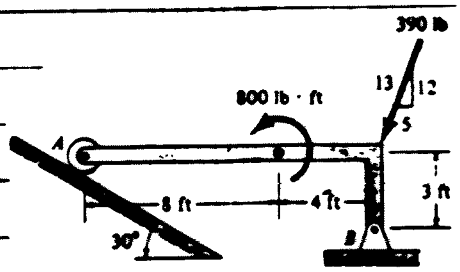
#14

Examples
Equilibrium of Rigid Bodies in 2-D

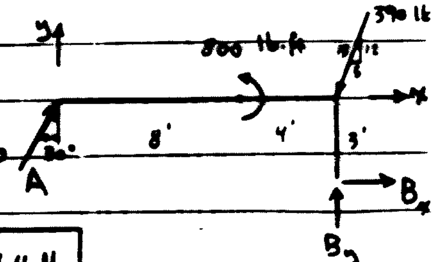
Example 1:

Given:
The structure shown



Req.d.:
The reactions at A and B

Soln.:
First, draw the FBD ⟹



$\circlearrowright \Sigma M_B = 0 \Rightarrow 390(\frac{5}{13})(3) + 800 - A\sin 30°(3) - A\cos 30°(12) = 0$

$\Rightarrow \boxed{A = 105 \text{ lb}}$

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow 105\sin 30° - 390(\frac{5}{13}) + B_x = 0 \Rightarrow \boxed{B_x = 97.4 \text{ lb}}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

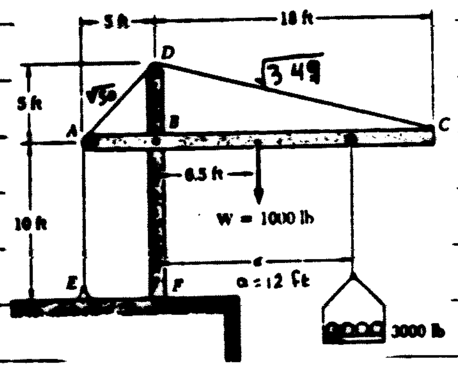
$105\cos 30° - 390(\frac{12}{13}) + B_y = 0 \Rightarrow \boxed{B_y = 269 \text{ lb}}$

[3 eqs. & 3 unknowns

Note that in this problem, it is the easiest to start with $\Sigma M_B = 0$ as above.

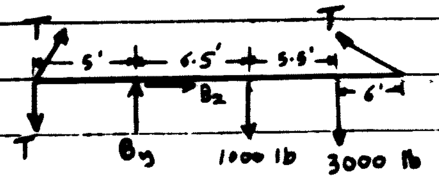
Example 2:

Given:
The figure shown



Req.d.:
Consider member ABC; find the tension in the cable and the "reaction" at B.

Soln.:
From the FBD shown,



$\circlearrowright \Sigma M_B = 0 \Rightarrow$

$T(\frac{5}{\sqrt{349}})18 - 3(12) - 6.5 + 5T - \frac{5(5)}{\sqrt{50}}T = 0$

$6.282T = 42.5 \Rightarrow \boxed{T = 6.76 \text{ k}}$

Note that smooth pulleys are assumed at A and D.

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$-T(\frac{18}{\sqrt{349}}) + \frac{5}{\sqrt{50}}T + B_x = 0 \Rightarrow \boxed{B_x = 1.73 \text{ k} \rightarrow}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$T(\frac{5}{\sqrt{349}}) - T + \frac{5}{\sqrt{50}}T - 4 + B_y = 0 \Rightarrow \boxed{B_y = 4.17 \text{ k} \uparrow}$

- 1000 lb = 1 kip
- 3 eqs. & 3 unknowns

14

Example 3:

Given:

The figure shown

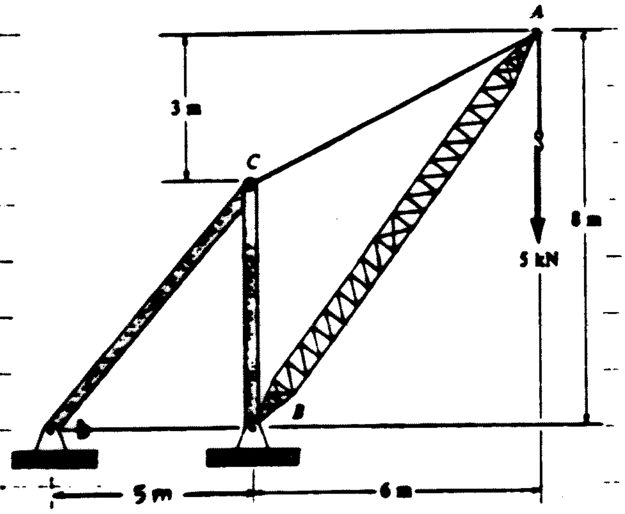
Reqd.:

Reaction at B

T_{AC}

Soln.:

First, the FBD is drawn.

