

Problem 1:

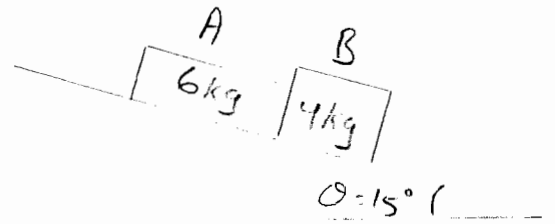
Given:- $\left. \begin{array}{l} \mu_s = 0.2 \\ \mu_k = 0.15 \end{array} \right\} \text{Package A}$ $\left. \begin{array}{l} \mu_s = 0.3 \\ \mu_k = 0.25 \end{array} \right\} \text{Package B}$ 

Fig P.

Required:-

- check any motion
- the friction on each package

Solution:-

a) First, check the sliding of package B only

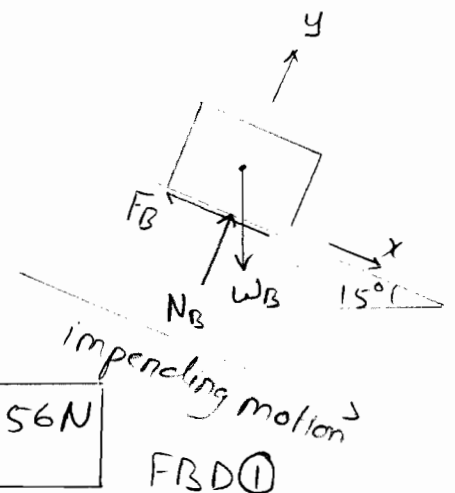
(Why?!))

In FBD ①

 $\sum F_x = 0 \Rightarrow$ (Why x in this direction?!))

$$W_B \sin 15^\circ - F_B = 0$$

$$4(9.81) \sin 15^\circ = F_B \Rightarrow \boxed{F_B = 10.156 \text{ N}}$$



$$\uparrow \Sigma F_y = 0$$

$$N_B - W_B \cos 15^\circ \Rightarrow N_B = 37.903 \text{ N}$$

check $F_B \leq \mu_B N_B$

$$\mu_B N_B = 0.3 (37.903) = 11.371 \text{ N}$$

$$\Rightarrow F_B = 11.371 \text{ N} < 10.540 \text{ N}$$

\Rightarrow B will not slide alone.

Important Note:

$$F_B \neq \mu_B N_B \quad (\text{Why?!!!})$$

Second, Check the sliding of package A and B together.

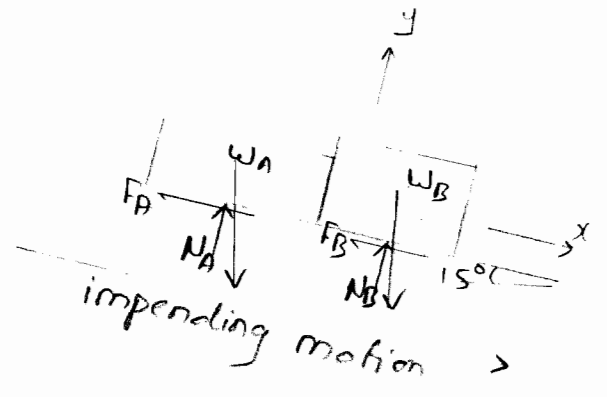
(Why did not we check the sliding of package A only ?!!!)

In FBD ②

As before $N_B = 37.903 \text{ N}$
For N_A ,

$$\uparrow \Sigma F_y = 0$$

$$N_A = W_A \cos 15^\circ \Rightarrow N_A = 56.854 \text{ N}$$



FBD ②

For F_A and F_B ,

$$\downarrow \sum F_x = 0$$

$$\Rightarrow W_A \sin 15 + W_B \sin 15 - F_A - F_B = 0$$

$$\Rightarrow (F_A + F_B) = 25.390 \text{ N}$$

$$\begin{aligned} (F_A + F_B)_{\max} &= \mu_{sA} N_A + \mu_{sB} N_B \\ &= 0.2(56.854) + 0.3(37.903) \\ &= 22.742 \text{ N} \end{aligned}$$

$$\Rightarrow 25.390 > (F_A + F_B)_{\max} = 22.742 \text{ N}$$

\Rightarrow Packages A and B will both move

Note: that there is at least one other method for the solution. (What is it?!!)

