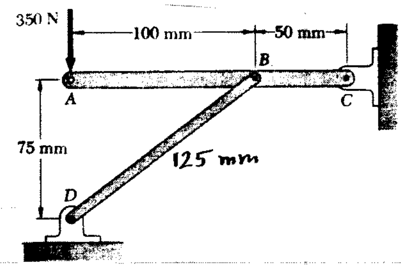
#18

Examples
Frames & Machines

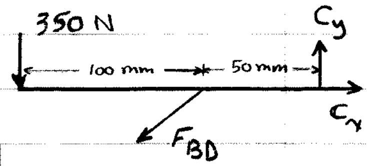
Example 1:



Given:
The frame shown

Req'd.:
The force in BD and the reaction at C.

Soln.:



Note that BD is a two-force member.
In the FBD shown,

$\circlearrowright^{+} \sum M_C = 0 = 350(150) + \frac{75}{125} F_{BD} (50) \Rightarrow$ $\boxed{F_{BD} = 1750 \text{ N (C)}}$

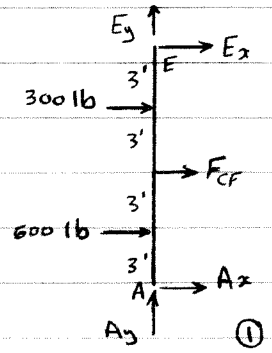
$\xrightarrow{+} \sum F_x = 0 \Rightarrow$

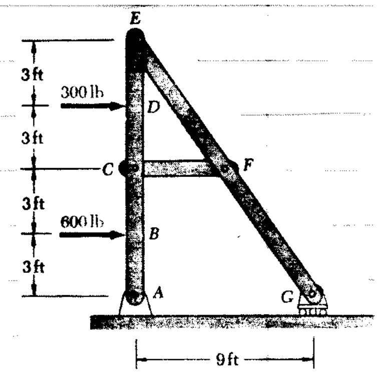
$C_x + 1750\left(\frac{100}{125}\right) = 0 \Rightarrow C_x = -1400 \text{ N} \Rightarrow$ $\boxed{C_x = 1400 \text{ N} \leftarrow}$

$+\uparrow \sum F_y = 0 \Rightarrow$

$-350 + 1750\left(\frac{75}{125}\right) + C_y = 0 \Rightarrow C_y = -700 \text{ N} \Rightarrow$ $\boxed{C_y = 700 \text{ N} \downarrow}$

Example 2:





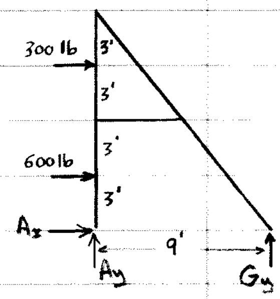
Given:
The frame shown

Req'd.:
All the forces in member AE

Soln.:

First, the FBD of AE is drawn in ①. (CF is a two-force member.)
In that FBD, there are 5 unknowns ⇒ Can not solve.
⇒ Find the reactions A_x and A_y first ⇒ go back to ①.
⇒ In FBD ②, $\xrightarrow{+} \sum F_x = 0 \Rightarrow A_x = 900 \text{ lb} \leftarrow$

$\circlearrowright^{+} \sum M_G = 0 = -300(9) - 600(3) - 9A_y \Rightarrow A_y = 500 \text{ lb} \downarrow$



Now, in FBD ①,

$\boxed{A_x = 900 \text{ lb} \leftarrow}$; $\boxed{A_y = 500 \text{ lb} \downarrow}$

$+\uparrow \sum F_y = 0 \Rightarrow$

$-500 + E_y = 0 \Rightarrow$ $\boxed{E_y = 500 \text{ lb} \uparrow}$; $\circlearrowright^{+} M_E = 0 \Rightarrow -900(12) + 600(9) + 6F_{CF} + 300(3) = 0$

