

Equilibrium of Particles in 3-D

Eqs. and FBD's are as in the 2-D above, but add the third (z) direction. \Rightarrow

$$\sum F_x = 0 \quad (1)$$

$$\sum F_y = 0 \quad (2)$$

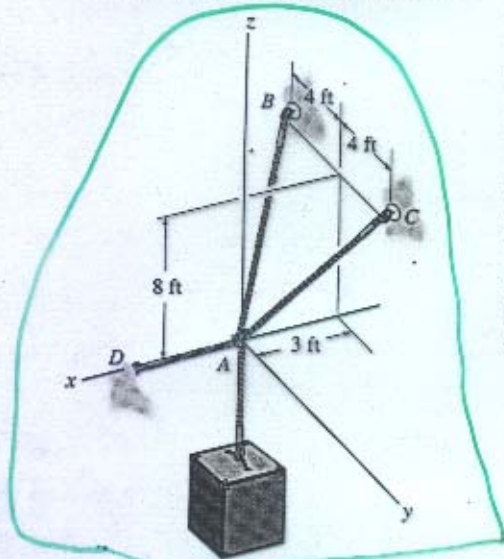
$$\sum F_z = 0 \quad (3)$$

3 eqs. \Rightarrow ?? unknowns ??

Review & Practice

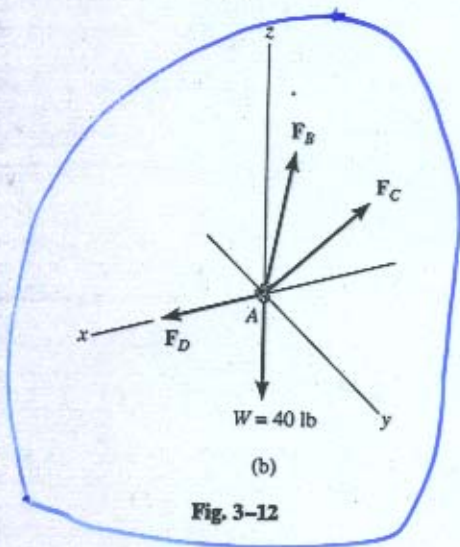
FBD, FBD, FBD, FBD

EXAMPLE 3-7



space (physical) diagram

(a)



(b)

Fig. 3-12

FBD

Determine the force developed in each cable used to support the 40-lb crate shown in Fig. 3-12a.

Solution

Free-Body Diagram. As shown in Fig. 3-12b, the free-body diagram of point A is considered in order to “expose” the three unknown forces in the cables.

Equations of Equilibrium. First we will express each force in Cartesian vector form. Since the coordinates of points B and C are $B(-3 \text{ ft}, -4 \text{ ft}, 8 \text{ ft})$ and $C(-3 \text{ ft}, 4 \text{ ft}, 8 \text{ ft})$, we have

$$F_B = F_B \left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}} \right]$$

$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$

$$F_C = F_C \left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}} \right]$$

$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$

$$F_D = F_D\mathbf{i}$$

$$W = \{-40\mathbf{k}\} \text{ lb}$$

Equilibrium requires

$$\Sigma \mathbf{F} = 0; \quad F_B + F_C + F_D + W = 0$$

$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k} - 0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = 0$$

Equating the respective $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components to zero yields

$$\Sigma F_x = 0; \quad -0.318F_B - 0.318F_C + F_D = 0 \quad (1)$$

$$\Sigma F_y = 0; \quad -0.424F_B + 0.424F_C = 0 \quad (2)$$

$$\Sigma F_z = 0; \quad 0.848F_B + 0.848F_C - 40 = 0 \quad (3)$$

Equation 2 states that $F_B = F_C$. Thus, solving Eq. 3 for F_B and F_C and substituting the result into Eq. 1 to obtain F_D , we have

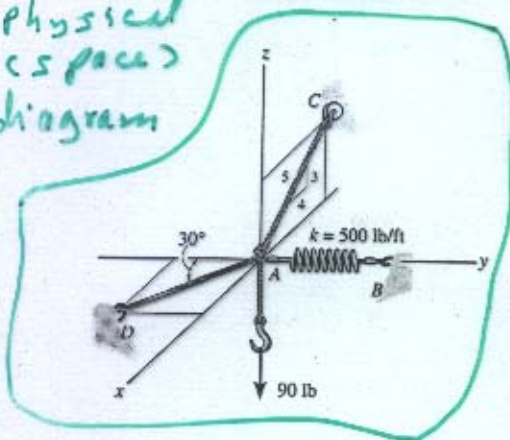
$$F_B = F_C = 23.6 \text{ lb} \quad \text{Ans.}$$

$$F_D = 15.0 \text{ lb} \quad \text{Ans.}$$

3 eqs. & 3 unknowns

EXAMPLE 3-5

Physical (space) diagram

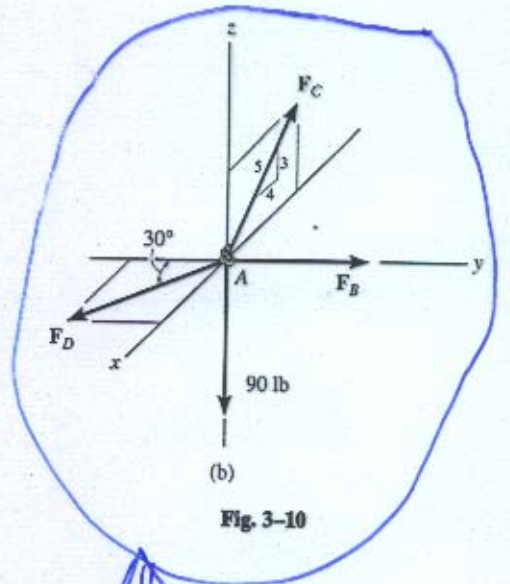


A 90-lb load is suspended from the hook shown in Fig. 3-10a. The load is supported by two cables and a spring having a stiffness $k = 500$ lb/ft. Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x - y plane and cable AC lies in the x - z plane.

