King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING

CE 201 STATICS (102)

First Major Examination

Tuesday 15/3/2011
7:00 p.m. → 9:00 p.m.

Name : ..... **I.D. # :** .....

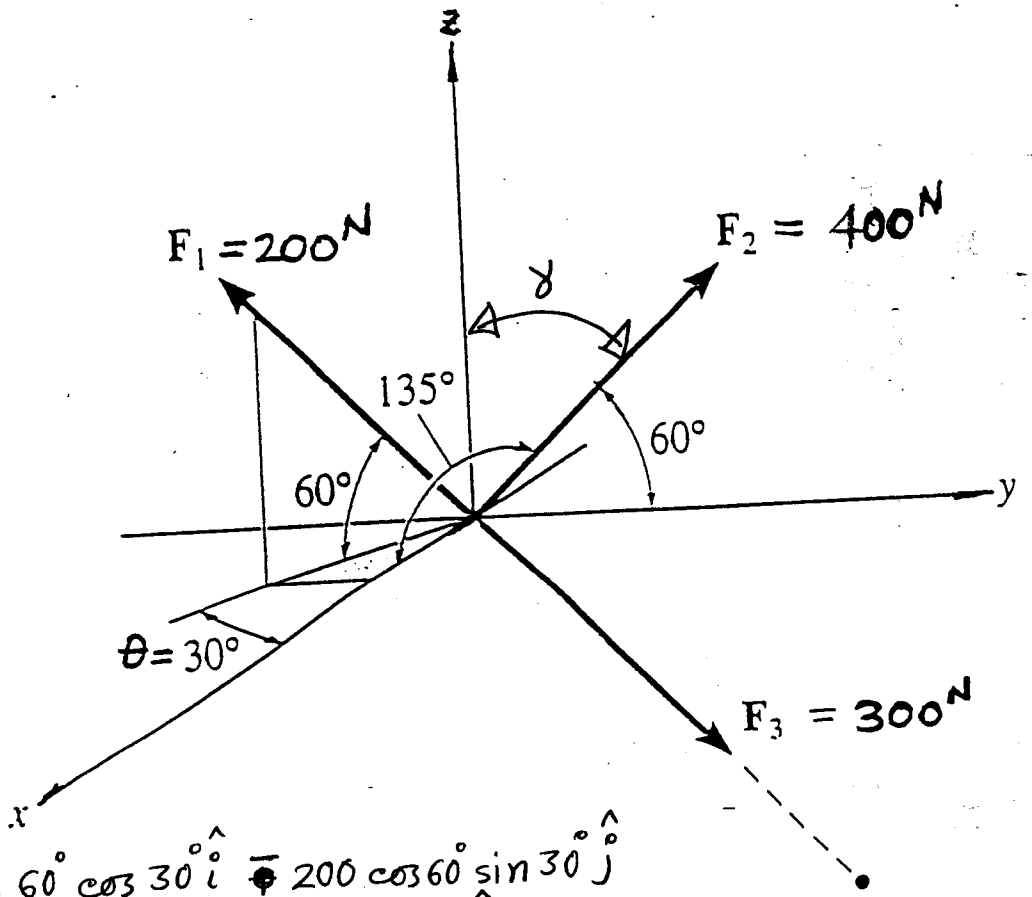
SECTION: ☐

Question	Points	Grade
1	25	
2	25	
3	25	
4	25	
TOTAL	**100**	

Good luck !

Question # 1 (25 points)

Find the resultant of the three forces shown below.



$$\bar{F}_1 = 200\cos 60^\circ \cos 30^\circ \hat{i} - 200\cos 60^\circ \sin 30^\circ \hat{j} + 200\sin 60^\circ \hat{K}$$

$$= 86.6\,\hat{i} - 50\,\hat{j} + 173\,\hat{K}$$

$$\bar{F}_2 = 400\cos 135^\circ \hat{i} + 400\cos 60^\circ \hat{j} + 400\cos\gamma\,\hat{K}$$

we know, $\cos^2 135^\circ + \cos^2 60^\circ + \cos^2\gamma = 1$ $\qquad \gamma = 60^\circ$

$\therefore \cos\gamma = \pm 0.5$, $\quad + \gamma = 120^\circ \leftarrow$ not valid

$$\therefore \bar{F}_2 = -282.8\,\hat{i} + 200\,\hat{j} + 200\,\hat{K}$$

$$\bar{r}_{F_3} = (4-0)\hat{i} + (4-0)\hat{j} + (-2-0)\hat{K}$$

$$|\bar{r}_{F_3}| = 6\text{ m}$$

$$\bar{F}_3 = 300 * \frac{\bar{r}_{F_3}}{|\bar{r}_{F_3}|} = 200\,\hat{i} + 200\,\hat{j} - 100\,\hat{K}$$

$$\therefore \bar{F}_R = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 3.8\,\hat{i} + 350\,\hat{j} + 273\,\hat{K}$$

$$|\bar{F}_R| = \sqrt{3.8^2 + 350^2 + 273^2} = 443.90\text{ N}$$ Ans.

