

Moments of Inertia "I"

(Second moment of the Area)

In the previous chapter (centroid), $\Delta W \propto \Delta A$
 $\Rightarrow \int \Delta A$ and $\int y \Delta A$ were found and calculated.

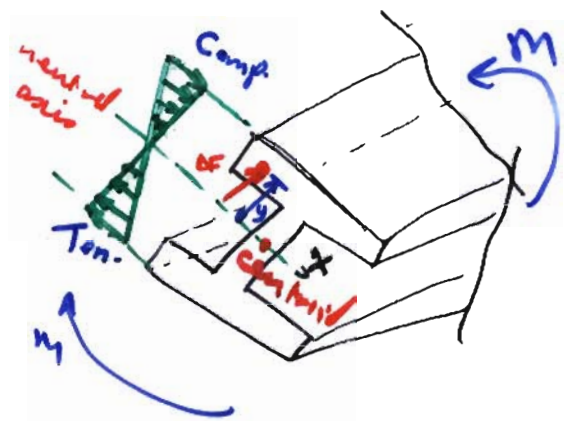
Sometimes, ΔF magnitude depends on ΔA as well as the distance from ΔA to some given axis.

$$\Delta F = ky \Delta A$$

The resultant R of the forces ΔF over the entire cross-section is

$$\begin{aligned} R &= \int \Delta F = \int ky \, dA \\ &= k \int y \, dA \end{aligned}$$

$$\begin{aligned} \int y \, dA &= \bar{y} A = \text{first moment of the cross-sectional area about the } x\text{-axis} \\ &= 0 \quad (\text{since the centroid is located on the } x\text{-axis.}) \end{aligned}$$



The moment about the x -axis:

$$\begin{aligned} \Delta M_x &= (\Delta F) y = (ky \Delta A) y \\ &= ky^2 \Delta A \end{aligned}$$

$$\begin{aligned} M_x &= \int \Delta M_x \\ &= \int_A ky^2 \, dA = R \int_A y^2 \, dA \end{aligned}$$

$\int y^2 \, dA$ = moment of inertia (or second moment of the cross-sectional area) wrt to the x -axis

$$= I_x \quad \leftarrow \text{always } (+)$$

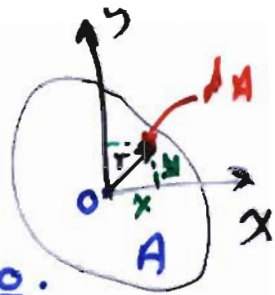
$$\left. \begin{aligned} I_x &= \int y^2 dA \\ I_y &= \int x^2 dA \end{aligned} \right\} \leftarrow \text{units} = l^4 \text{ (e.g. } m^4, \text{ in}^4, \dots \text{etc)}$$

I_x and I_y can be determined by **integration**.
This is pure math, as you can see the handout.

Polar moment of Inertia J_o :

$$J_o = \int r^2 dA$$

J_o is the polar moment of inertia of A wrt O .



$$\begin{aligned} J_o &= \int r^2 dA \\ &= \int (x^2 + y^2) dA = \int y^2 dA + \int x^2 dA \end{aligned}$$

$$\Rightarrow J_o = I_x + I_y$$

Radius of gyration k :