Solution

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig. 3-10b.



Equations of Equilibrium. By inspection, each force can easily be resolved into its x , y , z components, and therefore the three scalar equations of equilibrium can be directly applied. Considering components directed along the positive axes as "positive," we have

$$\begin{aligned} \Sigma F_x = 0; & \quad F_D \sin 30^\circ - \frac{4}{5} F_C = 0 & (1) \\ \Sigma F_y = 0; & \quad -F_D \cos 30^\circ + F_B = 0 & (2) \\ \Sigma F_z = 0; & \quad \frac{3}{5} F_C - 90 \text{ lb} = 0 & (3) \end{aligned}$$

Solving Eq. 3 for F_C , then Eq. 1 for F_D , and finally Eq. 2 for F_B , yields

$$\begin{aligned} F_C &= 150 \text{ lb} & \text{Ans.} \\ F_D &= 240 \text{ lb} & \text{Ans.} \\ F_B &= 208 \text{ lb} & \text{Ans.} \end{aligned}$$

The stretch of the spring is therefore

$$\begin{aligned} F_B &= k s_{AB} \\ 208 \text{ lb} &= 500 \text{ lb/ft} (s_{AB}) \\ s_{AB} &= 0.416 \text{ ft} & \text{Ans.} \end{aligned}$$

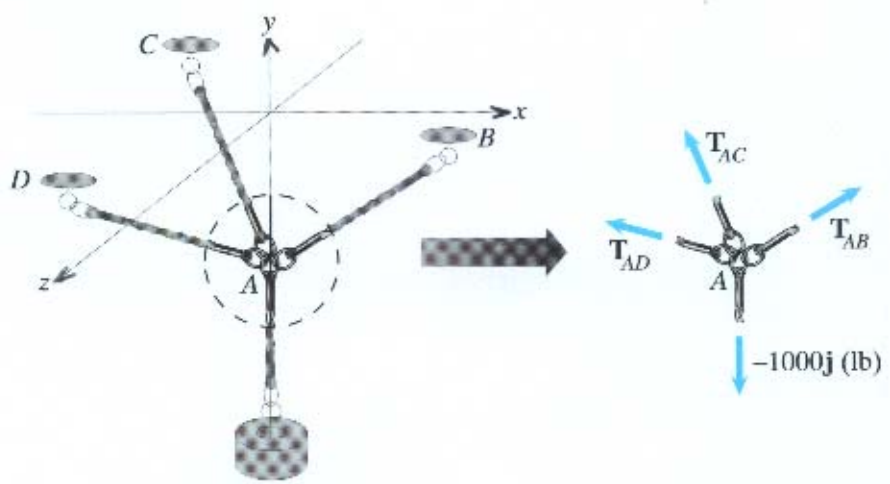
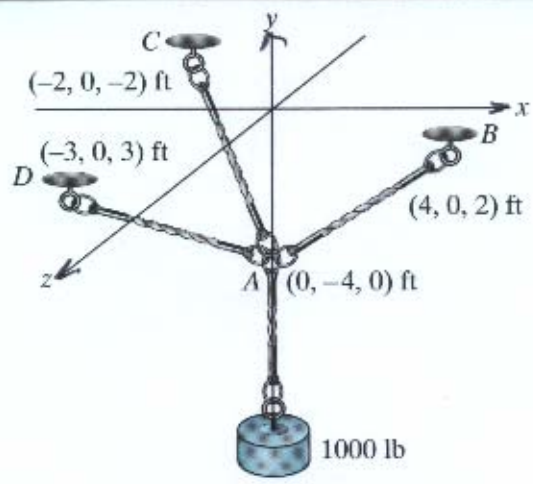
FBD

3 eq. & 3 unknowns

5. Equilibrium equations applied to free-body diagram of three-dimensional problem. (Example 3.5)

in 3-D Particle

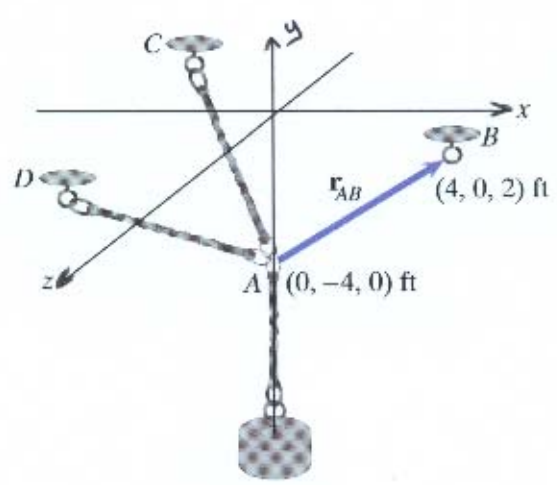
$$\begin{aligned} \sum F_x &= 0 & (1) \\ \sum F_y &= 0 & (2) \\ \sum F_z &= 0 & (3) \end{aligned}$$



FBD

(a) Isolating part of the cable system.

(b) The completed free-body diagram showing the forces exerted by the tensions in the cables.



(c) The position vector