$$\alpha = \cos^{-1}\frac{3.8}{443.9} = 89.5^\circ, \quad \beta = \cos^{-1}\frac{350}{443.9} = 37.96^\circ$$

$$\gamma = \cos^{-1}\frac{273}{443.9} = 52.05^\circ$$

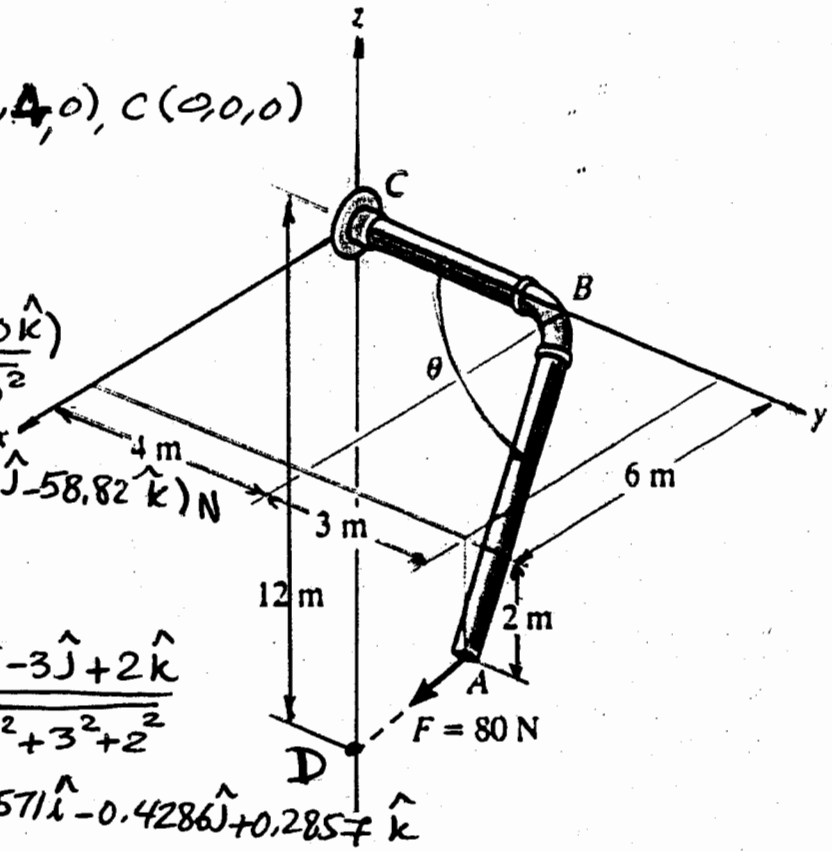
Ans.

Question # 2 (25 points)

In the pipe and force assembly shown below:

A) Determine the projection of force (F) along axis (AB).

B) Determine angle (θ) between (BA & BC).



A) Coordinates :

$A(6,7,-2)$, $B(0,4,0)$, $C(0,0,0)$

$D(0,0,-12)$

$\vec{F} = 80\,\vec{u}_{AD}$

$= 80\left(\frac{-6\hat{i}-7\hat{j}-10\hat{k}}{\sqrt{6^2+7^2+10^2}}\right)$

$= (-35.29\hat{i} - 41.17\hat{j} - 58.82\hat{k})\ N$

$F_{AB} = \vec{F} \cdot \vec{u}_{AB}$

$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{-6\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{6^2+3^2+2^2}}$

$= -0.8571\hat{i} - 0.4286\hat{j} + 0.2857\hat{k}$

$F_{AB} = (-35.29\hat{i} - 41.17\hat{j} - 58.82\hat{k}) \cdot (-0.8571\hat{i} - 0.4286\hat{j} + 0.2857\hat{k})$

$= 31.09\ N$

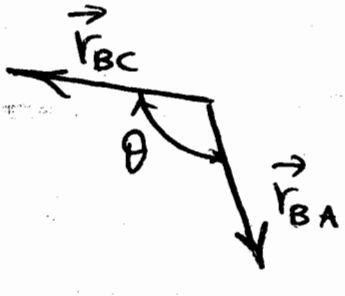
$\vec{F}_{AB} = F_{AB}\,\vec{u}_{AB} = (-26.65\hat{i} - 13.32\hat{j} + 8.88\hat{k})\ N$

