Equilibrium of a Particle in 2-D

$\Downarrow$

$$\vec{R} = 0$$

$\Rightarrow$

$$(\sum F_x)\vec{i} + (\sum F_y)\vec{j} = 0$$

$\Rightarrow$

$$\boxed{\sum F_x = 0} \quad (1)$$

$$\boxed{\sum F_y = 0} \quad (2)$$

* Newton First Law (?!)

$\Downarrow$

- no dimensions
- dimensions not important
- "small" body

etc.

but: If all forces meet at one point then it can be considered as Particle

concurrent coplanar forces

For now consider 2-D only

$\Downarrow$

forces are coplanar

$\Downarrow$

later consider 3-D

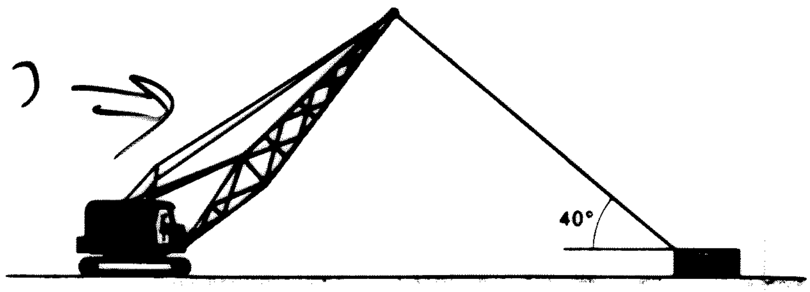
Free Body Diagram or FBD

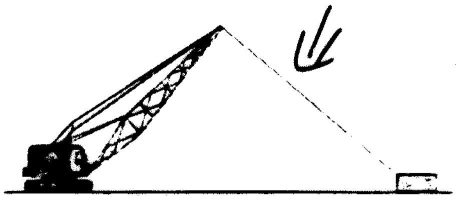
Example 3.1

The cable of the crane in Fig. 3.19 is attached to a stationary caisson of mass 300 kg. The tension in the cable is 1 kN. Determine the normal and friction forces exerted on the caisson by the ground.

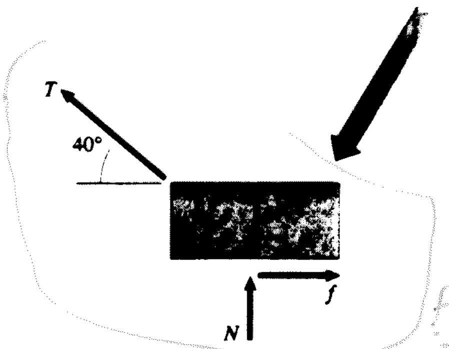
Figure 3.19

Space (physical) diagram



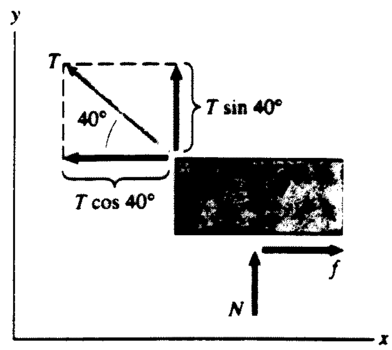


(a) Isolating the caisson.



FBD

(b) The completed free-body diagram shows the known and unknown external forces.



(c) Introducing a coordinate system and resolving T into its components.

STRATEGY

Since the caisson is in equilibrium, we can determine the normal and friction forces by drawing its free-body diagram and using Eqs. (3.3).

SOLUTION

Draw the Free-Body Diagram We isolate the caisson from its surroundings (Fig. a), then complete the free-body diagram by showing the external forces acting on it (Fig. b). The forces are the weight $W = mg = (300\text{ kg})(9.81\text{ m/s}^2) = 2943\text{ N}$, the force $T = 1000\text{ N}$ exerted by the cable, and the normal force N and friction force f exerted by the ground.

Apply the Equilibrium Equations By introducing the coordinate system shown in Fig. (c) and resolving the force exerted by the cable into x and y components, we obtain the equilibrium equations

$$\Sigma F_x = f - T\cos 40^\circ = 0,$$

$$\Sigma F_y = T\sin 40^\circ + N - W = 0.$$

The friction force is

$$f = T\cos 40^\circ = (1000)\cos 40^\circ = 766\text{ N},$$

and the normal force is

$$N = W - T\sin 40^\circ = 2943 - (1000)\sin 40^\circ = 2300\text{ N}.$$

DISCUSSION

You should always try to understand the physical interpretation of the equations you use. In this example we can see from the free-body diagram (Fig. b) that the caisson can be in equilibrium only if the friction force is balanced by the horizontal component of the force exerted by the cable: $f = T\cos 40^\circ$. We can also see that the normal force and the vertical component of the force exerted by the cable must balance the weight of the caisson: $N + T\sin 40^\circ = W$. These are the same equilibrium equations we obtained in a more formal way by setting the sums of the forces in the x and y directions equal to zero.