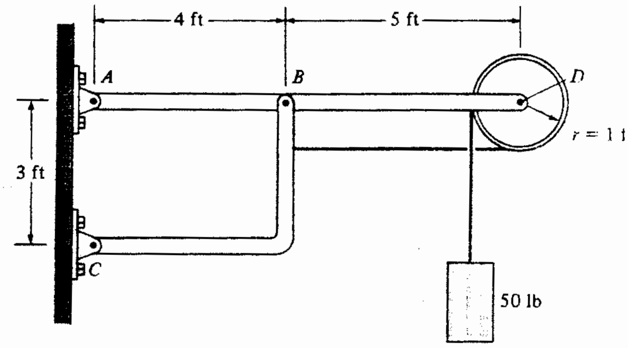
$\Rightarrow$ $\boxed{F_{CF} = 750 \text{ lb} \rightarrow \text{(T)}}$ $\Rightarrow \sum F_x = 0 \Rightarrow -900 + 600 + 750 + 300 + E_x = 0 \Rightarrow$ $\boxed{E_x = 750 \text{ lb} \leftarrow}$

#18

Example 3 :

Given :

The frame shown

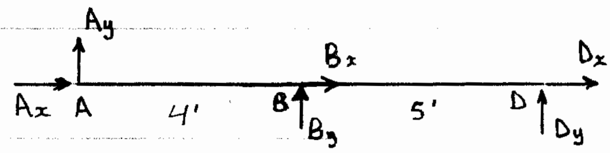


Req.d. :

The forces exerted by the pins on member ABD

Sol.n. :

From FBD ①, it can be seen that ABD has 6 unknown forces acting on it. ⇒ can't solve

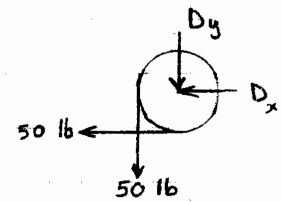


①

⇒ Take the pulley's FBD ②.

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow D_x = 50 \text{ lb} \rightarrow$

$+\uparrow \Sigma F_y = 0 \Rightarrow D_y = 50 \text{ lb} \uparrow$



②

⇒ Go back to FBD ① :

From the results above, (equal & opposite forces)

$\boxed{D_x = 50 \text{ lb} \leftarrow}$

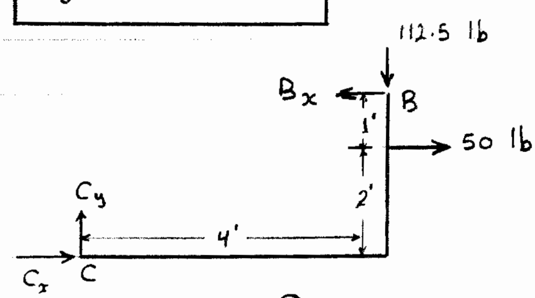
$\boxed{D_y = 50 \text{ lb} \downarrow}$

Now, still 4 unknowns, but we can solve for some of them.

$+\circlearrowleft \Sigma M_B = 0 = -50(5) - 4A_y = 0 \Rightarrow$ $\boxed{A_y = -62.5 \text{ lb} = 62.5 \text{ lb} \downarrow}$

$+\uparrow \Sigma F_y = 0 \Rightarrow -62.5 - 50 + B_y = 0 \Rightarrow$ $\boxed{B_y = 112.5 \text{ lb} \uparrow}$

Only one remaining eq. ($\Sigma F_x = 0$) and two unknowns (A_x & B_x). ⇒ We have to go to member BC (FBD ③). ⊛



③

$+\circlearrowleft \Sigma M_c = 0 = 3B_x - 50(2) - 112.5(4)$

$\Rightarrow B_x = 183.3 \text{ lb} \leftarrow$

Now, go back again to FBD ① :

From the result above (member BC), in member ABD, $\boxed{B_x = 183 \text{ lb} \rightarrow}$

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow -50 + 183 + A_x = 0 \Rightarrow$ $\boxed{A_x = -133 \text{ lb} = 133 \text{ lb} \leftarrow}$

