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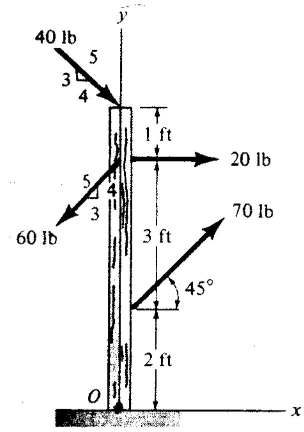
Examples

Resultants of a Force and Couple System

Example 1 :



Given :

The loads on the post shown

Req'd. :

The equivalent force-couple system at O

Sol'n. :

$$\Sigma F_x = 20 + 70\cos 45^\circ + 40\left(\tfrac{4}{5}\right) - 60\left(\tfrac{3}{5}\right)$$
$$= 65.50 \text{ lb} \rightarrow$$

$$\Sigma F_y = 70\sin 45^\circ - 40\left(\tfrac{3}{5}\right) - 60\left(\tfrac{4}{5}\right)$$
$$= -22.50 \text{ lb} = 22.50 \text{ lb} \downarrow$$

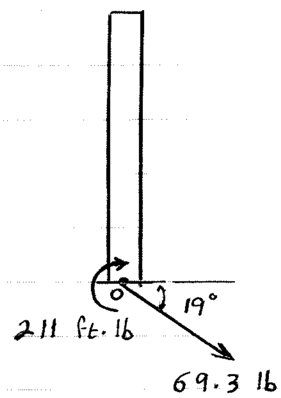
$$R = \sqrt{F_x^2 + F_y^2} = \sqrt{(65.50)^2 + (-22.50)^2} \Rightarrow \boxed{R = 69.3 \text{ lb}}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \frac{-22.5}{65.5} \Rightarrow \boxed{\theta = -19.0^\circ = 19.0^\circ}$$

$$\circlearrowleft\!+\ \Sigma M_o = -70\cos 45^\circ (2) - 20(5) - 40\left(\tfrac{4}{5}\right)6 + 60\left(\tfrac{3}{5}\right)5$$

$$\Rightarrow \boxed{M_o = -211 \text{ ft-lb} = 211 \text{ ft.lb} \circlearrowright}$$

The equivalent force-couple system is shown in the figure



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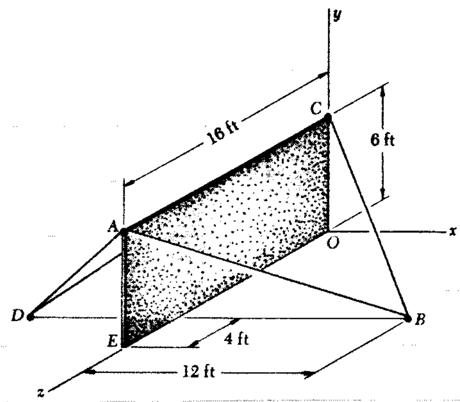
Example 2:

Given:

The cables connected to the wall shown

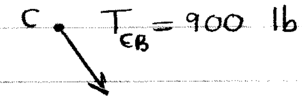


Reqd.:

Replace T_{CB} by a force-couple system located at

a) point O

b) point E



Soln.:

a) $B(12, 0, 12)$

$C(0, 6, 0)$

$\vec{CB} = 12\vec{i} - 6\vec{j} + 12\vec{k} \Rightarrow CB = 18 \text{ ft} \Rightarrow$

$\vec{u}_{CB} = \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}$

$\Rightarrow \vec{T}_{CB} = 900\left(\frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}\right)$

$\vec{F}_o = \vec{T}_{CB} \Rightarrow$ $\boxed{\vec{F}_o = 600\vec{i} - 300\vec{j} + 600\vec{k} \quad (\text{lb})}$

$$\vec{M}_o = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 6 & 0 \\ 600 & -300 & 600 \end{vmatrix} \Rightarrow \boxed{\vec{M}_o = 3600\vec{i} - 3600\vec{k} \quad (\text{ft.lb})}$$

b) $E(0, 0, 16)$

$\vec{EC} = C - E = 0\vec{i} + 6\vec{j} - 16\vec{k}$

As above, $\vec{F}_E = \vec{T}_{CB} \Rightarrow$ $\boxed{\vec{F}_E = 600\vec{i} - 300\vec{j} + 600\vec{k} \quad (\text{lb})}$

$$\vec{M}_E = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 6 & -16 \\ 600 & -300 & 600 \end{vmatrix} \Rightarrow \boxed{\vec{M}_E = -1200\vec{i} - 9600\vec{j} - 3600\vec{k} \quad (\text{ft.lb})}$$

If there are more than one force (tension), then the procedures above should be repeated for each force, and the force as well as the moment vectors are added so that we end up with a single $\vec{F}$-$\vec{M}$ system.