

CE 201 Sections (485) [0717]
H.W # 7

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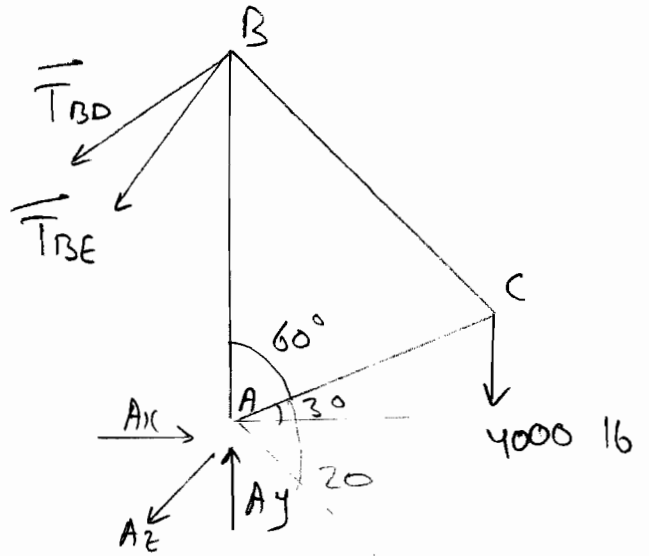
Problem 1

Given:

Fig P₁

load = 4000 lb

$\phi = 20^\circ$



FBD

Required:

Tension in each cable & Reactions at A

Solution

$$A(0, 0, 0)$$

$$B(0, 12, 0)$$

$$C(\cos 30^\circ \cos 20^\circ, \sin 30^\circ, \cos 30^\circ \sin 20^\circ)$$

$$D(-8, 0, 6)$$

$$E(-8, 0, -6)$$

$$\vec{BD} = -8\vec{i} - 12\vec{j} + 6\vec{k} \implies BD = \sqrt{244} \text{ ft}$$

$$\vec{BE} = -8\vec{i} - 12\vec{j} - 6\vec{k} \implies BE = \sqrt{244} \text{ ft}$$

$$\vec{AC} = 12(\cos 30^\circ \cos 20^\circ \vec{i} + \sin 30^\circ \vec{j} + \cos 30^\circ \sin 20^\circ \vec{k})$$

$$\vec{AC} = 9.7656\vec{i} + 6\vec{j} + 3.5544\vec{k}$$

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$$\therefore \vec{T}_{BD} = \frac{T_{BD}}{BD} (-8\vec{i} + 12\vec{j} + 6\vec{k})$$

$$\vec{T}_{BD} = \frac{T_{BD}}{\sqrt{244}} (-8\vec{i} + 12\vec{j} + 6\vec{k})$$

$$\vec{T}_{BE} = \frac{T_{BE}}{BE} (-8\vec{i} - 12\vec{j} - 6\vec{k})$$

$$\vec{T}_{BE} = \frac{T_{BE}}{\sqrt{244}} (-8\vec{i} - 12\vec{j} - 6\vec{k})$$

$$\text{Load}(\vec{F}) = 4000\vec{j}$$

$$\sum \vec{M}_A = 0$$

You can combine

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ \frac{-8}{\sqrt{244}} T_{BD} & \frac{-12}{\sqrt{244}} T_{BD} & \frac{6}{\sqrt{244}} T_{BD} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ \frac{-8}{\sqrt{244}} T_{BE} & \frac{-12}{\sqrt{244}} T_{BE} & \frac{-6}{\sqrt{244}} T_{BE} \end{vmatrix}$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9.7656 & 6 & 3.5544 \\ 0 & -4000 & 0 \end{vmatrix}$$

$$= 4.6093 T_{BD} \vec{i} + 6.1458 T_{BD} \vec{k} - 4.6093 T_{BE} \vec{i} + 6.1458 T_{BE} \vec{k} + 14217.6 \vec{i} - 39062.4 \vec{k}$$

$$= (4.6093 T_{BD} - 4.6093 T_{BE} + 14217.6) \vec{i} +$$

$$(6.1458 T_{BD} + 6.1458 T_{BE} - 39062.4) \vec{k}$$

Solving them together

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$$28.3278 T_{BD} - 28.3278 T_{BE} + 87378.5 = 0$$