b) Since both packages move, then

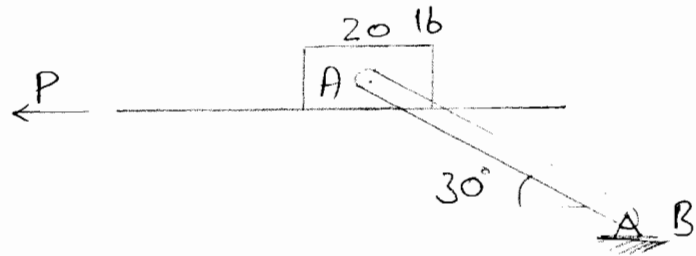
$$\begin{aligned} F_A &= F_{A_k \max} = (\mu_k N)_A \\ &= (0.15)(56.854) \end{aligned}$$

$$\Rightarrow \boxed{F_A = 8.528 \text{ N}}$$

$$\begin{aligned} F_B &= F_{B_k} = (\mu_k N)_B \\ &= (0.25)(37.903) \Rightarrow \boxed{F_B = 9.476 \text{ N}} \end{aligned}$$

Problem 2:

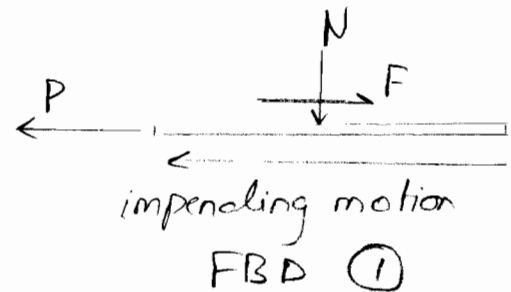
Given: 20 lb block
 $\mu_s = 0.25$
 Fig P2



Required:- magnitude of the force P required to move the belt to the left.

Solution:-

From FBD ① [conveyor belt],
 we draw FBD ② [block]



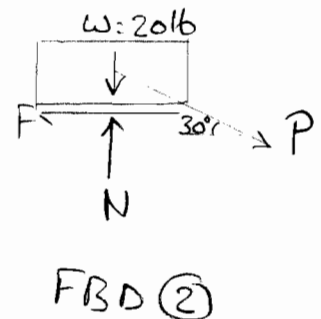
In FBD ②

$$\begin{aligned} \rightarrow \Sigma F_x = 0 \\ P \cos 30^\circ - F = 0 \Rightarrow P = F / \cos 30^\circ \end{aligned}$$

$$\begin{aligned} \uparrow \Sigma F_y = 0 \\ N - 20 - P \sin 30^\circ = 0 \end{aligned}$$

$$\Rightarrow N - 20 - \frac{F \sin 30^\circ}{\cos 30^\circ} = 0$$

$$\Rightarrow N - 20 - F \tan 30^\circ = 0$$



Since we need to move the belt to the left, it means that we will first have impending motion where

