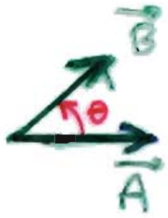
Dot Product

("Scalar " ")

$\vec{A} \cdot \vec{B} = A B \cos\theta$



Thus, the dot product is the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and of the cosine of the angle θ formed by $\vec{A}$ and $\vec{B}$.

It is
- * Commutative : $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- * Distributive : $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
- * Associative : $\vec{A} \cdot (\vec{B} \cdot \vec{C}) = ???!!$

Does <u>not</u> apply [no meaning]

Cartesian (Rectangular) Components:

$\vec{i} \cdot \vec{i} = (1)(1) \cos 0° = 1$

$\Rightarrow \vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$

$\vec{i} \cdot \vec{j} = (1)(1) \cos 90° = 0$

$\Rightarrow \vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 0$

Thus, $\vec{A} \cdot \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \cdot (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$

$= A_x B_x \underbrace{(\vec{i} \cdot \vec{i})}_{1} + A_x B_y \underbrace{(\vec{i} \cdot \vec{j})}_{0} + \cdots$

$\Rightarrow \cdots$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

<u>Two</u> applications of dot product:

① Angle formed by two vectors

$\left.\begin{aligned} \vec{A} \cdot \vec{B} &= AB \cos\theta \\ \vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z \end{aligned}\right\} \Rightarrow \cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{A_x B_x + A_y B_y + A_z B_z}{A\ B}$

$\frac{\vec{A} \cdot \vec{B}}{AB} = \vec{u}_A \cdot \vec{u}_B$

② Projection "or component" of a vector on a given axis / line

$A_{OL} = A \cos\alpha$

$= A \frac{\vec{A} \cdot \vec{OL}}{A\ OL}$

$= \frac{A}{A} \vec{A} \cdot \frac{\vec{OL}}{OL} = \vec{A} \cdot \vec{u}_{axis}$

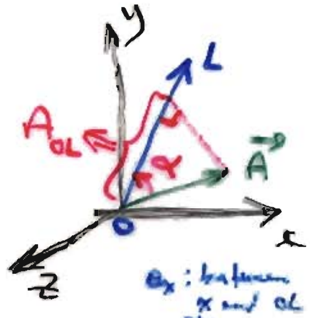
$\Rightarrow A_{OL} = A_x u_{x_{axis}} + A_y u_y + A_z u_z = A_x \cos\theta_x + A_y \cos\theta_y + A_z \cos\theta_z$

$A_{OL} = A_{\parallel}$

$A^2 = A_{\perp}^2 + A_{\parallel}^2$

$\Rightarrow$

$A_{\perp} = \sqrt{A^2 - A_{\parallel}^2}$



θ_x : between x and OL