Examples

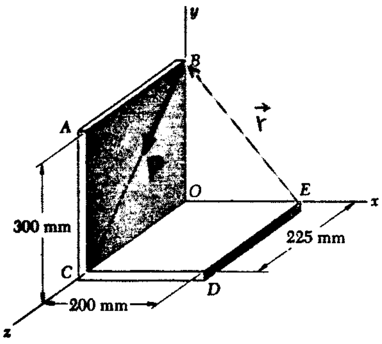
Moment of a Force (Vector Formulation)

#9

Example 1:

Given:

The force $P = 200\,N$ applied to the bent plate shown



Req.d.:

The moment of P about point E

Soln.:

$B(0,300,0)$, $E(200,0,0)$

$\Rightarrow r_x = 0 - 200 = -200$; $r_y = 300$ mm , $r_z = 0$

$P_x = 0$; $P_y = -200\left(300/\sqrt{(300)^2 + (225)^2}\right) = -200(300/375) = -160$ N

$P_z = 200\,(225/375) = 120$ N

$$\vec{M}_E = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -200 & 300 & 0 \\ 0 & -160 & 120 \end{vmatrix} = [300(120) - 0(-160)]\,\vec{i} - [-200(120) - 0(0)]\,\vec{j} + [-200(-160) - 0(300)]\,\vec{k}$$

$$\Rightarrow \boxed{\vec{M}_E = 36\,\vec{i} + 24\,\vec{j} + 32\,\vec{k} \quad (N.m)}$$

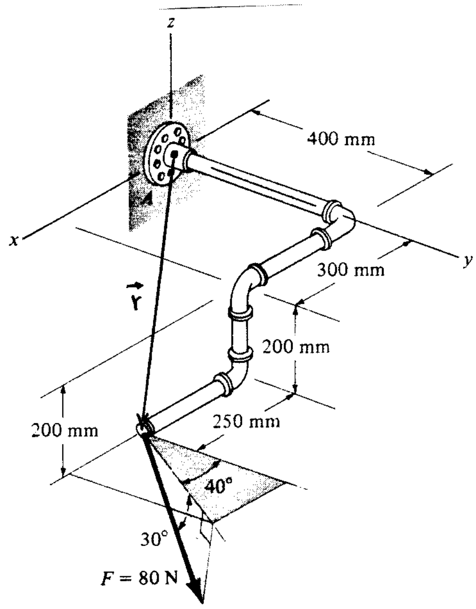
{{Rework the prob. using $\vec{EC}$ instead of $\vec{EB}$}} ⇒?

* Remember that $\vec{r}$ is the vector directed from the "point" to any point on the line of action of $\vec{F}$.

Example 2:

Given:

The figure shown



Req.d.:

$\vec{M}_A$

Soln.:

$$\vec{F} = 80(\cos 30^\circ \sin 40^\circ\,\vec{i} + \cos 30^\circ \cos 40^\circ\,\vec{j} - \sin 30^\circ\,\vec{k})$$

$$= 44.53\,\vec{i} + 53.07\,\vec{j} - 40\,\vec{k}$$

$$\vec{r} = (300+250)\,\vec{i} + 400\,\vec{j} - 200\,\vec{k}$$

$$\vec{M}_A = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 550 & 400 & -200 \\ 44.53 & 53.07 & -40 \end{vmatrix} = [400(-40) - (-200)(53.07)]\,\vec{i} - [550(-40) - (-200)(44.53)]\,\vec{j} + [550(53.07) - 400(44.53)]\,\vec{k}$$

$$\Rightarrow \boxed{\vec{M}_A = 5.39\,\vec{i} + 13.1\,\vec{j} + 11.4\,\vec{k} \quad (N.m)}$$

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Example 3 :

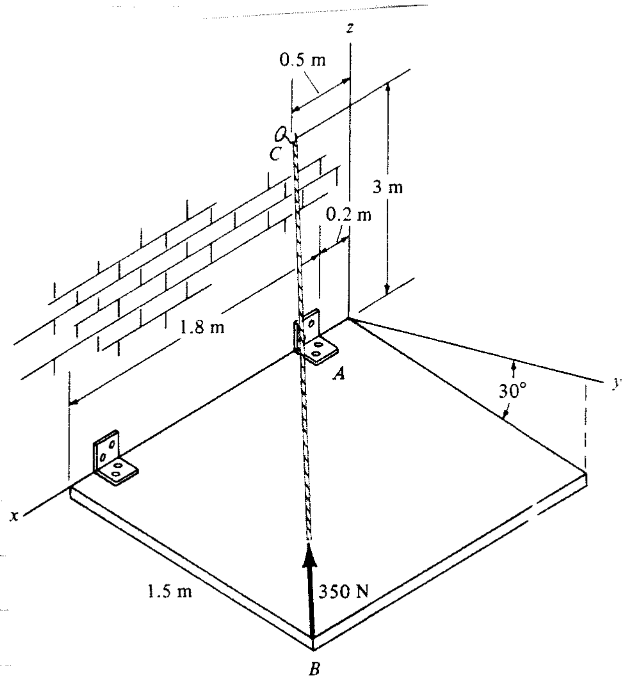
Given :

The figure shown

Req.d. :

The moment about A

Sol.n. :



A (0.2 , 0 , 0)