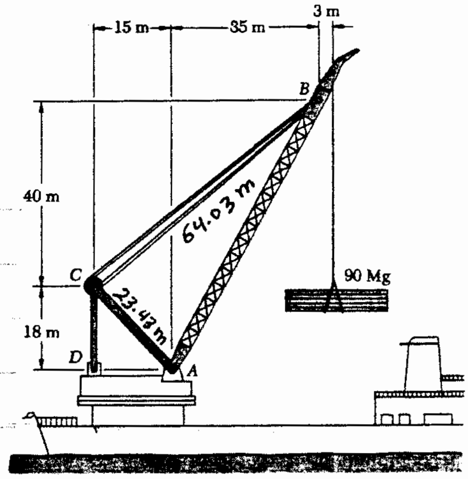
⊛ You may take the entire frame instead. Why & how ?!

18

Example 4 :

Given :

The marine crane shown



Reqd. :

a) The force in the link CD

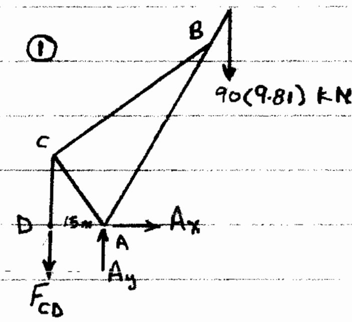
b) The force in the brace AC

c) The force exerted at A on the boom AB

Sol.n. :

Note that all members connected to joint C (CD, CA, CB) are two-force members.

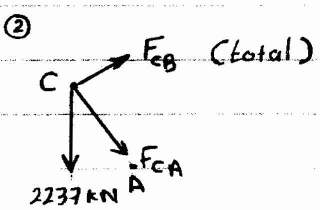
a) Consider FBD ① :



$\circlearrowleft + \; \Sigma M_A = 0 \Rightarrow$

$-90(9.81)(38) + 15 F_{CD} = 0 \Rightarrow$ $\boxed{F_{CD} = 2237 \text{ kN (T)}}$

b) Now, consider FBD ② :



$\circlearrowleft + \; \Sigma M_A = 0 \Rightarrow$

$2237(15) - \frac{50}{64.03} F_{CB} (18) - \frac{40}{64.03} F_{CB} (15) = 0$

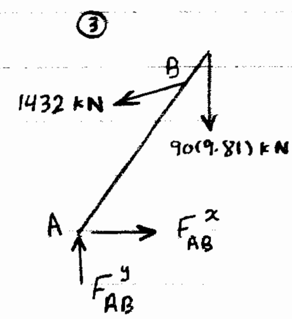
$\Rightarrow F_{CB} = 1432 \text{ kN} \quad (T)$

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$1432 \left(\frac{50}{64.03}\right) + \frac{15}{23.43} F_{CA} = 0$

$\Rightarrow$ $\boxed{F_{CA} = -1747 \text{ kN} = 1747 \text{ kN (C)}}$

c) From FBD ③, [[Note that F_{AB}^x & F_{AB}^y are <u>not</u> A_x & A_y]] Why?!



$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$F_{AB}^x - 1432 \left(\frac{50}{64.03}\right) = 0 \Rightarrow$ $\boxed{F_{AB}^x = 1118 \text{ kN} \rightarrow}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$F_{AB}^y - 1432 \left(\frac{40}{64.03}\right) - 90(9.81) = 0 \Rightarrow$ $\boxed{F_{AB}^y = 1777 \text{ kN} \uparrow}$

$$\left.\begin{aligned} \Rightarrow F_{AB} &= \sqrt{(1118)^2 + (1777)^2} \\ \theta &= \tan^{-1} \frac{1777}{1118} \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} F_{AB} &= 2100 \text{ kN} \\ \theta &= 57.8^\circ \end{aligned}}$$

" Final answer in 3 S.F.

