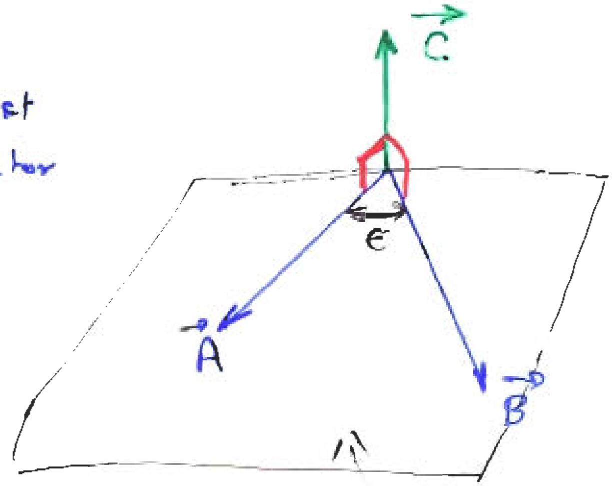
Cross Product

« Vector " »

$$\vec{C} = \vec{A} \times \vec{B}$$



plane containing $\vec{A}$ and $\vec{B}$

The result of a cross (vector) product of two vectors $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ with the following properties:

1) Magnitude :

$$C = A\,B\,\sin\theta$$

2) Line of action :

$\perp$ (perpendicular) to the plane containing $\vec{A}$ and $\vec{B}$

Sense :

right hand rule (RHR)

Laws of Operations :

1) $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$= -\vec{B} \times \vec{A}$ « RHR »

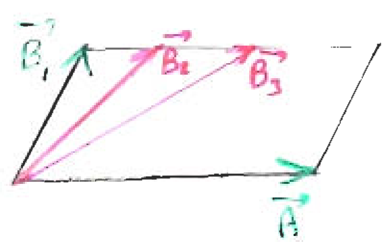
2) $n(\vec{A} \times \vec{B}) = (n\vec{A}) \times \vec{B} = \vec{A} \times (n\vec{B}) = (\vec{A} \times \vec{B})\,n$

n is a scalar

3) $\vec{A} \times (\vec{B} + \vec{D}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{D})$

4) $\vec{A} \times \vec{B}_1 \overset{?}{=} \vec{A} \times \vec{B}_2 \overset{?}{=} \vec{A} \times \vec{B}_3$

no ! no !



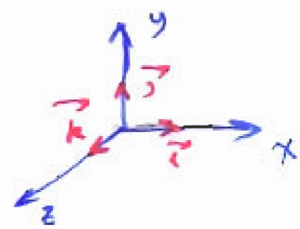
5) $(\vec{A} \times \vec{B}) \times \vec{D} \overset{?}{=} \vec{A} \times (\vec{B} \times \vec{D})$

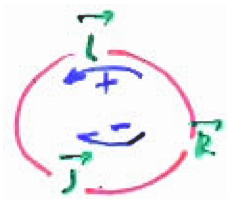
* Cartesian Vector / Rectangular Coordinate

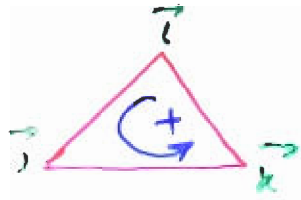
$$\vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{i}=-\vec{k}$$

$$\vec{j}\times\vec{k}=\vec{i} \qquad \vec{k}\times\vec{j}=-\vec{i}$$

$$\vec{k}\times\vec{i}=\vec{j} \qquad \vec{i}\times\vec{k}=-\vec{j}$$







$$\vec{i}\times\vec{i}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}=0$$

$$\vec{A}\times\vec{B}=(A_x\vec{i}+A_y\vec{j}+A_z\vec{k})\times(B_x\vec{i}+B_y\vec{j}+B_z\vec{k})$$
$$=A_xB_x(\vec{i}\times\vec{i})+A_xB_y(\vec{i}\times\vec{j})+\cdots$$
$$=0+A_xB_y\vec{k}+\cdots$$

$\Rightarrow$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

((determinant))