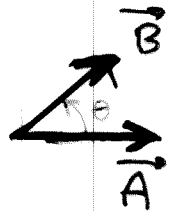


# Dot Product (Scalar Product)

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Thus, the dot product is the product of the magnitudes of  $\vec{A}$  and  $\vec{B}$  and of the cosine of the angle  $\theta$  formed by  $\vec{A}$  and  $\vec{B}$ .



- It is
- \* Commutative :  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
  - \* Distributive :  $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
  - \* Associative :  $\vec{A} \cdot (\vec{B} \cdot \vec{C}) = ?$  ( )  
Does not apply [no meaning]

## Cartesian (Rectangular) Components :

$$\vec{i} \cdot \vec{i} = (1)(1) \cos 0^\circ = 1$$

$$\Rightarrow \vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$$

$$\vec{i} \cdot \vec{j} = (1)(1) \cos 90^\circ = 0$$

$$\Rightarrow \vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 0$$

Thus, 
$$\vec{A} \cdot \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \cdot (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

$$= A_x B_x (\underbrace{\vec{i} \cdot \vec{i}}_1) + A_x B_y (\underbrace{\vec{i} \cdot \vec{j}}_0) + \dots$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

## Two applications of dot product :

① Angle formed by two vectors

$$\left. \begin{aligned} \vec{A} \cdot \vec{B} &= AB \cos \theta \\ \vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z \end{aligned} \right\} \Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{A_x B_x + A_y B_y + A_z B_z}{AB}$$

$$\Rightarrow \vec{u}_A \cdot \vec{u}_B$$

② Projection ("component") of a vector on a given axis / line

$$A_{OL} = A \cos \alpha$$

$$= A \frac{\vec{A} \cdot \vec{OL}}{A \cdot OL}$$

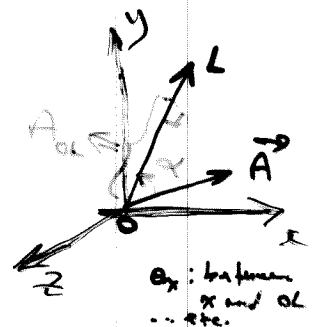
$$= \frac{A}{A} \vec{A} \cdot \frac{\vec{OL}}{OL} = \vec{A} \cdot \vec{u}_{axis}$$

$$\Rightarrow A_{OL} = A_x u_{x,axis} + A_y u_{y,axis} + A_z u_{z,axis} = A_x \cos \theta_x + A_y \cos \theta_y + A_z \cos \theta_z$$

$$A_{OL} = A_{||}$$

$$A^2 = A_x^2 + A_y^2 + A_z^2$$

$$\Rightarrow A_x = \sqrt{A^2 - A_y^2 - A_z^2}$$



$\theta_x$ : between  $x$  and  $OL$  ... etc.