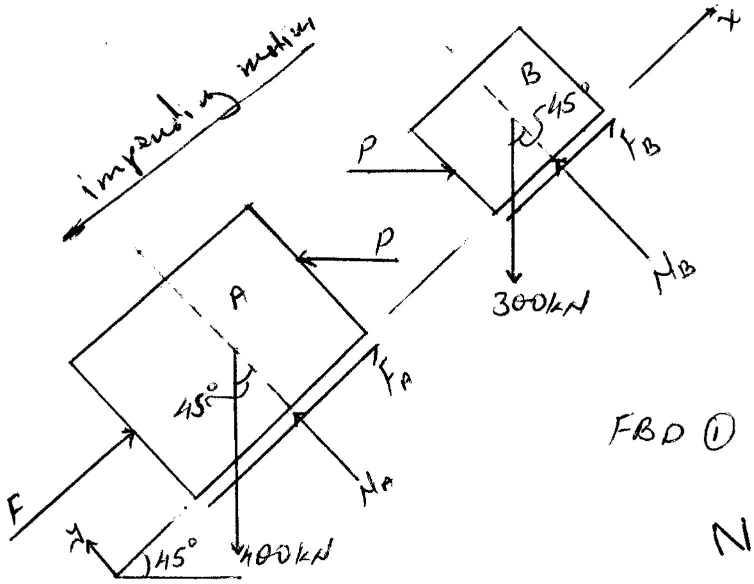
Note that the two boxes must move. (i.e. one box only will not move) why?!

Problem 3

Given: Block A and B as shown in figure P3 of question sheet.
Required: Range of values of force F, for which the blocks will remain in statical equilibrium.

Solution.

First; calculate F_{min} to prevent the blocks from sliding down



$\mu_A = 0.3$

$\mu_B = 0.5$

FBD ①

Note that
$F_{fric.} = \mu N$ (why?!)

x - along incline

Block A.

$\overset{+}{\nearrow} \sum F_x = 0 \Rightarrow F - 400\sin 45^\circ + F_A - P\sin 45^\circ = 0$

$\Rightarrow F - 400\sin 45^\circ + 0.3 N_A - P\sin 45^\circ = 0$ — ①

$+\uparrow \sum F_y = 0 \Rightarrow N_A - 400\cos 45^\circ + P\cos 45^\circ = 0$

$\Rightarrow N_A = \cos 45^\circ (400 - P) = 0$ — ②

Block B.

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow P\sin 45^\circ + F_B - 300\sin 45^\circ = 0$

$\Rightarrow P\sin 45^\circ + 0.5 N_B - 300\sin 45^\circ = 0$ — ③

$+\uparrow \sum F_y = 0 \Rightarrow N_B - P\cos 45^\circ - 300\cos 45^\circ = 0$

$\Rightarrow N_B = \cos 45^\circ (P + 300)$ — ④

Solving the above equations;

④ into ③ $\Rightarrow P\sin 45^\circ + 0.5\cos 45^\circ (P + 300) - 300\sin 45^\circ = 0$

$\Rightarrow P(\sin 45^\circ + 0.5\cos 45^\circ) + 150(\cos 45^\circ - 2\sin 45^\circ) = 0$

$\Rightarrow P = 100\,kN$

T into ② $\Rightarrow N_A = \cos 45 (400 - 100)$

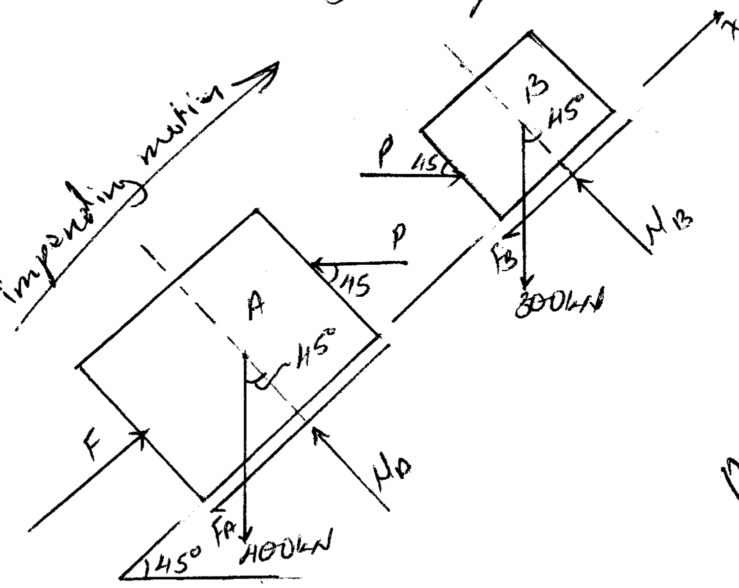
$\Rightarrow N_A = 212.132\,kN$

N_A and P into ① $\Rightarrow F - 400 \sin 45 + 0.3(212.132) - 100 \cos 45 = 0$

$\Rightarrow F = 289.9\,kN$

$\therefore F_{min} = 289.9\,kN$

Then Calculate Fmax, just required to move the blocks up.