$$28.3278 T_{BD} + 28.3278 T_{BE} - 180050.3 = 0$$

$$56.6556 T_{BD} = 92671.8$$

$$\boxed{T_{BD} = 1635.7 \text{ lb}}$$

$$\boxed{T_{BE} = 4720.2 \text{ lb}}$$

$$\Sigma F_x = 0 \rightarrow +$$

$$A_x - \frac{8}{\sqrt{244}} T_{BD} - \frac{8}{\sqrt{244}} T_{BE} = 0$$

$$A_x = \boxed{3255.2 \text{ lb}}$$

$$\Sigma F_y = 0 \uparrow$$

$$A_y - \frac{12}{\sqrt{244}} T_{BD} - \frac{12}{\sqrt{244}} T_{BE} - 4000 = 0$$

$$\boxed{A_y = 8882.7 \text{ lb}}$$

$$\Sigma F_z = 0 \downarrow$$

$$A_z + \frac{6}{\sqrt{244}} T_{BD} - \frac{6}{\sqrt{244}} T_{BE} = 0, \quad \boxed{A_z = 1184.8 \text{ lb}}$$

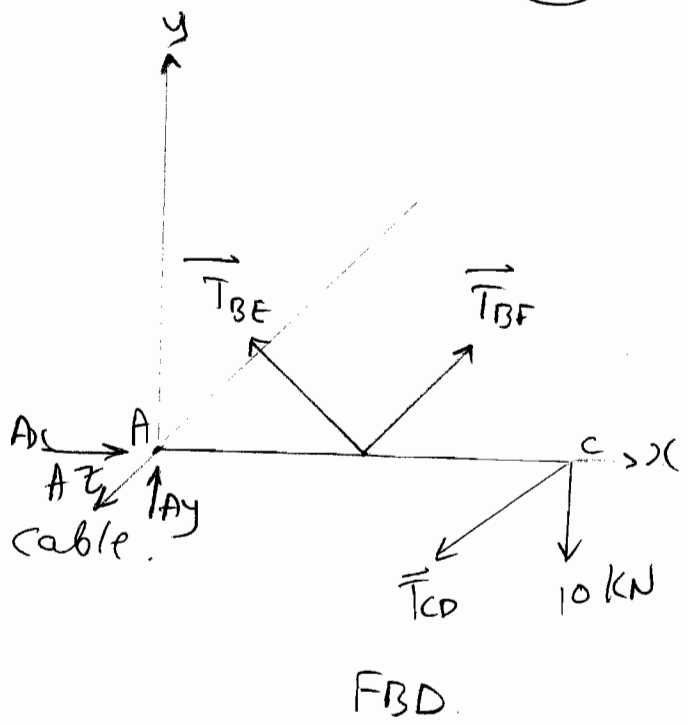
Problem 2

Given:

Fig P2

Required:

Determine tension in each cable.



Solution:

$A(0,0,0)$

$B(1,0,0)$

$C(2.5,0,0)$

$D(0,0,1.5)$

$E(0,1,0)$

$F(0,0,-2)$

$\vec{AC} = 2.5\vec{i}$

$\vec{BE} = -\vec{i} + \vec{j} \implies BE = 2$

$\vec{BF} = -\vec{i} - 2\vec{k} \implies BF = \sqrt{5}$

$\vec{CD} = -2.5\vec{i} + 1.5\vec{k} \implies CD = \sqrt{8.5}$

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$$\vec{F} = -10\vec{j}$$

$$\vec{T}_{BE} = \frac{T_{BE}}{\sqrt{2}} (-\vec{i} + \vec{j})$$

$$\vec{T}_{BF} = \frac{T_{BF}}{\sqrt{5}} (-\vec{i} - 2\vec{k})$$

$$\vec{T}_{CD} = \frac{T_{CD}}{\sqrt{8.5}} (-2.5\vec{i} + 1.5\vec{k})$$

1 By Taking Moment around A:

$$\sum \vec{M}_A = 0$$

can combine

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -\frac{T_{BE}}{\sqrt{2}} & \frac{T_{BE}}{\sqrt{2}} & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -\frac{T_{BF}}{\sqrt{5}} & 0 & -\frac{2T_{BF}}{\sqrt{5}} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2.5 & 0 & 0 \\ 0 & -10 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2.5 & 0 & 0 \\ -\frac{2.5T_{CD}}{\sqrt{8.5}} & 0 & \frac{1.5T_{CD}}{\sqrt{8.5}} \end{vmatrix}$$