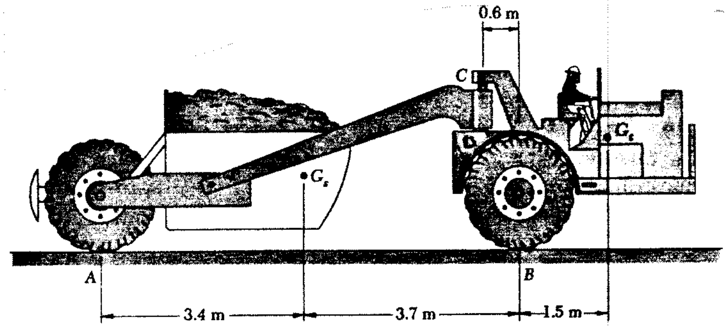
Intermediate answers in 4 S.F."

at least

#18

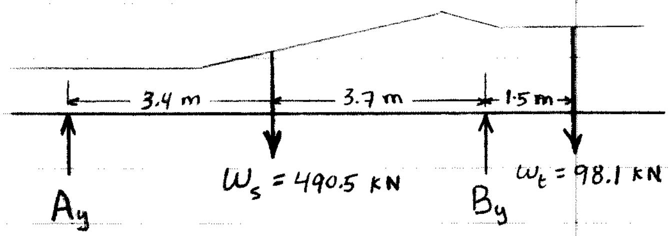
Example 5 :

The tractor and scraper units shown are connected by a vertical pin located 0.6 m behind the tractor wheels. The distance from C to D is 0.75 m. The center of gravity of the 10-Mg tractor unit is located at G_t. The scraper unit and load have a total mass of 50 Mg and a combined center of gravity located at G_s. Knowing that the machine is at rest, with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the tractor unit at C and D.



Sol'n. :

a) The FBD is drawn first for the entire system (①).



$\circlearrowleft + \sum M_B = 0 \Rightarrow$

$-7.1(\underbrace{2\,A_y}_{\text{two scraper wheels (symmetric)}}) + 490.5(3.7) - 98.1(1.5) = 0$

$W_s = 50(10)^3(9.81) = 490\,500$ N
$= 409.5$ kN

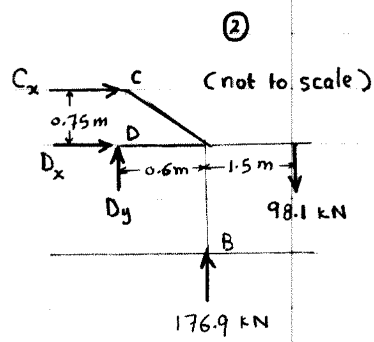
$W_t = 10(9.81) = 98.1$ kN

$\Rightarrow$ $\boxed{A_y = 117.4 \text{ kN at each scraper wheel}}$

$+\uparrow \sum F_y = 0 \Rightarrow -490.5 - 98.1 + 2(117.4) + 2\,B_y = 0$

$\Rightarrow$ $\boxed{B_y = 176.9 \text{ kN at each tractor wheel}}$

b) Note that in FBD ②, there is no C_y. See the support condition in the original figure.



In FBD ②, $\circlearrowleft + \sum M_D = 0 \Rightarrow$

$2(176.9)0.6 - 98.1(2.1) - 0.75\,C_x = 0$

$\Rightarrow$ $\boxed{C_x = 8.36 \text{ kN} \rightarrow}$

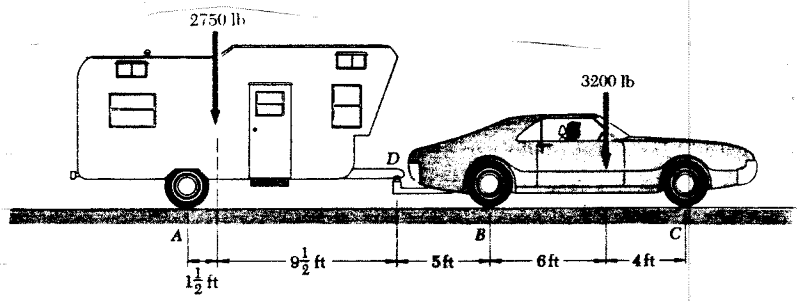
$\xrightarrow{+} \sum F_x = 0 \Rightarrow 8.36 + D_x = 0 \Rightarrow$ $\boxed{D_x = -8.36 \text{ kN} = 8.36 \text{ kN} \leftarrow}$