$$F = F_{\max} = \mu_s N$$

Thus,

$$N - 20 - 0.25 N \tan 30 = 0$$

$$\Rightarrow N = 23.374 \text{ lb}$$

$$\Rightarrow F = 0.25(23.374) = 5.8434 \text{ N}$$

In FBD (1),

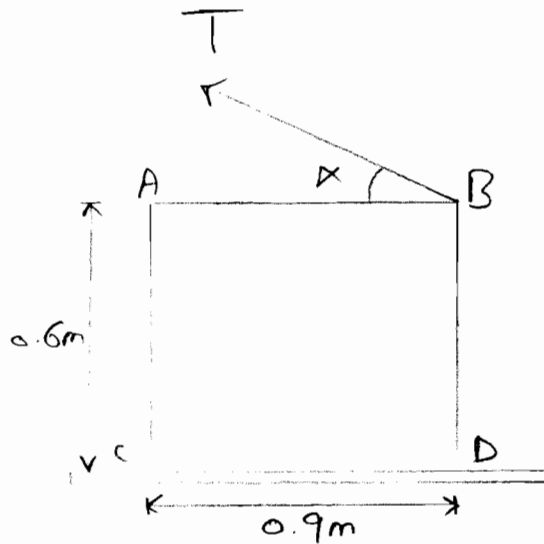
$$\rightarrow \Sigma F_x = 0$$

$$F - P = 0$$

$$\boxed{P = 5.843 \text{ N}}$$

Problem 3:-

Given: mass = 30 kg
 $\mu_s = 0.35$
 $\alpha = 30^\circ$
 Fig P3



Required:-

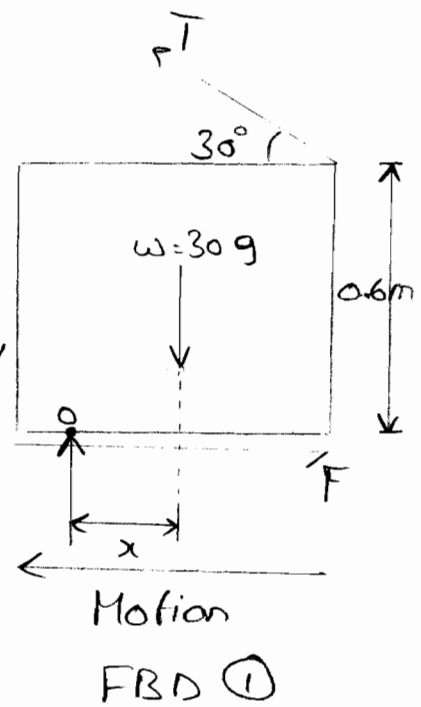
- a) The tension T required to move the crate.
- b) Whether the crate will slide or tip.

Solution:

FBD is drawn as shown.

There are more than one way to solve this problem; one method is below; (try others!)

Since we need to move the crate, we must have impending motion first.



Thus, $F = F_{max} = \mu_s N$ _____ ①

OR $x = x_{max} = 0.45m$ _____ ②

Let's assume ① about controls (i.e., sliding will occur before tipping).

[We have to check our assumption latter on as you will see.]

$$\rightarrow \sum F_x = 0 \quad F - T \cos 30 = 0 \quad \text{--- ③}$$

$$\uparrow \sum F_y = 0 \quad N - 30(9.81) + T \sin 30 = 0 \quad \text{--- ④}$$

From ① into ③

$$0.35 N - T \cos 30 = 0$$

$$\Rightarrow N = T \frac{\cos 30}{0.35} \quad \text{--- ⑤}$$

From ⑤ into ④

$$T \left(\frac{\cos 30}{0.35} \right) - 30(9.81) + T \sin 30 = 0$$

$$\Rightarrow T = 98.95 \text{ N}$$

Now we need to check $x \leq 0.45 \text{ m}$ with this value of T .

$$+\downarrow \Sigma M_o = 0$$

$$98.95 \cos 30^\circ (0.6) + 98.95 \sin 30^\circ (0.45 + x) - 30(9.81)(x) = 0$$

$$\Rightarrow x = 0.3009 \text{ m} < 0.45 \text{ m} \Rightarrow \text{OK.}$$

The crate will slide before it tips

$$\Rightarrow \boxed{T = 98.95 \text{ N}}$$

Note: (If x came out to be $> 0.45 \text{ m}$,
What would you do ??!!)

Problem 4:-

Given:- $W = 3000 \text{ lb}$
 $\mu_s = 0.5$

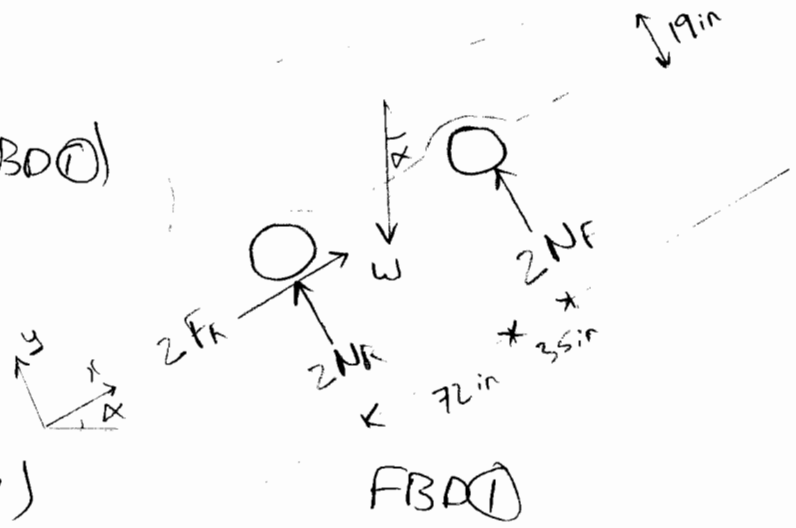
Required:- The largest value of angle α if:-

- a) the car has rear-wheel drive.
- b) the car has front-wheel drive.
- c) The car has four-wheel drive.

Solution:-

a) rear-wheel drive (FBD 1)

In this case there is friction in the rear wheel only. (Why?!!)