B) $\theta = \cos^{-1}\left(\frac{\vec{r}_{BA} \cdot \vec{r}_{BC}}{r_{BA}\, r_{BC}}\right) = \cos^{-1}\left[\frac{(6\hat{i}+3\hat{j}-2\hat{k}) \cdot (-4\hat{j})}{(7)(4)}\right] = 115.38^\circ$

Or $\theta = \cos^{-1}(\vec{u}_{BA} \cdot \vec{u}_{BC})$

$= \cos^{-1}\left[(0.8571\hat{i} + 0.4286\hat{j} - 0.2857\hat{k}) \cdot (-\hat{j})\right]$

$= \cos^{-1}(-0.4286) = 115.38^\circ$



Question # 3 (25 points)

A block of weight (W) is suspended by a (25 cm) long cord (AC), and by two springs (AB & AD). The original length of these two springs is (22.5 cm), and their stiffness coefficients are:

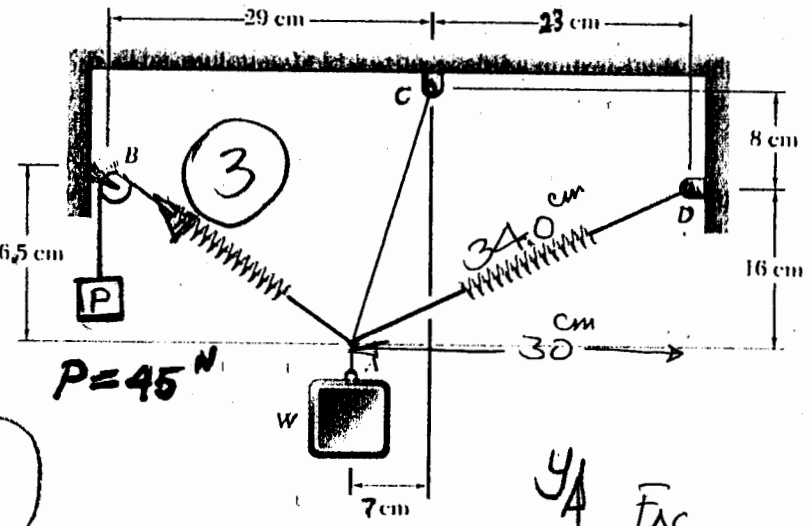
$k_{AB} = 9$ N/cm

$k_{AD} = 3$ N/cm

A) Determine the weight of the block (W).

B) Determine the tension in cord (AC).

Solution



$$L_{AD} = \sqrt{(30)^2 + (16.5)^2}$$

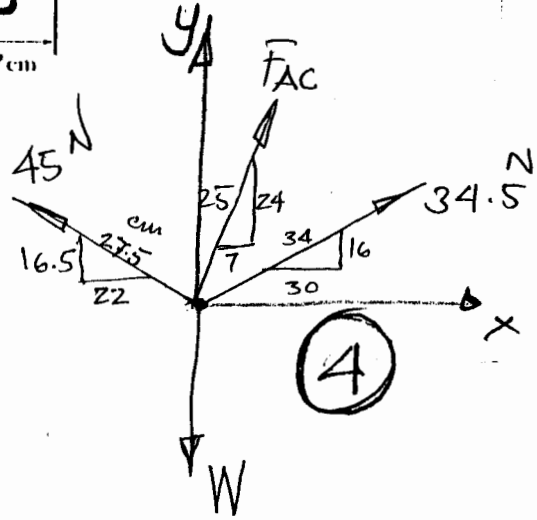
$$= 34 \text{ cm}$$

$$\Delta_{AD} = 34 - 22.5$$

$$= 11.5 \text{ cm}$$

(4)

$$F_{AD} = K_{AD}\,\Delta_{AD} = 3 \times 11.5 = 34.5 \text{ N}$$



(4)

Take joint (A) under equil.:

$\Sigma F_x = 0 \quad \rightarrow +$ (7)

$$\frac{30}{34} \times 34.5 + \frac{7}{25} F_{AC} - \frac{22}{27.5} \times 45 = 0$$

$$F_{AC} = \frac{25}{7}\left[36 - 30.44\right] = \boxed{19.85 \text{ N}}$$

$\Sigma F_y = 0 \quad +\uparrow$ (7)

