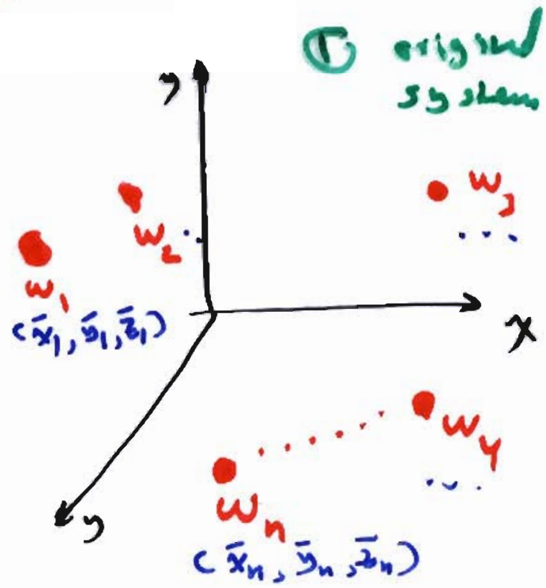
Center of Gravity

Centroid

To replace the system of particles / bodies by a simple system composed of only one body, two conditions must be satisfied

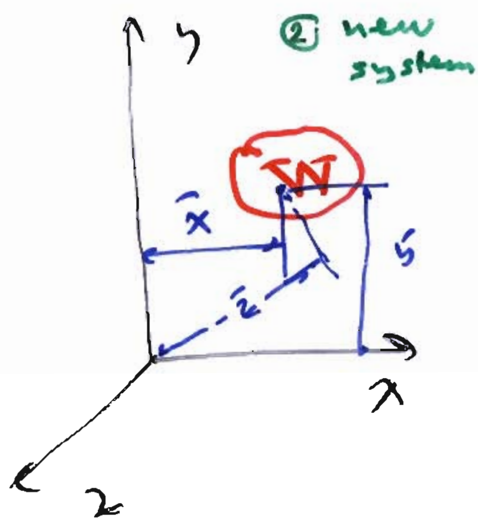


((as far as Statics is concerned)). Thus the two systems will be equivalent.

① Σ Forces in the two systems must be equal ⇒

ΣF_y :

$$W = \sum_{i=1}^{n} w_i$$



② Σ Moments in the two systems must be equal.

ΣM_x :

$$\bar{z}\, W = \sum_{i=1}^{n} \bar{z}_i\, w_i$$

$$\Rightarrow \bar{z} = \frac{\sum_{i=1}^{n} \bar{z}_i\, w_i}{\underbrace{W}_{\Sigma w_i}}$$

Similarly :

$$M_z \Rightarrow \bar{x} = \frac{\sum_{i=1}^{n} x_i\, w_i}{W}$$

If w_i gets smaller and smaller, then $w_i \rightarrow \Delta w_i \rightarrow dw_i$

$\Rightarrow W = \sum_{i=1}^{\infty} (\Delta w)_i \Rightarrow W = \int dw$; similarly, $\bar{x} = \frac{\int \tilde{x}\, dw}{\int dw}$, $\bar{y} \ldots$, $\bar{z} = \ldots$

27-b

C. for "Particles"

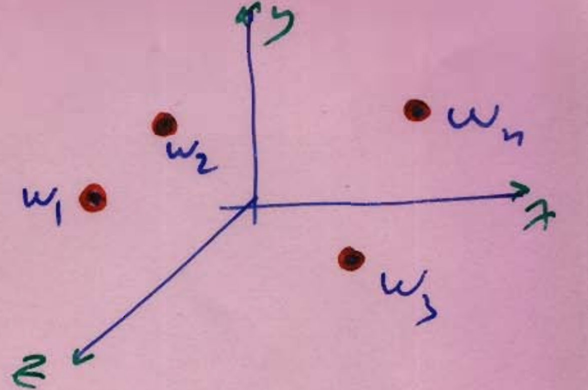
Centroid

$$\bar{x} \Sigma W_i = \sum_{i=1}^{n} \bar{x}_i W_i$$

$$\Rightarrow \quad \bar{x} = \frac{\Sigma \bar{x}_i W_i}{\Sigma W_i}$$

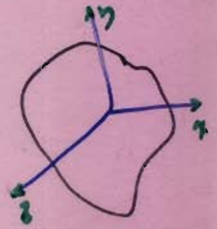
$$\bar{y} = \ldots$$

$$\bar{z} = \ldots$$



C.G. for bodies:

$$\bar{x} = \frac{\int \bar{x}\, dw}{\int dw} \; ; \quad \bar{y} = \ldots , \quad \bar{z} = \ldots$$



C.M.

$$\bar{x} = \frac{\int \bar{x}\, \rho\, dv}{\int \rho\, dv} \; ; \quad \bar{y} = \ldots , \quad \bar{z} = \ldots$$

Centroid

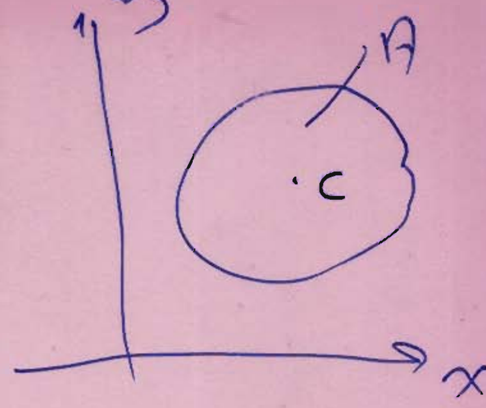
$$\bar{x} = \frac{\int_V \bar{x}\, dV}{\int_V dV} , \quad \bar{y} = \ldots , \quad \bar{z} = \ldots \Leftarrow V$$

$$\bar{x} = \frac{\int_A \bar{x}\, dA}{\int_A dA} \quad - \; - - \; — \quad \Leftarrow A$$

$$\bar{x} = \frac{\int_L \bar{x}\, dL}{\int_L dL} \quad . \; - \; - \; — \quad \Leftarrow L$$

$$\bar{x}A = \int x\,dA$$

$$\bar{y}A = \int y\,dA$$



$\int x\,dA$ = first moment of the area wrt y-axis

$\int y\,dA$ = " " " " " x-"

If the centroid of an area is located on a coordinate axis, then the first moment of the area wrt that axis is ZERO

Symmetry $\Longleftarrow$ Take Advantage of

Centroid for Composite Bodies

Volumes
Areas

We can come back to the original definition. $\Rightarrow$

use $\boxed{\Sigma\Delta}$ instead of $\boxed{\int}$. $\Rightarrow$

$$\bar{x} = \frac{\Sigma \bar{x}_i V_i}{\Sigma V_i}$$

$\vdots$

$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i}$$

$\vdots$

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