can combine

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$$\left(\frac{T_{BE}}{\sqrt{2}} - 25 \right) \vec{i} + \left(\frac{2T_{BF}}{\sqrt{5}} - \frac{2.5 \times 1.5}{\sqrt{8.5}} T_{CD} \right) \vec{j} = 0$$

$$\therefore T_{BE} = 25 \times \sqrt{2} = 35.355$$

$$T_{BE} = 35.355 \text{ KN}$$

$$T_{BF} = T_{BE}$$

$$T_{BF} = 35.355 \text{ KN}$$

$$T_{CD} = 24.585 \text{ KN}$$

$$\therefore T_{BE} = T_{BF} = 35.4 \text{ KN}$$

$$T_{CD} = 24.6 \text{ KN}$$

Problem 3

Given:

$\omega = 20 \text{ kg}$, No axial thrust at A

Fig P3, Assume hinges at A & B are properly aligned.

Required:

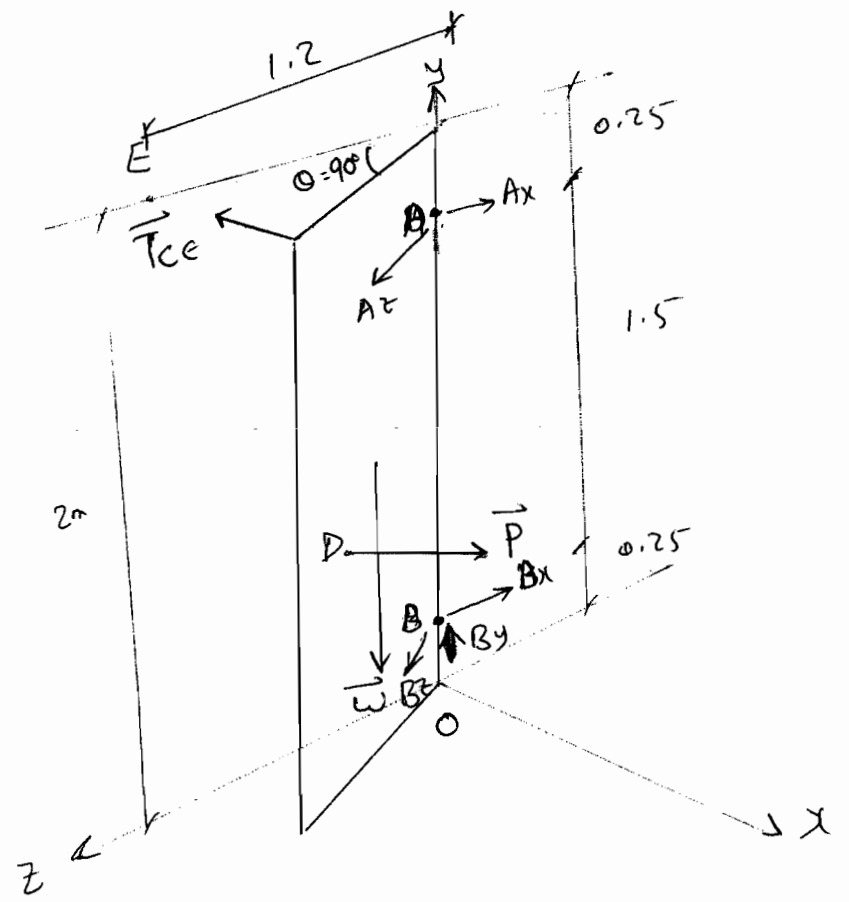
$\therefore M_A = M_B = 0$

Magnitude of P &

reactions A & B when $\theta = 90^\circ$

Solution:

- A (0, 1.75, 0)
- B (0, 0.25, 0)
- C (1.2, 2, 0)
- D (1.05, 0.9, 0)
- E (0, 2, 1.2)
- G (0.6, 1, 0)
- O (0, 0, 0)



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By taking Moment about OA

$$\sum M_{OA} = 0$$

$$\vec{U}_{OA} \cdot (\vec{OD} \times \vec{P}) + \vec{U}_{OA} \cdot (\vec{OG} \times \vec{W}) + \vec{U}_{OA} \cdot (\vec{OC} \times \vec{T}_{CE}) = 0$$

$$\vec{U}_{OA} = 1.75 \vec{j}$$

$$\vec{OD} = 1.05 \vec{i} + 0.9 \vec{j}$$

$$\vec{P} = -P \vec{k}$$

$$\vec{OG} = 0.6 \vec{i} + \vec{j}$$

$$\vec{W} = -20 \times 9.81 \vec{j} = -196.2 \vec{j}$$

$$\vec{OC} = 1.2 \vec{i} + 2 \vec{j}$$

$$\vec{T}_{CE} = \frac{T_{CE} (-1.2 \vec{i} + 1.2 \vec{k})}{\sqrt{2.88}}$$

$$M_{OA} = \begin{vmatrix} 0 & 1.75 & 0 \\ 1.05 & 0.9 & 0 \\ 0 & 0 & -P \end{vmatrix} + \begin{vmatrix} 0 & 1.75 & 0 \\ 0.6 & 1 & 0 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1.75 & 0 \\ 1.2 & 2 & 0 \\ \frac{-T_{CE}(1.2)}{\sqrt{2.88}} & 0 & \frac{T_{CE}(1.2)}{\sqrt{2.88}} \end{vmatrix}$$

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$$\text{Since } T_{ce} = 15 \times 9.81 = 147.15 \text{ N}$$

$$\therefore \sum M_{OA} = 0$$

$$= -1.75(-P \times 1.05) - 1.75 \frac{(1.2 \times 1.2 \times 147.15)}{\sqrt{2.88}} = 0$$

$$\boxed{P = 118.915 \text{ N}}$$

$$\sum M_B = 0$$

can combine

$$= (\vec{BA} \times \vec{Ax}) + (\vec{BA} \times \vec{Az}) + (\vec{BC} \times \vec{T}_{ce}) + (\vec{BG} \times 196.2) + (\vec{BD} \times 118.915) = 0$$

$$\vec{BA} = 1.5\vec{j}$$

$$\vec{BC} = 1.2\vec{i} + 1.75\vec{j}$$

$$\vec{BG} = 0.6\vec{i} + 0.75\vec{j}$$

$$\vec{BD} = 1.05\vec{i} + 0.65\vec{j}$$

PTO \rightarrow

can combine

$$= \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \left| \begin{array}{ccc} 1 & j & k \\ 0 & 1.5 & 0 \\ A_x & 0 & 0 \end{array} \right| & + & \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \left| \begin{array}{ccc} 1 & j & k \\ 0 & 1.5 & 0 \\ 0 & 0 & A_z \end{array} \right| & + & \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \left| \begin{array}{ccc} 1 & j & k \\ 1.2 & 1.75 & 0 \\ -104.051 & 0 & 104.051 \end{array} \right| \\ \\ \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \left| \begin{array}{ccc} 1 & j & k \\ 0.6 & 0.75 & 0 \\ 0 & -196.2 & 0 \end{array} \right| & + & \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \left| \begin{array}{ccc} 1 & j & k \\ 1.05 & 10.65 & 0 \\ 0 & 0 & -118.913 \end{array} \right| \end{matrix} \end{matrix}$$

$$= - (1.5 A_x) \vec{k} + (1.5 A_z) \vec{i} + (1.75 \times 104.051) \vec{i} - (1.2 \times 104.051) \vec{j} + (1.75 \times 104.051) \vec{k} + (0.6 \times 196.2) \vec{k} + (0.65 \times 118.913) \vec{i} + (1.05 \times 118.913) \vec{j} = 0$$

