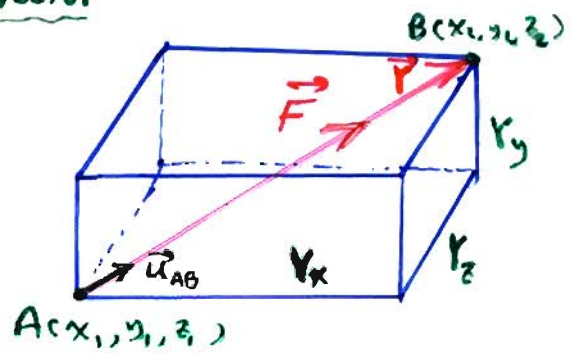
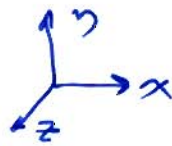


Force Directed along a Line (3-D)

$$\vec{r} = \vec{AB} = B(\dots) - A(\dots) \quad \Leftarrow \text{Position Vector}$$

$$= r_x \vec{i} + r_y \vec{j} + r_z \vec{k}$$

$$= (x_2 - x_1) \vec{i} + (y_2 - y_1) \vec{j} + (z_2 - z_1) \vec{k}$$



$$\vec{u}_r = \frac{\vec{r}}{r}$$

$$= \frac{1}{r} (r_x \vec{i} + r_y \vec{j} + r_z \vec{k})$$

$$\vec{u}_F = \frac{\vec{F}}{F}$$

Since \vec{F} and \vec{r} are on the same line,

$$\vec{u}_r = \vec{u}_F$$

$$\Rightarrow \vec{F} = F \vec{u}_F$$

$$= F \vec{u}_r$$

$$= F \left[\frac{1}{r} (r_x \vec{i} + r_y \vec{j} + r_z \vec{k}) \right] = \frac{F}{r} (r_x \vec{i} + r_y \vec{j} + r_z \vec{k})$$

Thus:

$$F_x = \frac{F}{r} r_x$$

$$= \frac{r_x}{r} F$$

$$F_y = \frac{r_y}{r} F$$

$$F_z = \frac{r_z}{r} F$$

$$\Rightarrow \cos \theta_x = \frac{F_x}{F} = \frac{\frac{r_x}{r} F}{F} = \frac{r_x}{r}$$

$$\cos \theta_y = \frac{r_y}{r}$$

$$\cos \theta_z = \frac{r_z}{r}$$

$$\vec{R} = (\sum F_x) \vec{i} + (\sum F_y) \vec{j} + (\sum F_z) \vec{k}$$

$$= R_x \vec{i} + R_y \vec{j} + R_z \vec{k}$$

For the direction of the resultant:

$$\cos \theta_x = \frac{R_x}{R} \quad ; \quad \cos \theta_y = \frac{R_y}{R} \quad ; \quad \cos \theta_z = \frac{R_z}{R}$$

* Note that graphical or trigonometric methods are NOT practical in 3-D