Note "2" in N and R. (Why?!!)

$$\sum F_x = 0 \implies 2F_R - W \sin \alpha = 0 \quad \text{--- (1)}$$

Since we need the steepest grade, it means impending motion is assumed

$$\implies F = \mu_s N$$

$$2F_R = \mu_s (2N_R) \implies F_R = \mu_s N_R \quad \text{--- (2)}$$

⇒ From (2) in (1)

$$2(M_s N_R) = W \sin \alpha$$

$$\Rightarrow N_R = (W \sin \alpha) / 2M_s \text{ — (3)}$$

Taking $\downarrow \Sigma M_F = 0$ (Why Front?!!)

$$(W \sin \alpha)(19) + (W \cos \alpha)(35) - 2N_R(72+35) = 0$$

$$\Rightarrow N_R = \frac{W}{2(107)} (19 \sin \alpha + 35 \cos \alpha) \text{ — (4)}$$

From eq (3) into (4)

$$\frac{W}{2M_s} \sin \alpha = \frac{W}{2(107)} (19 \sin \alpha + 35 \cos \alpha) \text{ — (5)}$$

Multiply both sides of (5) by $\frac{2M_s(107)}{W \cos \alpha}$

$$107 \tan \alpha = M_s (19 \tan \alpha + 35)$$

Rearranging:

$$(107 - 19M_s) \tan \alpha = 35M_s$$

$$\Rightarrow \boxed{\tan \alpha = \frac{35M_s}{107 - 19M_s}}$$

By replacing M_s by 0.5

$$\boxed{\alpha = 10.18^\circ}$$

b) Front-wheel drive [FBD ②]

(Similar to FBD (1) except the friction is at the front wheel)

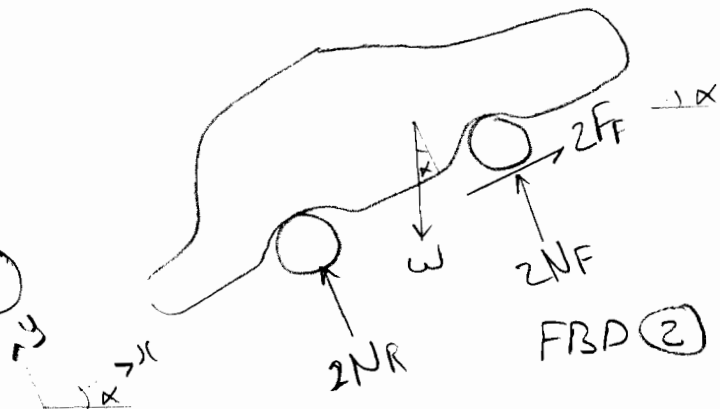
$$\rightarrow \Sigma F_x = 0 \quad 2F_f - W \sin \alpha = 0 \quad \text{--- ①}$$

Since we need the steepest grade, it means impending motion is assumed

$$\Rightarrow F = \mu_s N$$

$$\Rightarrow 2F_f = \mu (2N_f)$$

$$F_f = \mu_s N_f \quad \text{--- ②}$$



From ② into ①

$$2(\mu_s N_f) = W \sin \alpha \quad \text{--- ③}$$

$$N_f = (W \sin \alpha) / 2\mu_s$$

taking $\Sigma M_R = 0$

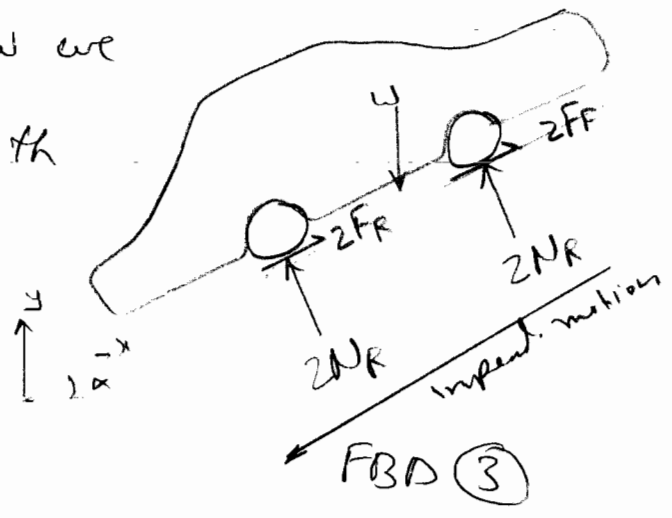
$$N_f = \frac{W}{2(107)} (-19 \sin \alpha + 72 \cos \alpha) \quad \text{--- ④}$$

$$\Rightarrow \tan \alpha = \frac{72}{107/\mu_s + 19}$$

Since $\mu_s = 0.5 \therefore \alpha = 17.17^\circ$

c) Four-wheel drive: [FBD 3]

As before, but now we have friction at both front and rear wheels as shown in FBD 3



$\rightarrow \Sigma F_x = 0$

$2F_F + 2F_R - W \sin \alpha = 0$

$\Rightarrow 2\mu_s N_F + 2\mu_s N_R - W \sin \alpha = 0 \text{ --- (A)}$

$\downarrow \Sigma M_F = 0$ [as in part (a)!]