$+\uparrow \sum F_y = 0 \Rightarrow 2(176.9) - 98.1 + D_y = 0$

$\Rightarrow$ $\boxed{D_y = -255.7 \text{ kN} = 255.7 \text{ kN} \downarrow}$

\# 18

Example 6:



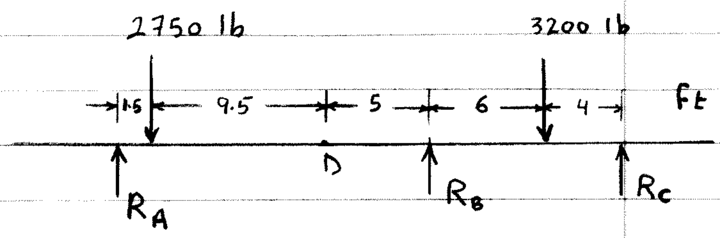
Given:

The automobile and trailer shown at rest

Req.d.:

a) The reactions at each of the six wheels

b) The additional load on each of the automobile wheels due to the trailer



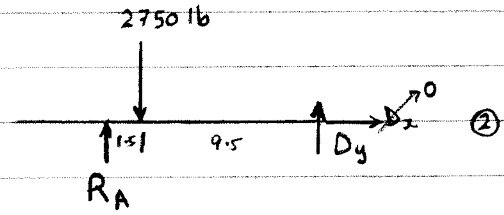
①

Soln.:

a) First, FBD ① is drawn.

Note: There are 3 unknowns ⇒ We can't find them (Why? Try!!). $\Sigma F_x = 0$ is useless!

⇒ FBD ② is drawn.



$\circlearrowleft\!+ \ \Sigma M_D = 0$ in ②

$2750(9.5) - 11(2\ R_A) = 0$

two wheels at A

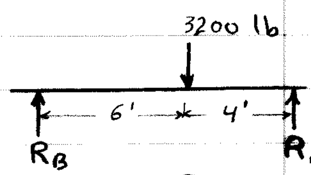
⇒ $\boxed{R_A = 1188 \text{ lb/wheel}}$

In FBD ①, $\circlearrowleft\!+ \ \Sigma M_C = 0 = -1188(26)(2) + 2750(24.5) - 10(\underbrace{2}_{\text{two wheels}} R_B) + 3200(4) = 0$

⇒ $\boxed{R_B = 920 \text{ lb/wheel}}$

$+\uparrow \Sigma F_y = 0 = 1188(2) - 2750 + 920(2) - 3200 + 2R_c$ ⇒ $\boxed{R_c = 867 \text{ lb/wheel}}$

b) Now consider the automobile w/o trailer:



③

car only

In FBD ③, $\circlearrowleft\!+ \ M_C = 0$

$3200(4) - 10(2\ R_B)$ ⇒ $R_B = 640$ lb

$\Delta R_B = R_B^a - R_B^b = 920 - 640$ ⇒ $\boxed{\Delta R_B = 280 \text{ lb / wheel}}$

$+\uparrow \Sigma F_y = 0$ ⇒

$2(640) + 2\ R_c - 3200 = 0$ ⇒ $R_c = 960$ lb

$\Delta R_c = R_c^a - R_c^b = 867 - 960$ ⇒ $\boxed{\Delta R_c = -93 \text{ lb / wheel}}$

change due to trailer

note minus in ΔR_c ?!!