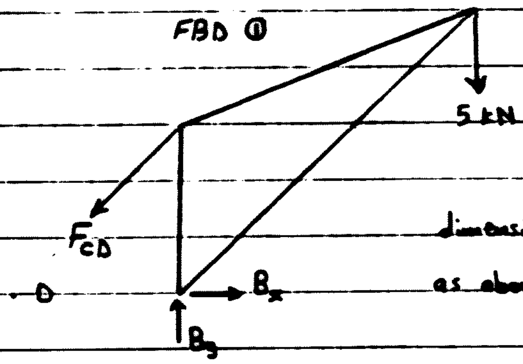


Note that CD is a two-force member.

From FBD ①,

$\circlearrowleft^{+} \Sigma M_D = 0 \Rightarrow$

$-5(11) + 5B_y = 0 \Rightarrow \boxed{B_y = 11 \text{ kN} \uparrow}$



$+\uparrow \Sigma F_y = 0 \Rightarrow$

$11 - 5 - F_{CD}\left(\frac{5}{\sqrt{50}}\right) = 0 \Rightarrow F_{CD} = 8.485 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$B_x - 8.485\left(\frac{5}{\sqrt{50}}\right) = 0$

(3 eqs. & 3 unknowns)

$\Rightarrow \boxed{B_x = 6 \text{ kN} \rightarrow}$

Note that you may start with $\Sigma M_B = 0 \Rightarrow F_{CD} \Rightarrow \Sigma F_x = 0 \Rightarrow B_x$

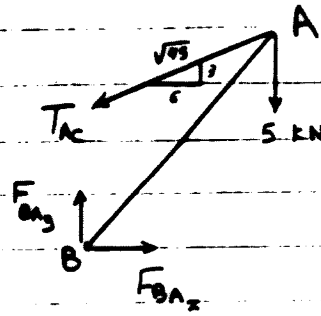
$\Rightarrow \Sigma F_y = 0 \Rightarrow B_y$

In FBD ②,

$\circlearrowleft^{+} \Sigma M_B = 0 \Rightarrow$

$-5(6) - T_{AC}\left(\frac{3}{\sqrt{45}}\right)(6) + T_{AC}\left(\frac{6}{\sqrt{45}}\right)(8) = 0$

$\Rightarrow \boxed{T_{AC} = 6.71 \text{ kN}}$



FBD ②

member AB

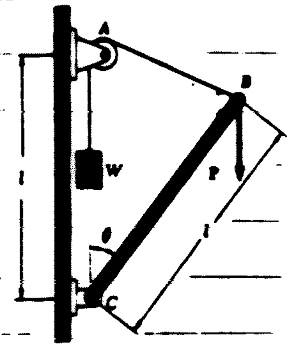
* Note that the reactions at B in FBD② are not B_x & B_y as in FBD①. Why?!

#14

Example 4:

Given:

The figure shown

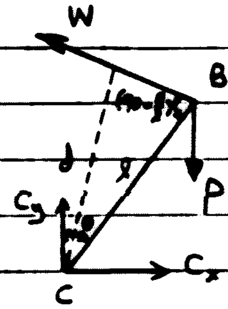


Req'd:

a) Expression for θ ; b) θ when $P = 2W$

Sol'n:

From the FBD, $\circlearrowleft \Sigma M_c = 0 \Rightarrow$



a) $-Pl\sin\theta + Wl\sin(90 - \frac{\theta}{2}) = 0 \Rightarrow$

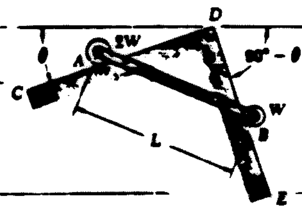
$-P\sin\theta + W\cos\frac{\theta}{2} = 0 \Rightarrow$

$-2P\sin\frac{\theta}{2}\cos\frac{\theta}{2} + W\cos\frac{\theta}{2} = 0 \Rightarrow$

$\sin\frac{\theta}{2} = \frac{W}{2P} \Rightarrow \boxed{\theta = 2\sin^{-1}\frac{W}{2P}}$

b) $P = 2W \Rightarrow \theta = 2\sin^{-1}\frac{W}{4W} = 2\sin^{-1}\frac{1}{4} \Rightarrow \boxed{\theta = 29.0^\circ}$

