

#20

Examples Internal Forces (3-D)

Example 1:

Given:

The concrete block shown

$$\gamma_{\text{concrete}} = 150 \text{ lb/ft}^3 \quad (\text{specific weight})$$

Req'd.:

The internal forces at a horizontal section taken through point O

Sol'n.:

In the FBD shown,

$$W = \gamma V = 150(2 \times 1 \times 1.5)$$

$$= 450 \text{ lb}$$

Scalar Solution

$$\rightarrow \sum F_x = 0 = 200 + F_x$$

$$\Rightarrow F_x = V_x = -200 \text{ lb} = 200 \text{ lb} \nearrow$$

$$\rightarrow \sum F_y = 0 = -400 + F_y$$

$$\Rightarrow F_y = V_y = 400 \text{ lb}$$

$$\uparrow \sum F_z = 0 = -450 + F_z = 0$$

$$\Rightarrow F_z = A_z = 450 \text{ lb}$$

$$\curvearrowright \sum M_x = 0 = 400(2) + M_x \Rightarrow$$

$$M_x = -800 \text{ ft}\cdot\text{lb}$$

← bending moment

$$\curvearrowright \sum M_y = 0 = 200(2) + M_y \Rightarrow$$

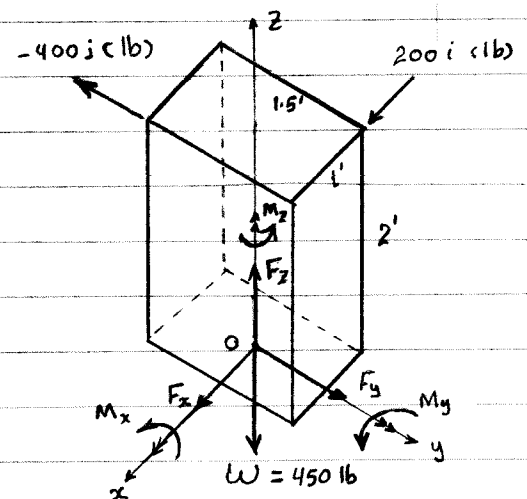
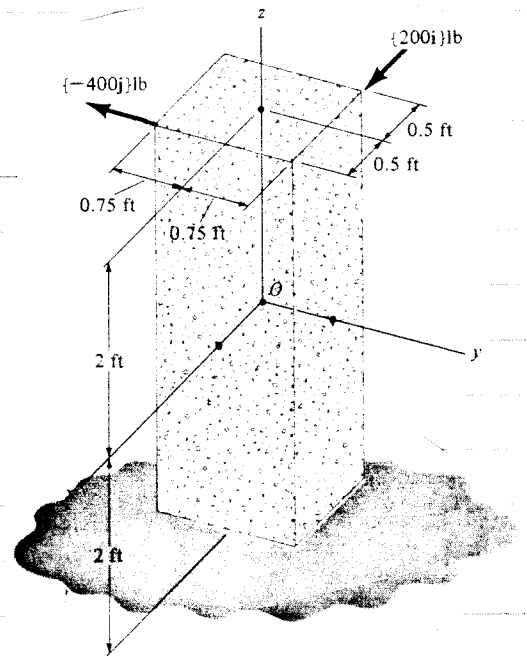
$$M_y = -400 \text{ ft}\cdot\text{lb}$$

← bending moment

$$\curvearrowright \sum M_z = 0 = -200(0.75) - 400(0.5) + M_z \Rightarrow$$

$$M_z = 350 \text{ ft}\cdot\text{lb}$$

← twisting moment



Note that in this 3-D problem, scalar analysis is much easier than vector analysis. Try the vector analysis yourself.

Example 2 :

Given:

The pipe assembly shown
Weight is neglected.

Reqd.:

The internal forces at a vertical section through C

Soln.:

I) Scalar Analysis

In the FBD shown,

$$\Sigma F_x = 0 = F_x^c - 26\left(\frac{5}{13}\right) \Rightarrow F_x^c = V_x^c = 10 \text{ lb}$$

$$\Sigma F_y = 0 \Rightarrow F_y^c = A_y^c = 0$$

$$\Sigma F_z = 0 = F_z^c - 80 - 26\left(\frac{12}{13}\right) \Rightarrow F_z^c = V_z^c = 104 \text{ lb}$$

$$\Sigma M_x^c = 0 = M_x^c - 30 - 80(2) - 26\left(\frac{12}{13}\right)(2)$$

$$\Rightarrow M_x^c = 238 \text{ lb}\cdot\text{ft} \quad (\text{RHR used})$$

$$\Sigma M_y^c = 0 = M_y^c + 26\left(\frac{12}{13}\right)(3) \Rightarrow M_y^c = -72 \text{ lb}\cdot\text{ft}$$

$$\Sigma M_z^c = 0 = M_z^c + 26\left(\frac{5}{13}\right)(2) \Rightarrow M_z^c = -20 \text{ lb}\cdot\text{ft}$$

II) Vector Analysis

$$\Sigma \vec{F} = \vec{0} \Rightarrow \vec{F}_1 + \vec{F}_2 + \vec{F}_c = \vec{0}$$

$$\Rightarrow [-80\vec{k}] + \left[\underbrace{-26\left(\frac{5}{13}\right)}_{-10}\vec{i} + \underbrace{-26\left(\frac{12}{13}\right)}_{-24}\vec{k} \right] + \vec{F}_c = \vec{0} \Rightarrow \vec{F}_c = [10\vec{i} + 0\vec{j} + 104\vec{k}] \text{ lb}$$

$$\Sigma \vec{M} = \vec{0} \Rightarrow (\vec{r}_{CB} \times \vec{F}_1) + (\vec{r}_{CA} \times \vec{F}_2) + \vec{M}_c - 30\vec{i} = \vec{0}$$

$$\Rightarrow \vec{M}_c - 30\vec{i} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ 0 & 0 & -80 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 0 \\ -10 & 0 & -24 \end{vmatrix} = \vec{0}$$

$$\vec{M}_c - 30\vec{i} + (-160\vec{i}) + (-48\vec{i} - (-72)\vec{j} - (-20)\vec{k}) = \vec{0}$$

$$\Rightarrow \vec{M}_c + (-30 - 160 - 48)\vec{i} + (72)\vec{j} + (20)\vec{k} = \vec{0} \Rightarrow$$

$$\vec{M}_c = 238\vec{i} - 72\vec{j} - 20\vec{k} \text{ (lb}\cdot\text{ft)}$$