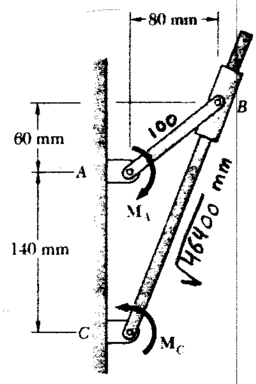
#18

Example 7 :

Given :

The two rods and frictionless collar at B shown

$M_A = 20$ N.m



Reqd. :

a) M_c required for equilibrium

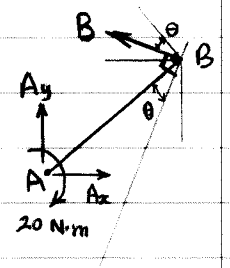
b) The corresponding reaction at C

Soln. :

Note that there are 4 reactions and M_c as unknowns. ⇒ We can not use the equilibrium of the entire system to solve for the unknowns (3 eqs. & 5 unks).

⇒ Consider rod AB (FBD ①)

(Note that we can not start with rod BC. Why ?)



①

a) $\circlearrowleft^{+} \Sigma M_A = 0 \Rightarrow$

$-20 + B(0.8542)0.1 = 0$

$\Rightarrow B = 234.1$ N

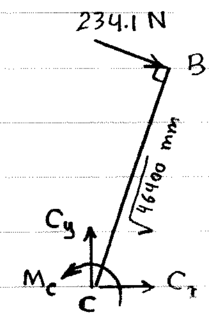
$$\cos\theta = \frac{10000 + 46400 - (140)^2}{200\sqrt{46400}} = 0.8542$$

Now, take member BC (FBD ②)

$\circlearrowleft^{+} \Sigma M_c = 0 \Rightarrow$

$M_c - 234.1\left(\sqrt{46400}/1000\right) = 0$

$\Rightarrow \boxed{M_c = 50.4 \text{ N.m}}$



②

b) In FBD ②,

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$C_x + 234.1\left(\frac{140+60}{\sqrt{46400}}\right)$

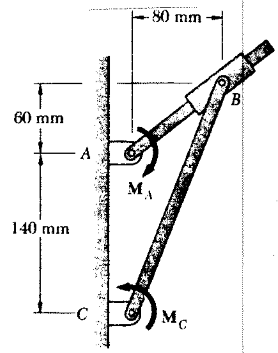
$\Rightarrow \boxed{C_x = -217 \text{ N} = 217 \text{ N} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$C_y - 234.1\left(\frac{80}{\sqrt{46400}}\right) = 0 \Rightarrow \boxed{C_y = 86.9 \text{ N} \uparrow}$

18

Example 8 :



Given:

The two rods and smooth collar at B shown

$M_A = 20$ N.m

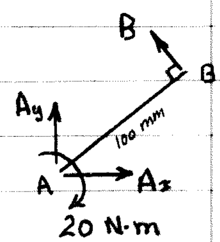
Reqd. :

a) M_c required for equilibrium

b) The corresponding reaction at C

Soln. :

a) First, consider rod AB (FBD ①)



①

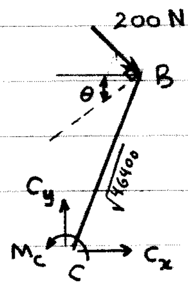
$\circlearrowleft + \; \Sigma M_A = 0 \Rightarrow$

$B(0.1) - 20 = 0 \Rightarrow B = 200$ N

Now, Draw FBD for member BC (②)

$\cos\theta = 0.8$

$\sin\theta = 0.6$



②

$\circlearrowleft + \; \Sigma M_c = 0 \Rightarrow$

$M_c - 200(0.6)(0.2) - 200(0.8)(0.08) = 0$

$\Rightarrow \boxed{M_c = 36.8 \text{ N.m} \circlearrowright}$

b) $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$C_x + 200(0.6) = 0$

$\Rightarrow C_x = -120$ N $\Rightarrow \boxed{C_x = 120 \text{ N} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$C_y - 200(0.8) = 0$

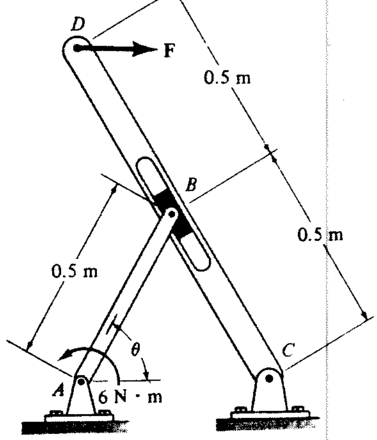
$\Rightarrow \boxed{C_y = 160 \text{ N.m} \uparrow}$

Compare this example with the previous one (#7) in "all aspects".