Example 5:



Given:

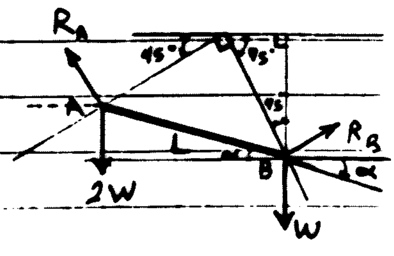
The figure shown , $\theta = 45^\circ$

Req'd:

α = the angle that the rod AB forms with the horizontal

Sol'n:

The FBD is drawn first



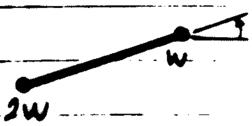
$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow R_A\cos 45^\circ + R_B\cos 45^\circ = 0$

$\Rightarrow R_A = R_B$ ①

$+\uparrow \Sigma F_y = 0 \Rightarrow -3W + R_A\sin 45^\circ + R_B\sin 45^\circ = 0$

$\Rightarrow R_A + R_B = 3W/\sin 45^\circ$ ②

From ① into ② , $R_B = 3W/2\sin 45^\circ$



$\circlearrowleft \Sigma M_A = 0 \Rightarrow (R_B\sin 45^\circ - W)L\cos\alpha + (R_B\cos 45^\circ)L\sin\alpha = 0$

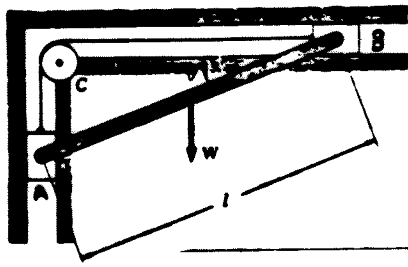
$\left(\frac{3W}{2\sin 45^\circ}\sin 45^\circ - W\right)\cos\alpha + \frac{3W}{2\sin 45^\circ}\cos 45^\circ\sin\alpha = 0$

$\frac{1}{2}W\cos\alpha + \frac{3}{2}W\sin\alpha = 0 \Rightarrow$

$\frac{1}{2} + \frac{3}{2}\tan\alpha = 0 \Rightarrow \tan\alpha = -\frac{1}{3} \Rightarrow \boxed{\alpha = -18.4^\circ = 18.4^\circ \circlearrowright}$ as shown

#14

Example 6 :



Given :

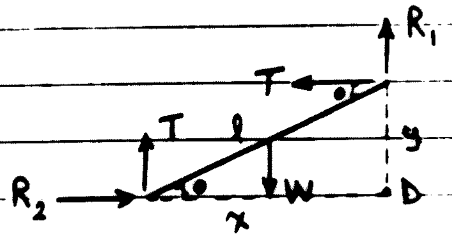
The figure shown

Blocks A and B move freely in the guides

Reqd. :

a) The tension in the cord in terms of W and θ

b) θ when $T = 2W$



Soln. :

a) From the FBD shown,

$\circlearrowleft \Sigma M_D = 0 \Rightarrow$ ← Why did we choose ΣM_D so ?!

$$W\frac{x}{2} + Ty - Tx = 0$$

$$x = l \cos\theta$$

$$y = l \sin\theta$$

$$\Rightarrow \frac{W}{2} l \cos\theta + Tl \sin\theta - Tl\cos\theta = 0$$

$$\frac{W}{2}\cos\theta + T(\sin\theta - \cos\theta) = 0$$

$$\Rightarrow T = \frac{-W\cos\theta}{2(\sin\theta - \cos\theta)} = \frac{-W\cos\theta/\cos\theta}{2(\sin\theta/\cos\theta - \cos\theta/\cos\theta)}$$

$$\Rightarrow \boxed{T = \frac{-W}{2(\tan\theta - 1)}}$$

b) $T = 2W$

$T = -W/2(\tan\theta - 1)$

$$\Rightarrow 2W = \frac{-W}{2(\tan\theta - 1)} \Rightarrow 4(\tan\theta - 1) = -1$$

$$\Rightarrow \tan\theta = \frac{-1}{4} + 1 = \frac{3}{4} \Rightarrow \boxed{\theta = 36.9^\circ}$$

■ Very imp. : The direction of the reaction can be assumed if it is not known (in any prob.). If the answer is ⊕, then the assumed dir. is correct if ⊖, then, it is the opp.