$\mu_A = 0.3$

$\mu_B = 0.5$

Note that

$F_{fric.} = \mu N$ (Why?!)

x - along incline.

Block A.

$\xrightarrow{+} \sum F_x = 0 \Rightarrow F - 400 \sin 45° - 0.3 N_A - P \sin 45° = 0$ — ⑤

$+\uparrow \sum F_y = 0 \Rightarrow$ Similar to equation ② $\Rightarrow N_A = \cos 45° (400 - P)$ - ②

Block B

$\xrightarrow{+} \sum F_x = 0 \Rightarrow P \sin 45° - 0.5 N_B - 300 \sin 45° = 0$ — ⑥

$+\uparrow \sum F_y = 0 \Rightarrow$ Similar to equation ④ $\Rightarrow N_B = \cos 45° (P + 300)$ - ④

Solving the above equation.

⑪ into ⑥ $\Rightarrow P \sin 45° - 0.5 \cos 45 (T + 300) - 300 \sin 45° = 0$

$\Rightarrow P(\sin 45° - 0.5 \cos 45°) - 150(\cos 45° + 2 \sin 45° = 0 \Rightarrow P = 900\,kN$

T into ② $\Rightarrow N_A = -353.553\,kN$

N_A and P into ⑤ $\Rightarrow F = 813.2\,kN$. $\Rightarrow F_{max} = 813.2\,kN$

Thus the range of values of force, $F \Rightarrow \boxed{289.9\,kN \leq F \leq 813.2\,kN}$

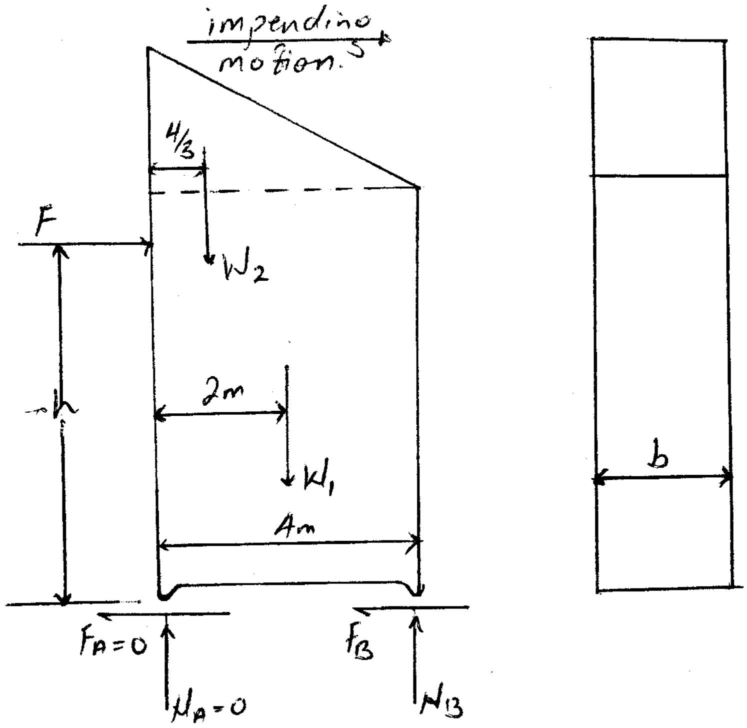
Problem 4

Given: A 20 kN object as shown in figure P4 of question paper.

Required: The largest value of h for which the object will slip before it tips over.

Solution.

The FBD is drawn first



$$W_1 = \frac{V_1}{V_{total}} W_{total}$$

$$= \frac{4(6)b}{4(6)b + \frac{4(2)b}{2}}(20)$$

$$= 17.143 \text{ kN}$$

$$W_2 = 20 - 17.143$$
$$= 2.857 \text{ kN}$$

Since we are looking for h_{max} such that slipping occurs before tipping, we conclude the following:

(A) $N_A = 0 \Rightarrow F_A = 0$ (as we need h_{max}, and thus tipping is "imminent")

(B) $F_B = F_{max} = \mu_B N_B$ (as we want slipping to occur before tipping).

