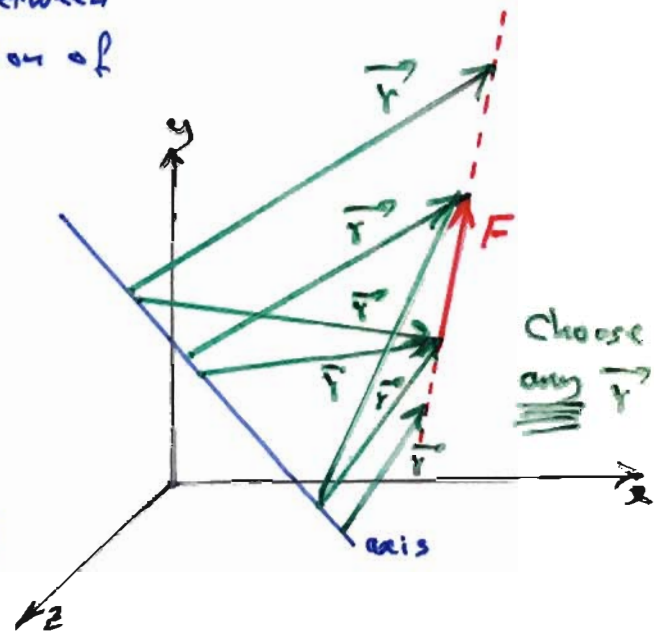


Moment about an Axis

1) Scalar Analysis:

If the perpendicular distance between the axis and the line of action of \vec{F} is known, then

$$M_{\text{axis}} = F d$$



2) Vector Analysis:

Most of the time, d is not known; thus, vector analysis is utilized.

First, let's find the moment of the force about any point on the axis, say P :

$$\Rightarrow \vec{M}_P = \vec{r} \times \vec{F}$$

This was discussed in the previous topic. Now remember that \vec{r} is the a position vector going from any point on the axis to any point on the line of action of \vec{F} .

Second, we can think of \vec{M}_P as any vector; we can find the projection (component) of \vec{M}_P on that axis using the Dot Product discussed earlier. \Rightarrow

$$M_{\text{axis}} = \vec{u}_{\text{axis}} \cdot \vec{M}_P \quad \leftarrow \text{[scalar]} \quad \leftrightarrow \text{Volume !!}$$

$$= \vec{u}_{\text{axis}} \cdot (\vec{r} \times \vec{F})$$

$$= (u_x \vec{i} + u_y \vec{j} + u_z \vec{k}) \cdot (M_x \vec{i} + M_y \vec{j} + M_z \vec{k})$$

This is called mixed triple product $[\vec{u}, \vec{r}, \vec{F}]$ with three vectors $\vec{u}, \vec{r}, \vec{F}$.

or

$$M_{\text{axis}} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

\Rightarrow In vector form:

$$\vec{M}_{\text{axis}} = M_{\text{axis}} \vec{u}_{\text{axis}}$$