$$\therefore 1.5 A_z + 182.089 - 77.295 = 0$$

$$\boxed{A_z = -69.863 \text{ N}}$$

$$-1.5 A_x + 182.089 - 117.720 = 0$$

$$\boxed{A_x = 42.913 \text{ N}}$$

$$\sum F_x = 0 \rightarrow -104.051 + A_x + B_x = 0$$

$$B_x = 104.051 - 42.913$$

$$\boxed{B_x = 61.138 \text{ N}}$$

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$$\sum F_z = 0 \swarrow + \quad -P + 104.051 + A_z + B_z = 0$$

$$B_z = 118.915 - 104.051 + 69.863$$

$$B_z = 84.727 \text{ N}$$

∴ Since No axial thrust at A

$$\therefore A_y = 0 \quad (\text{no } A_x)$$

$$\therefore \cancel{A_y} + B_y = 196.2$$

$$B_y = 196.2 \text{ N}$$

Problem 4

Given:

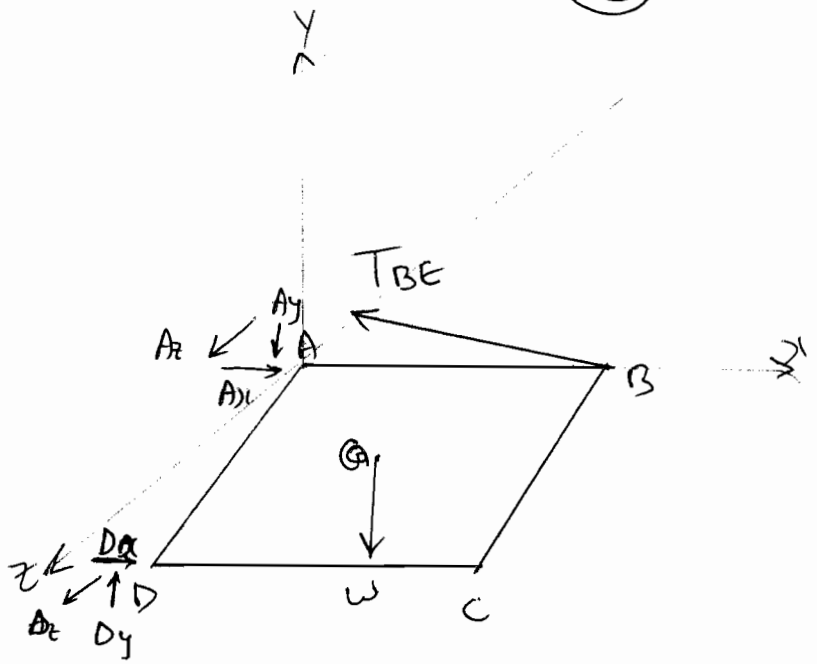
$w = 23 \text{ kg}$

Plate = $(325 \times 450) \text{ mm}$

Fig P4

Required:

Tension in the wire.



Solution:

The hinges along AD are assumed properly aligned. No moment about the axis

A (0, 0, 0)

B (450, 0, 0)

C (450, -125, 300)

D (0, -125, 300)

E (0, 225, 150)

G (225, -62.5, 150)

By taking moment around axis AD

$$\sum M_{AD} = 0$$

$$= \vec{U}_{AD} \cdot (\vec{AB} \times \vec{T}_{BE}) + \vec{U}_{AD} \cdot (\vec{AG} \times \vec{W})$$

$$\vec{U}_{AD} = \frac{-125\vec{j} + 300\vec{k}}{325}$$

$$\vec{AB} = 450\vec{i}$$

$$\vec{AG} = 225\vec{i} - 62.5\vec{j} + 150\vec{k}$$

$$\vec{T}_{BE} = T_{BE} \frac{-450\vec{i} + 225\vec{j} + 150\vec{k}}{525}$$

$$\sum M_{AD} = 0$$

$$= \begin{vmatrix} 0 & -125/325 & 300/325 \\ 450 & 0 & 0 \\ -450T_{BE}/525 & 225T_{BE}/525 & 150T_{BE}/525 \end{vmatrix} + \begin{vmatrix} 0 & -125/325 & 300/325 \\ 225 & -62.5 & 150 \\ 0 & -225.63 & 0 \end{vmatrix}$$