Thus: $+\uparrow \Sigma F_y = 0 \Rightarrow N_B = W_{total} = 20 \text{ kN}$

$\xrightarrow{+} F_x = 0 \Rightarrow F = F_B.$

$F_B = F_{max} = 0.36(20) = 7.2 \text{ kN} \quad \Rightarrow F = 7.2 \text{ kN}$

$\circlearrowleft \Sigma M_B = 0 \Rightarrow 17.143(2) + 2.857[\tfrac{2}{3}(4)] - 7.2(h) = 0$

$\Rightarrow \boxed{h = 5.82 \text{ m}}$

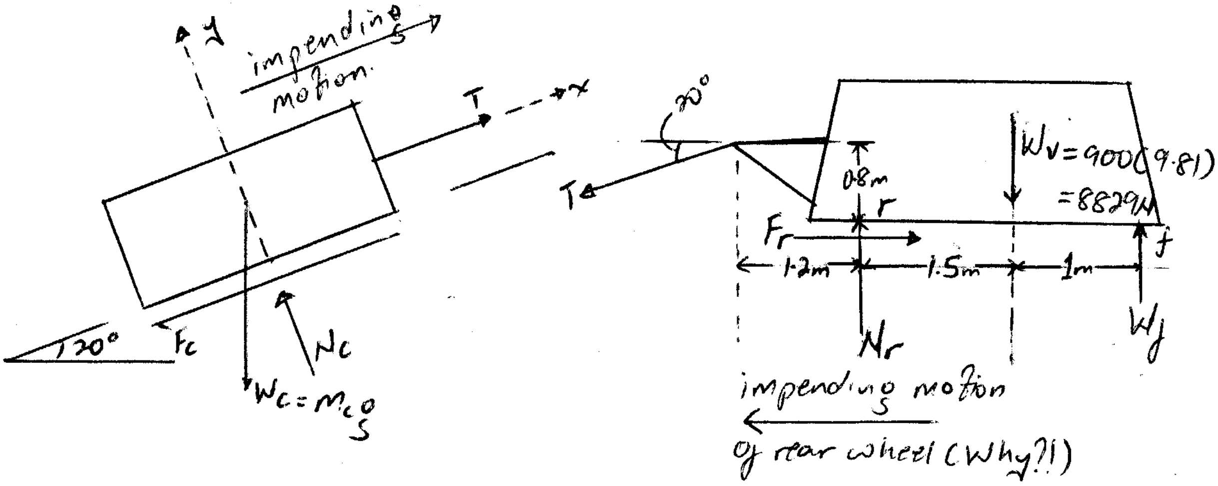
Problem 5

Given: Figures P5 as shown in the question sheet.

Required: The largest value of the mass of crate for which it will slip up the incline before the vehicle's tires slip.

Solution:

The FBDs of the crate and the vehicle are drawn separately (Why?!)



* Note that we assumed impending motion for both the crate and the vehicle)

* Note that no friction at the front wheel.

Why?! Read the problem statement carefully.

For the crate:

$+\uparrow \sum F_y = 0 \Rightarrow N_c = W_c \cos 20°$

$+\rightarrow \sum F_x = 0 \Rightarrow T - F_c - W_c \sin 20° = 0$

$F_c = \mu_c N_c$ (Why?!)

$\Rightarrow T = \mu_c N_c + W_c \sin 20°$

$= \mu_c W_c \cos 20° + W_c \sin 20°$

$= W_c (0.4 \cos 20° + \sin 20°)$; Let $(0.4 \cos 20° + \sin 20°) = A$,

$\Rightarrow T = W_c A$

For the vehicle:

$\circlearrowleft \sum M_f = 0 \Rightarrow$

$\Rightarrow 8829(1) + 0.8(W_c A)\cos 20^\circ + (1.2+2.5)(W_c A)\sin 20^\circ - 2.5 N_r = 0$

$$\Rightarrow N_r = \frac{8829 + (0.8\cos 20^\circ + 3.7\sin 20^\circ) A W_c}{2.5} \quad — ①$$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -W_c A \cos 20^\circ + F_r = 0$

$F_r = \mu_r N_r$ (Why?!)

$\Rightarrow W_c A \cos 20^\circ + 0.65 N_r = 0$

$$\Rightarrow N_r = \frac{\cos 20}{0.65} W_c A \quad — ②$$

From equations ① and ②

$$\frac{8829 + (0.8\cos 20^\circ + 3.7\sin 20) A W_c}{2.5} = \frac{\cos 20}{0.65} A W_c$$

$$\Rightarrow \frac{8829}{2.5} = \left[\frac{\cos 20}{0.65} - \frac{0.8\cos 20 + 3.7\sin 20}{2.5}\right] A W_c$$

$\Rightarrow 3531.6 = (1.4457 - 0.80689)(0.71790) W_c$

$\Rightarrow W_c = 7700.1\ N$

$$\Rightarrow m_c = \frac{W_c}{g} = \frac{7700.1}{9.81}$$

$$\Rightarrow \boxed{m_c = 785\ kg}$$