$\Rightarrow N_R = \frac{W}{2(107)} (19 \sin \alpha + 35 \cos \alpha) \text{ --- (B)}$

$\downarrow \Sigma M_R = 0$ [as in part (b)!]

$N_F = \frac{W}{2(107)} (-19 \sin \alpha + 72 \cos \alpha) \text{ (C)}$

Thus, from eq (B) and (C), into (A)

$$\frac{2M_s W}{2(107)} (-19 \sin \alpha + 72 \cos \alpha) +$$

$$+ \frac{2M_s W}{2(107)} (19 \sin \alpha + 35 \cos \alpha) - W \sin \alpha = 0 \quad \text{--- (D)}$$

Simplifying eq (D)

$$\frac{M_s}{107} (72 + 35) \cos \alpha - \sin \alpha = 0$$

$$\frac{M_s}{107} (107) \cos \alpha - \sin \alpha = 0$$

$$\sin \alpha = M_s \cos \alpha$$

Dividing by $\cos \alpha$, $\tan \alpha = M_s$

Replacing M_s by 0.5,

$$\tan \alpha = 0.5 \implies \text{span style="border: 1px solid black; padding: 2px;"> $\alpha = 26.57^\circ$$$

Conclusions:-

The following conclusions can be drawn:-

- 1) The steepest slope $(\alpha) \neq f(W)$!
- 2) α is proportional to μ ; that is the bigger the μ the bigger the slope α .

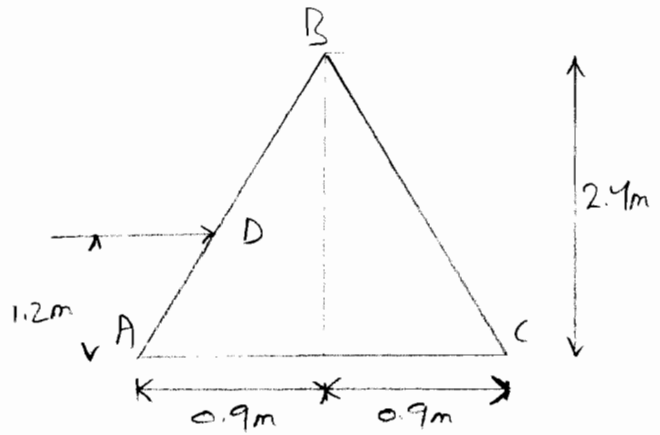
- 3 - In this particular case, the best car is the four wheel drive, followed by the front-wheel drive.
- 4 - Rear-wheel-drive could be better than front-wheel drive if the center of gravity (centroid) is closer to it (72" vs. 35").
- 5 - If you have a rear-wheel drive car, and you have a problem climbing the slope (when the road is slippery), then try to shift the centroid to the rear by adding more loads on the back (e.g trunk back seats).
- 6 - In four-wheel drive, the steepest slope is a function of μ_s only; it is not a function of weight, centroid, or geometry. It becomes like any single object on an incline in which $\tan \alpha = \mu_s$ (Always).

7. To be able to climb steeper slope, try to increase μ by making the tires and/or the road rougher « less smooth » (How?!!)

8. Note that driving on normal highways, you need (in general) to minimize the friction (for smooth driving, saving gas, going faster, --- etc); thus, rear-wheel drive cars are better (if the centroid is closer to the front wheels). But driving on steep slippery slopes, four-wheel & front-wheel drive cars are better.

Problem 5: -

Given: - mass = 20 kg
 $\mu_s = 0.4$
 Fig P5



Required:

- The largest magnitude of the Force P for which equilibrium will be maintained.
- The surface at which motion will impend.

Solution:

There are several possible motions !!