$$k_x = \sqrt{I_x/A}$$

$$k_y = \sqrt{I_y/A}$$

$$k_o = \sqrt{J_o/A}$$

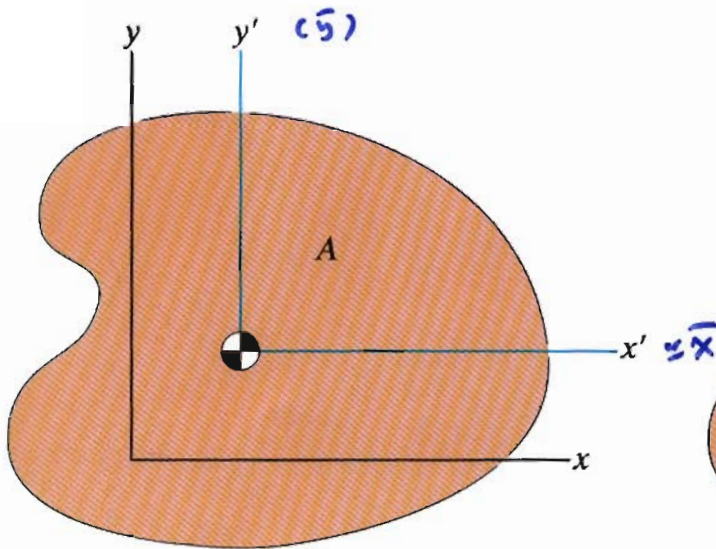
$$\Rightarrow k_o^2 = k_x^2 + k_y^2$$

35.

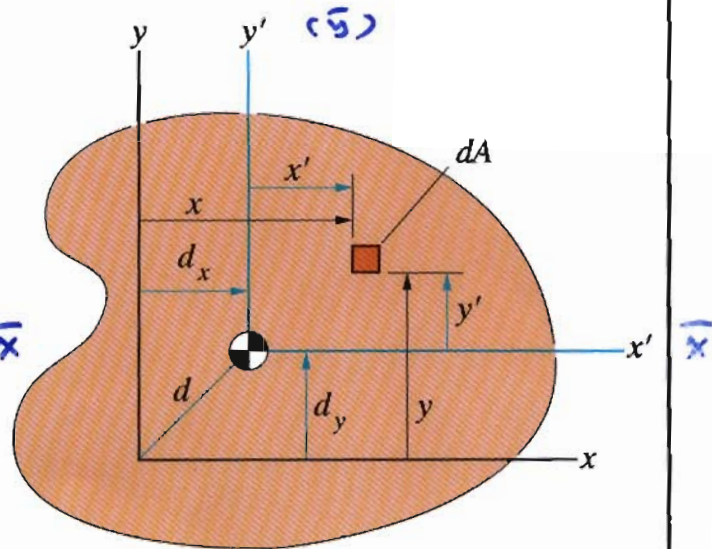
Parallel-axis theorem for the moment of inertia of an area about the x axis.

$$I_x = \int y^2 dA$$

$x' \cong \bar{x}$ is the centroidal axis.



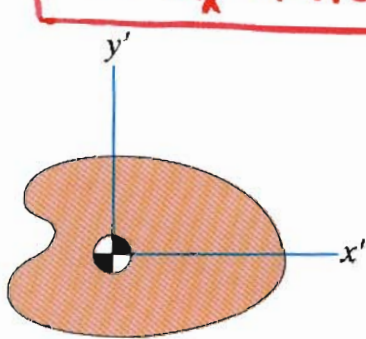
(a)



(b)

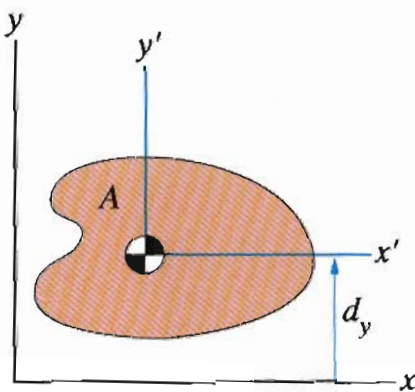
$$\begin{aligned} I_x &= \int y^2 dA = \int (y' + d_y)^2 dA = \int [(y')^2 + 2y'd_y + (d_y)^2] dA \\ &= \int y'^2 dA + \int 2d_y y' dA + \int d_y^2 dA \\ &= \int y'^2 dA + 2d_y \int y' dA + d_y^2 \int dA \end{aligned}$$

$$I_x = \bar{I}_{x'} + A d_y^2$$



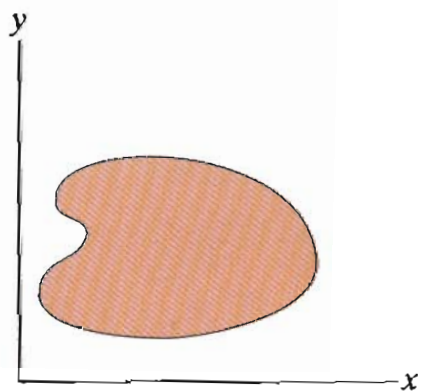
$I_{x'}$

+



$d_y^2 A$

=



I_x

Similarly:

$$I_y = \bar{I}_{y'} + A d_x^2$$

$$J_o = \bar{J}_o + A d^2$$

Moments of Inertia by the Composite Area Method

The total area A can be divided into areas $A_1, A_2, A_3, \dots, A_n$, then

$$\int y^2 dA = \int y^2 dA_1 + \int y^2 dA_2 + \dots + \int y^2 dA_n$$
$$= I_{x_{c1}} + I_{x_{c2}} + \dots + I_{x_{cn}}$$

$$= \left[(\bar{I}_{x'_{c1}} + A_1 d_{c1}^2) + \dots + (\bar{I}_{x'_{cn}} + A_n d_{cn}^2) \right]$$

$$I_x = \sum_{i=1}^n \left[\bar{I}_{x'_i} + (A d^2)_i \right]$$

also

$$\bar{I}_{x'} = \sum \left[\bar{I}_{x'_i} + (A d^2)_i \right]$$

What is the
distance?!!!

in 

See the definition
in the handout!

Similarly,

$$I_y \text{ or } \bar{I}_{y'} = \sum \left[\bar{I}_{y'_i} + (A d^2)_i \right]$$

See the table on the inside back cover of your textbook for the values of $\left\{ \begin{array}{l} \bar{I}_{x'} \text{ and } \bar{I}_{y'} \\ \text{or } I_x \text{ and } I_y \end{array} \right\}$ for common area shapes.

* Very Important:

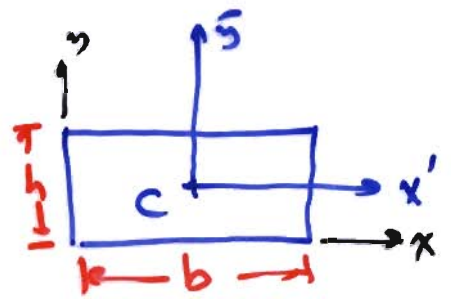
Do NOT get confused between $x'-y'$ and $x-y$!!!!

See, read, and understand the handout!!!

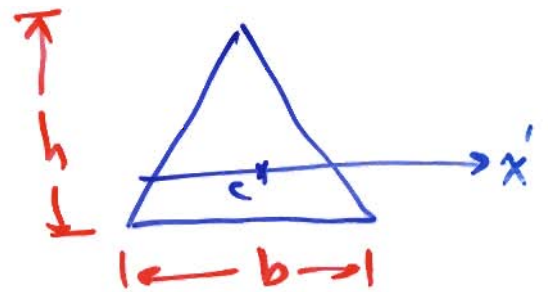
$$\bar{I}_{x'} = \frac{1}{12} b h^3$$

$$\bar{I}_{y'} = \frac{1}{12} h b^3$$

$$\left(\begin{array}{l} \bar{I}_x = \frac{1}{3} b h^3 \\ \bar{I}_y = \frac{1}{3} h b^3 \end{array} \right)$$

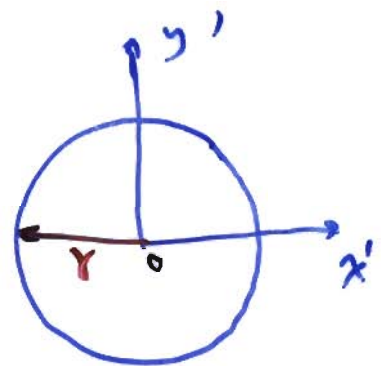


$$\bar{I}_{x'} = \frac{1}{36} b h^3$$

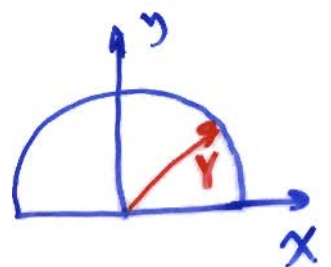


$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{\pi}{4} r^4$$

$$J_o = \frac{\pi}{2} r^4$$



$$I_x = I_y = \frac{\pi}{8} r^4$$



not centroidal axis \Rightarrow ?!!!