B (1.8 + 0.2 , 1.5 cos 30°, −1.5 sin 30°)

$\Rightarrow$ B (2 , 1.299 , −0.75)

C (0.5 , 0 , 3)

$\vec{M} = \vec{r} \times \vec{F}$

$\Rightarrow \vec{M}_A = \vec{AB} \times \vec{F}_B$

$\vec{AB} = 1.8\,\vec{i} + 1.299\,\vec{j} - 0.75\,\vec{k}$ (m)

$\vec{BC} = -1.5\,\vec{i} - 1.299\,\vec{j} + 3.75\,\vec{k}$ (m)

$\Rightarrow BC = 4.243$ m

$$\vec{F}_{BC} = \frac{F_{BC}}{BC}\vec{BC} = \frac{350}{4.243}\left(-1.5\vec{i} - 1.299\vec{j} + 3.75\vec{k}\right) \quad (N)$$

$$= -123.7\,\vec{i} - 107.2\,\vec{j} + 309.4\,\vec{k} \quad (N)$$

$$\Rightarrow \vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.8 & 1.299 & -0.75 \\ -123.7 & -107.2 & 309.4 \end{vmatrix}$$

$$\Rightarrow \boxed{\vec{M}_A = 322\,\vec{i} - 464\,\vec{j} - 32.3\,\vec{k} \quad (N.m)}$$

* Rework the problem using $\vec{AC}$ instead of $\vec{AB}$. Compare the answers and comment. Which is easier ?!
* Remember what is $\vec{r}$?!!

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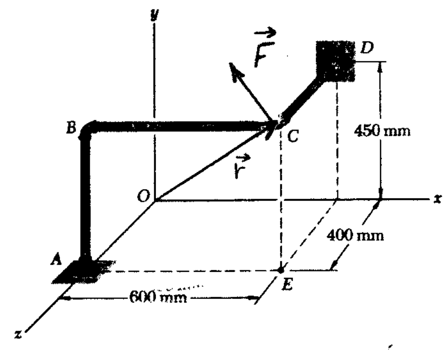
Example 4:



Given:

The pipe shown

A force $\vec{F}$ with unknown magnitude and direction is applied at C

$\vec{F}$ produces $M_x = +150$ N.m, $M_z = +90$ N.m } about O

Reqd.:

M_y

Soln.:

$\vec{M}_o = \vec{r}_{oc} \times \vec{F}_c = M_x \vec{i} + M_y \vec{j} + M_z \vec{k}$

C (0.6 , 0.45 , 0.4) m

$$\vec{M}_o = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.6 & 0.45 & 0.4 \\ F_x & F_y & F_z \end{vmatrix} = (0.45 F_z - 0.4 F_y)\vec{i} - (0.6 F_z - 0.4 F_x)\vec{j} + (0.6 F_y - 0.45 F_x)\vec{k}$$

$$M_x = -0.4 F_y + 0.45 F_z \Rightarrow -0.4 F_y + 0.45 F_z = 150 \quad (1)$$

$$M_y = 0.4 F_x - 0.6 F_z \Rightarrow 0.4 F_x - 0.6 F_z = M_y \quad (2)$$

$$M_z = -0.45 F_x + 0.6 F_y \Rightarrow -0.45 F_x + 0.6 F_y = 90 \quad (3)$$

Note that there are 4 unknowns and only 3 equations; however, we can solve for M_y without the need to solve for the other unknowns as follows:

6 x eq.(1) + 4 x eq.(3) gives

$$-1.8 F_x + 0 F_y + 2.7 F_z = 1260 \quad (4)$$

4.5 x eq.(2) gives

$$1.8 F_x + 0 F_y - 2.7 F_z = 4.5 M_y \quad (5)$$

Now adding (4) and (5) yields

$$0 = 1260 + 4.5 M_y \Rightarrow \boxed{M_y = -280 \text{ N.m}}$$

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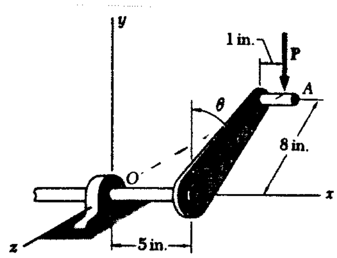
Example 5 :



Given :

The figure shown

P produces $M_x = -160$ in-lb, $M_z = -240$ in-lb about O

Req'd. :

The force P and the angle θ

Sol'n. :

$\vec{P} = -P\vec{j}$

$\Rightarrow P_x = 0 \quad , \quad P_y = -P \quad , \quad P_z = 0$

$O\,(0,0,0)$

$A\,(6\,,\,8\cos\theta\,,\,-8\sin\theta)$

$\vec{M}_o = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$

$$\vec{M}_o = \vec{r}_{OA} \times \vec{P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 8\cos\theta & -8\sin\theta \\ 0 & -P & 0 \end{vmatrix}$$

$$= (-8P\sin\theta)\vec{i} - 0\vec{j} - (6P)\vec{k}$$

$M_x = -8P\sin\theta = -160$ in-lb

$M_z = -6P = -240$ in-lb

$\Rightarrow P = \dfrac{240}{6} \Rightarrow \boxed{P = 40 \text{ lb}}$

$\Rightarrow \sin\theta = \dfrac{160}{8\,(40)} = 0.5$

$\Rightarrow \boxed{\theta = 30°}$