(a) Sliding of A and C, FBD is drawn as shown

Since we are assuming motion of A & C

$F = \mu N$ @ A and C

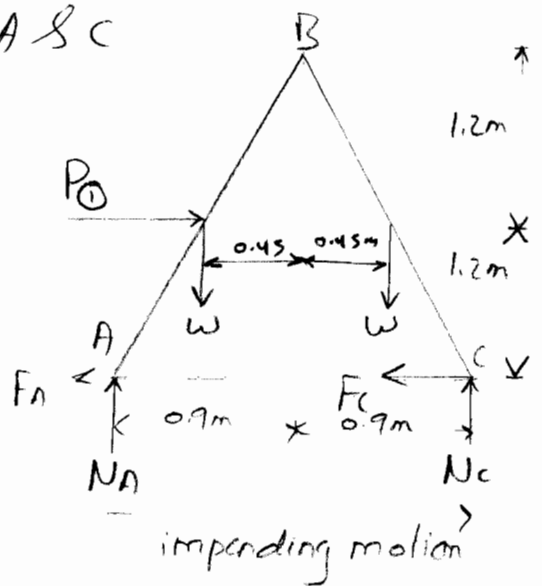
* $\sum F_x = 0$

$P - F_A - F_C = 0$

$P - 0.4N_A - 0.4N_C = 0$ — (1)

$\uparrow \sum F_y = 0$

$N_A + N_C - 2W = 0$ — (2)



impending motion

$$+\left(\sum M_c = 0\right)$$

$$0.45W + 1.35W = 1.8N_A = 1.2P = 0$$

$$1.8W - 1.8N_A - 1.2P = 0 \quad \text{--- (3)}$$

$$\text{From (3), } P = 1.5W - 1.5N_A \quad \text{--- (4)}$$

$$\text{From (4) into (1), } 1.5W - 1.5N_A - 0.4N_A - 0.4N_c = 0$$

$$\Rightarrow 1.5W - 1.9N_A - 0.4N_c = 0 \quad \text{--- (5)}$$

$$\text{From (2), } N_A = 2W - N_c \quad \text{--- (6)}$$

$$\text{From (6) into (5), } 1.5W - 1.9(2)W - 1.9(-N_c) - 0.4N_c = 0$$

$$\Rightarrow -2.3W + 1.5N_c = 0$$

$$\Rightarrow N_c = 1.53333W \quad \text{--- (7)}$$

$$\text{From (7) into (2), } N_A = 2W - 1.53333W$$

$$N_A = 0.466667W \quad \text{--- (8)}$$

$$\begin{aligned} \text{From (8) into (4), } P &= 1.5W - 1.5(0.466667W) \\ &= 0.65W = 0.65(20)(9.81) \end{aligned}$$

$$\boxed{P_{\text{O}} = 127.53 \text{ N}}$$

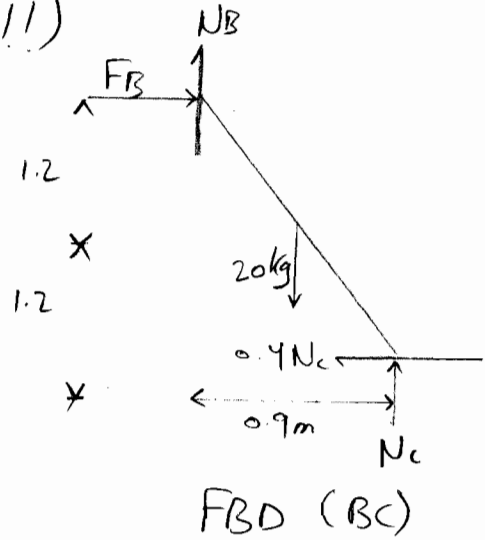
⑥ Sliding of A and C ((Another Method))

FBD of BC is known. (Why 2!!)

Since motion is at C and no motion at B

$$F_C = \mu N_C = 0.4 N_C$$

$$F_B < \mu N_B \\ = ?$$



$$\uparrow \sum M_B = 0$$

$$0.9 N_C - (2.4)(0.4)N_C - 20(0.4) = 0$$

$$\Rightarrow N_C = \frac{9.96}{0.9} = 11.07 \text{ g}$$

check $F_B \stackrel{?}{\leq} \mu N_B$

$$\rightarrow \sum F_x = 0 \quad N_B = 0.4 N_C = 4.427 \text{ g}$$

$$\uparrow \sum F_y = 0 \quad F_B = -N_C + 20 = 8.93 \text{ g}$$

$$F_{B \max} = \mu N_B = 1.77 \text{ g} < 8.93 \text{ g} \Rightarrow \text{Not ok}$$

Which means it is not possible to slide at A before sliding B

Thus, sliding of A and C will not occur first.

2) Tipping of both boards. (Why?!)

Due to the location and direction of P, the tipping will occur to the right at the instant the reactions at A (N and F) are becoming zero. (Why?!).