Note that in these two examples, the x-axis could have been chosen along AB or BC. Try it !

18

Example 9 :



Given :

The slider mechanism shown

$\theta = 60^\circ$

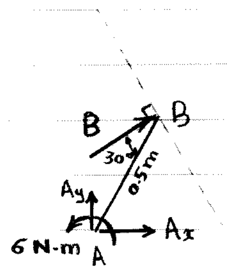
Req'd. :

The force F required to maintain equilibrium

Sol'n. :

Note that the FBD for the whole system can <u>not</u> be used to solve for F. (Why? See example 7.)

Take FBD ① (AB). Do not start with CD. Why?



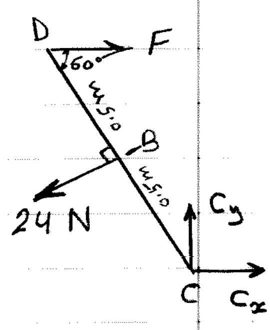
①

$\circlearrowleft^{+} \; \Sigma M_A = 0 \Rightarrow$

$6 - B \sin 30^\circ (0.5) = 0$

$\Rightarrow \; B = 24 \text{ N}$

Now, consider CD : FBD ②



②

$\circlearrowleft^{+} \; M_c = 0 \Rightarrow$

$24 (0.5) - F (1 \sin 60^\circ) = 0$

$\Rightarrow \; \boxed{F = 13.9 \text{ N} \rightarrow}$

Is it possible to maintain equilibrium w/o F as shown (location & direction)?

Is it possible to maintain equilibrium w/ F at D in th vertical direction? Explain.

18

Example 10:

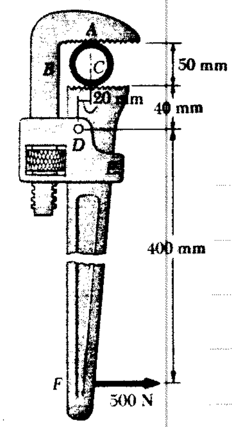
Given:

The Stillson wrench shown

Portions AB and DE are rigidly attached to each other.

Portion CF is connected by a pin at D.

No slipping occurs between the pipe and the wrench.

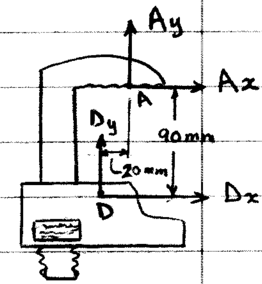


Reqd.:

The forces exerted on the pipe at A and C

Soln.:

In order to be able to see the forces at A and C, FBD's ① and ② are drawn.



①

In FBD ①, $\circlearrowleft^{+} \Sigma M_A = 0 \Rightarrow$

$0.09 D_x - 0.02 D_y = 0$ (1)

In FBD ②, $\circlearrowleft^{+} \Sigma M_C = 0 \Rightarrow$

$-0.04 D_x + 0.02 D_y + 500(0.44) = 0$ (2)