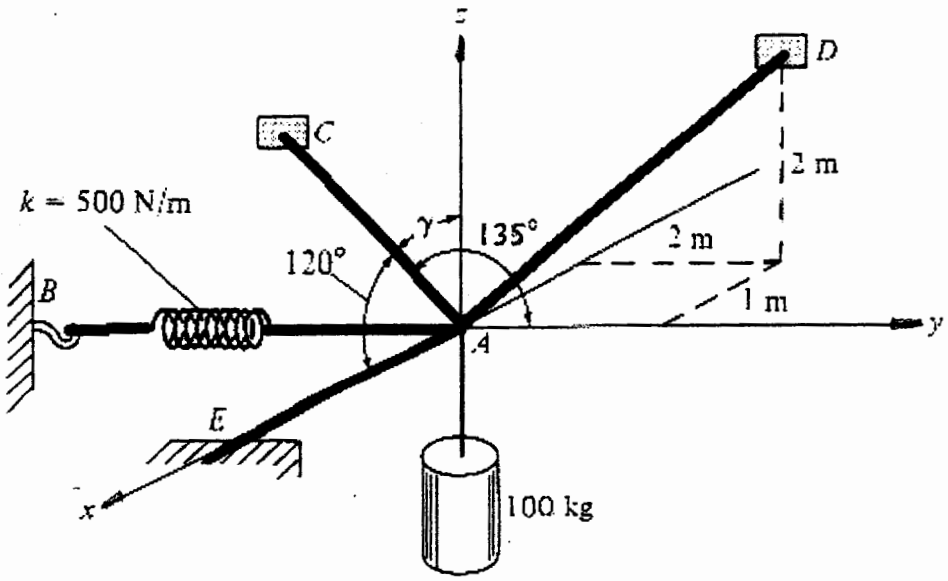
$$\frac{16}{34} \times 34.5 + \frac{24}{25} \times 19.85 + \frac{16.5}{27.5} \times 45 - W = 0$$

$$16.24 + 19.06 + 27 - W = 0$$

$$\boxed{W = 62.3 \text{ N}}$$

Question # 4 (25 points)

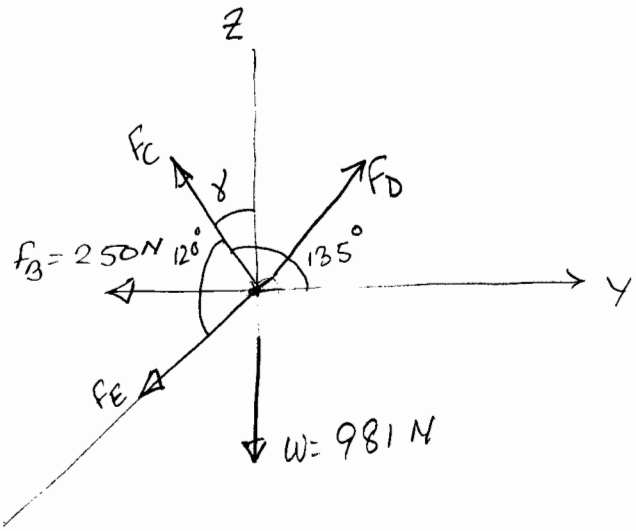
The 100 kg cylinder shown in the figure below is supported by four cables, *AB*, *AC*, *AD* and *AE*. If the spring in cable *AB* has a stiffness of $k = 500$ N/m and is stretched 0.5 m, determine the tension in each cable for equilibrium of the cable.



problem 4 :

$$F_B = 500\ N/m * 0.5\ m = 250N$$

$$W = 100\ kg = 9.81 * 100 = 981\ N$$



$$F_B = -F_B j = -250 j\ N$$

$$F_E = F_E i$$

$$W = -981\ kN$$

$$\vec{r}_D = (-1i + 2j + 2k)$$

$$|r_D| = \sqrt{(-1)^2 + 2^2 + 2^2} = 3$$

$$F_D = F_D \cdot \frac{\vec{r}_D}{|r_D|} = F_D\left(-\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k\right) = -0.333F_D i + 0.667F_D j + 0.667F_D k$$

For F_C, $\alpha = 120°$, $\beta = 135°$,

We know, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \cos^2 120° + \cos^2 135° + \cos^2\gamma = 1$$

$$\Rightarrow \cos\gamma = \pm 0.5$$

$$\therefore \gamma = 60°, 120°$$

By inspection cable AC, $\gamma = 60°$

$$\therefore F_C = F_C \cos 120° i + F_C \cos 135° j + F_C \cos 60° k$$

$$= -0.5F_C i - 0.707F_C j + 0.5F_C k$$

$$\Sigma F_x = 0 = F_E - 0.333 F_D - 0.5 F_C$$

$$\Sigma F_y = 0 = -250 + 0.667 F_D - 0.707 F_C$$

$$\Sigma F_z = 0 = -981 + 0.667 F_D + 0.5 F_C$$

Solving,

$$F_C = 605.6 \text{ N}$$

$$F_D = 1018.8 \text{ N}$$

$$F_E = 641.4 \text{ N}$$