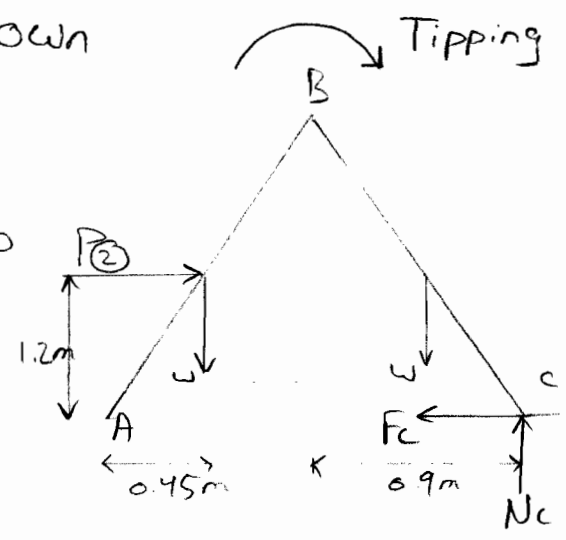
⇒ FBD is drawn as shown

∑Mc = 0 (Why?!)

⇒ -1.2 P₍₂₎ + 1.35 W + 0.45 W = 0

⇒ P₍₂₎ = 1.5 W

⇒ P₍₂₎ = 294.3 N



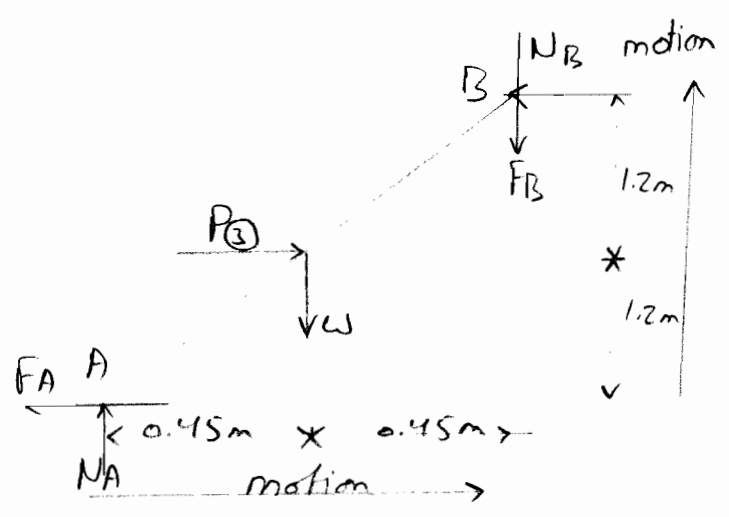
3) Motion at A and B :

FBD is drawn.

F = μN « impending motion is assumed »

F_A = 0.4 N_A

F_B = 0.4 N_B



→ ∑Fx = 0 ⇒ P₍₃₎ = N_B = 0.4 N_A = 0 — (1)

$$\uparrow \Sigma F_y = 0 \implies N_A - 0.4 N_B - W = 0 \quad \text{--- (2)}$$

$$\downarrow \Sigma M_B = 0 \implies 1.2 P_{(3)} + 0.45W - 0.9N_A - 2.4(0.4N_A) = 0$$

$$\implies 1.2 P_{(3)} + 0.45W - 1.86 N_A = 0 \quad \text{--- (3)}$$

From (1), $N_B = P_{(3)} - 0.4 N_A$ --- (4)

From (2), $N_A = W - 0.4 N_B$ --- (5)

From (4) into (5), $N_A = W - 0.4(P_{(3)} - 0.4N_A)$

$$\implies N_A = (W - 0.4 P_{(3)}) / 0.84 \quad \text{--- (6)}$$

From (6) into (3),

$$1.2 P_{(3)} + 0.45W - \frac{1.86}{0.84} (W - 0.4 P_{(3)}) = 0$$

$$\implies P_{(3)} = 0.84589 W$$

$$\boxed{P_{(3)} = 166.0 \text{ N}}$$

4) Motion at Band C:-

(Is it possible?! Is it possible to control???)