Example 3.2

The automobile engine block in Fig. 3.20 is suspended by a system of cables. The mass of the block is 200 kg. What are the tensions in cables AB and AC?

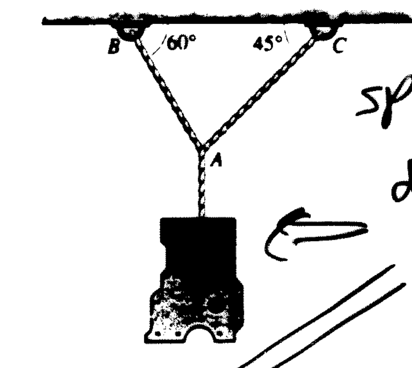


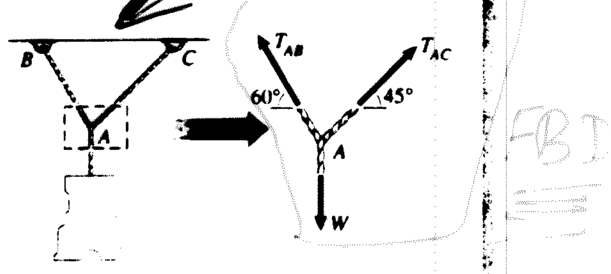
Figure 3.20

STRATEGY

We need a free-body diagram that is subjected to the forces we want to determine. By isolating part of the cable system near point A where the cables are joined, we can obtain a free-body diagram that is subjected to the weight of the block and the unknown tensions in cables AB and AC.

SOLUTION

Draw the Free-Body Diagram Isolating part of the cable system near point A (Fig. a), we obtain a free-body diagram subjected to the weight of the block $W = mg = (200\text{ kg})(9.81\text{ m/s}^2) = 1962$ N and the tensions in cables AB and AC (Fig. b).



(a) Isolating part of the cable system.

(b) The completed free-body diagram.

Apply the Equilibrium Equations We select the coordinate system shown in Fig. (c) and resolve the cable tensions into x and y components. The resulting equilibrium equations are

$$\Sigma F_x = T_{AC}\cos 45^\circ - T_{AB}\cos 60^\circ = 0,$$

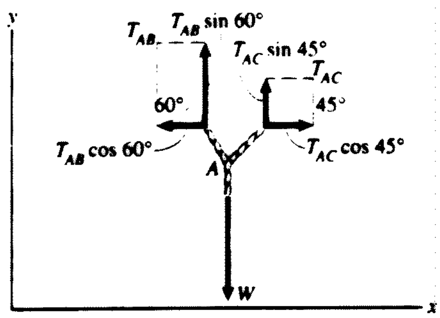
$$\Sigma F_y = T_{AC}\sin 45^\circ + T_{AB}\sin 60^\circ - 1962 = 0.$$

Solving these equations, we find that the tensions in the cables are $T_{AB} = 1436$ N and $T_{AC} = 1016$ N.

Alternative Solution. We can determine the tensions in the cables in another way that will also help you visualize the conditions for equilibrium. Since the sum of the three forces acting on our free-body diagram is zero, the vectors form a closed polygon when placed head to tail (Fig. d). You can see that the sum of the vertical components of the tensions supports the weight and that the horizontal components of the tensions must balance each other. The angle of the triangle opposite the weight W is $180^\circ - 30^\circ - 45^\circ = 105^\circ$. By applying the law of sines,

$$\frac{\sin 45^\circ}{T_{AB}} = \frac{\sin 30^\circ}{T_{AC}} = \frac{\sin 105^\circ}{1962},$$

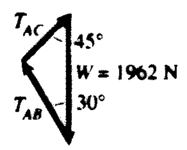
we obtain $T_{AB} = 1436$ N and $T_{AC} = 1016$ N.



(c) Selecting a coordinate system and resolving the forces into components.

DISCUSSION

How were we able to choose a free-body diagram that permitted us to determine the unknown tensions in the cables? There are no definite rules for choosing free-body diagrams. You will learn what to do in many cases from the examples we will present, but you will also encounter new situations. It may be necessary to try several free-body diagrams before finding one that provides the information you need. Remember that forces you want to determine should appear as external forces on your free-body diagram, and your objective is to obtain a number of equilibrium equations equal to the number of unknown forces.



(d) The triangle formed by the sum of the three forces.