Adding (1) and (2) yeilds

$0.05 D_x + 500(0.44) = 0 \Rightarrow D_x = -4400 \text{ N}$

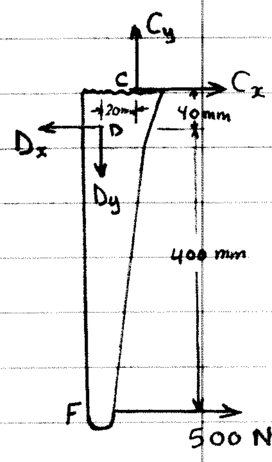
$\Rightarrow 0.09(-4400) = 0.02 D_y \Rightarrow D_y = -19\,800 \text{ N}$

in ①, $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$A_x + (-4400) = 0 \Rightarrow A_x = 4400 \text{ N } (\rightarrow)$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$A_y + (-19\,800) = 0 \Rightarrow A_y = 19\,800 \text{ N } (\uparrow)$



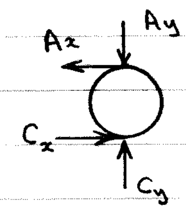
②

in ②, $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$C_x - (-4400) + 500 = 0 \Rightarrow C_x = -4900 \text{ N} = 4900 \text{ N } (\leftarrow)$

$+\uparrow \Sigma F_y = 0 \Rightarrow C_y - (-19800) = 0 \Rightarrow C_y = 19800 \text{ N } (\downarrow)$

Since the question asks for the forces exerted on the pipe, then in FBD ③ (pipe), they are equal and opposite.



③

$\Rightarrow$

$A_x = 4.4 \text{ kN} \leftarrow$	$A_y = 19.8 \text{ kN} \downarrow$
$C_x = 4.9 \text{ kN} \rightarrow$	$C_y = 19.8 \text{ kN} \uparrow$

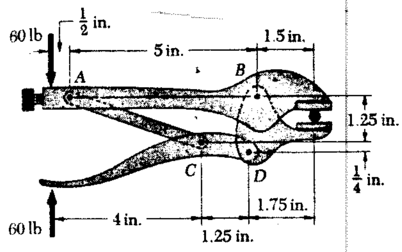
Notes: 1) Do not start with $\Sigma M_D = 0$ Why? Try it! 2) FBD ① & ② may be drawn with the pipe. Try. 3) Note how much force the 500 N load produces on the pipe (MACHINE).

#18

Example 11 :

Given:

The figure shown



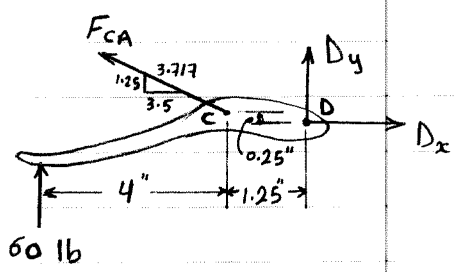
Req.d.:

The gripping force

Soln.:

To solve this problem, we have to start with FBD ①. Why?

In ①,



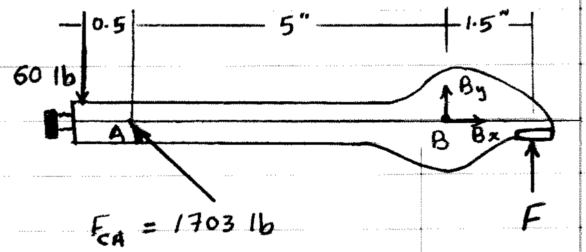
①

$\circlearrowleft + \; \Sigma M_D = 0 \Rightarrow$

$$-60(5.25) + \frac{3.5}{3.717}F_{CA}(0.25) - \frac{1.25}{3.717}F_{CA}(1.25) = 0$$

$\Rightarrow \quad F_{CA} = -1703 \text{ lb} = 1703 \text{ lb } (C) \Leftarrow$ two-force member

Now consider FBD ②:



②

$\circlearrowleft + \; \Sigma M_B = 0 \Rightarrow$

$$1.5F + 60(5.5) - 1703\left(\frac{1.25}{3.717}\right)5 = 0$$

$\Rightarrow \quad \boxed{F = 1689 \text{ lb}}$

Thus, the 60-lb force gave a force of 1689-lb, which is more than 28 times, at the point (location) of interest.

This is the job of the machines. They <u>transmit</u> and <u>modify</u> <u>forces</u>.