5) Motion at A only:-

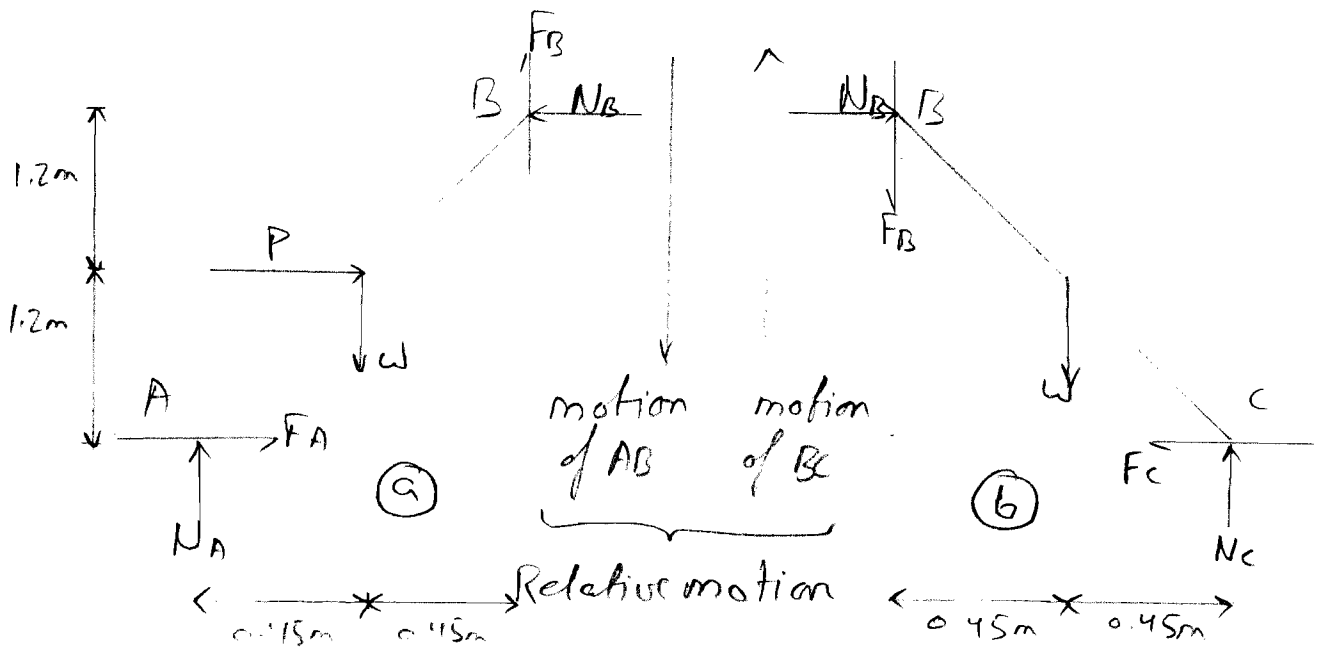
Is it possible? Try it yourself!

6) Motion of C only:-

Is it possible?! Try it yourself!

7) Motion at B only :-

The two FBDs are drawn as shown. Note that we had to separate AB and BC. (Why?!)



In FBD (b),

$$F_B = F_{max} = \mu N_B \quad (\text{Why?!})$$

$$= (0.4)(N_B)$$

$$F_c < F_{max} (= \mu N_c) \quad (\text{Why?!})$$

$$\sum M_c = 0$$

$$0.45W + (0.9)(0.4N_B) - 2.4N_B = 0$$

$$\Rightarrow N_B = \frac{0.45W}{2.09} = 43.279 \text{ N}$$

check $F_c \stackrel{?}{<} \mu N_c$

$$\rightarrow \Sigma F_x = 0$$

$$F_c = N_B = 43.279 \text{ N}$$

$$\uparrow \Sigma F_y = 0$$

$$N_c = W + 0.4(43.279) \\ = 213.5 \text{ N}$$

$$F_{c \text{ max}} = \mu N_c = (0.4)(213.5) \\ = 85.39 \text{ N} < F_c = 43.279 \text{ N}$$

\Rightarrow OK (i.e., no sliding at C)

In FBD @

$$F_B = F_{\text{max}} = \mu N_B$$

$$F_A < F_{\text{max}} = \mu N_A$$

$$\downarrow \Sigma M_A = 0$$

$$-0.45W - 1.2P_B + 2.4(43.279) \\ + 0.9(0.4)(43.279) = 0$$

$$P_B = 25.93 \text{ N}$$

check $F_A \stackrel{?}{<} F_{\text{max}}$

$$\uparrow \Sigma F_y = 0$$

$$N_A = W - F_B = 178.9 \text{ N}$$

$$\rightarrow \Sigma F_x = 0$$

$$F_A = N_B - P = 17.35 \text{ N}$$

$$F_{Amax} = \mu N_B = 0.4 (178.9) = 71.56 \text{ N}$$

$$F_A < F_{Amax} \implies \text{OK}$$

"No motion at A"

$$\underline{P_{max}} = \underline{P_{min}} (P_0, P_1, \dots, P_n) \quad (\text{Why?!})$$

$P_{max} = P_1 = 25.93 \text{ N}$