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$$= \frac{175}{325} \left(\frac{67500}{525} T_{BE} + \frac{135000}{525} T_{BE} \right) +$$

$$\frac{300}{325} \left(\frac{450 \times 225}{525} T_{BE} \right) + \frac{300}{325} (-50766.75) = 0$$

$$\Rightarrow 49.451 T_{BE} + 178.022 T_{BE} = 46861.615$$

$$T_{BE} = 206.009$$

$$\therefore \boxed{T_{BE} = 206.01 \text{ N.}}$$

Problem 5:

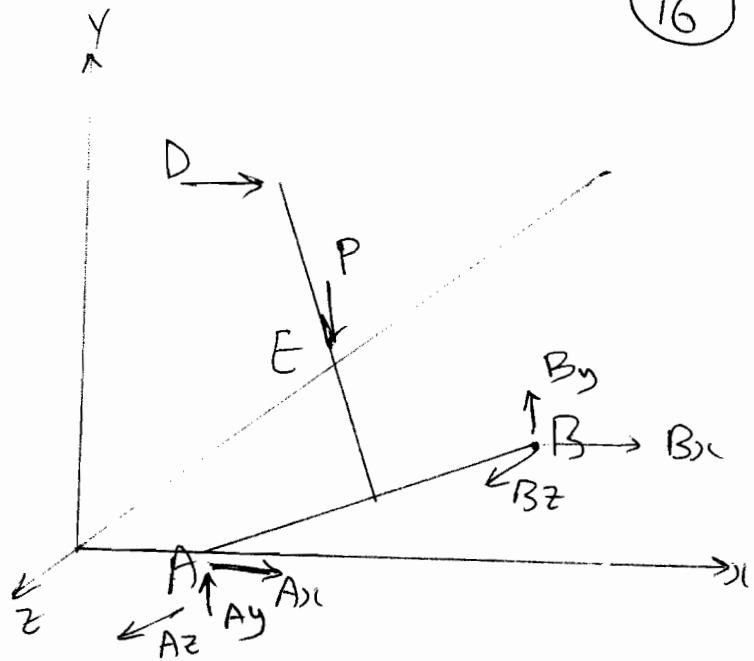
Given:

$$P = 400\text{N}$$

Fig P5

Required:

reaction at D



Solution: (The bearings at A and B are assumed properly aligned)

By taking moment around Axis AB

$$\sum M_{AB} = 0$$

$$= \vec{U}_{AB} \cdot (\vec{AE} \times \vec{P}) + \vec{U}_{AB} \cdot (\vec{AD} \times \vec{D}) = 0$$

$$A (125, 0, 0)$$

$$B (275, 0, -200)$$

$$C (200, 0, -100)$$

$$D (0, 250, -250)$$

$$E (100, 125, -175)$$

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$$\vec{U}_{AB} = \frac{150\vec{i} - 200\vec{k}}{250}$$

$$\vec{AE} = -25\vec{i} + 125\vec{j} - 175\vec{k}$$

$$P = -400\vec{j}$$

$$D = D\vec{i} \Rightarrow \text{Reaction at D.}$$

$$\vec{AD} = -125\vec{i} + 250\vec{j} - 250\vec{k}$$

$$\sum \vec{M}_{AB} = 0$$

$$= \begin{vmatrix} \frac{150}{250} & 0 & -\frac{200}{250} \\ -25 & 125 & -175 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \frac{150}{250} & 0 & -\frac{200}{250} \\ -125 & 250 & -250 \\ 0 & 0 & 0 \end{vmatrix}$$

$$= -\frac{150}{250} (70000) - \frac{200}{250} (10000) + (-200D) = 0$$

$$\boxed{D = 250 \text{ N.}}$$