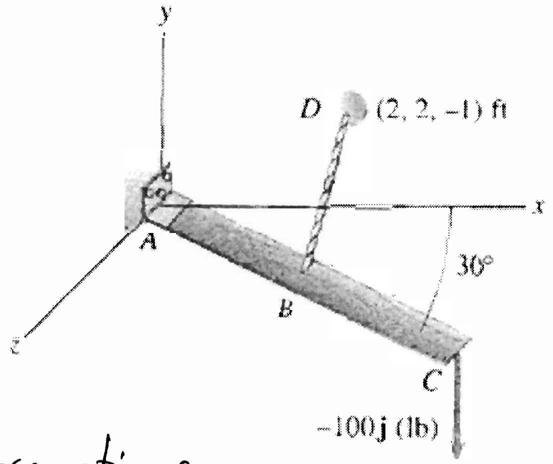
Problem 1:

Given :



The figure shown

Length of the bar = 4 ft

Required :

Tension in the cable and the reactions exerted on the bar by the hinge.

Solution:

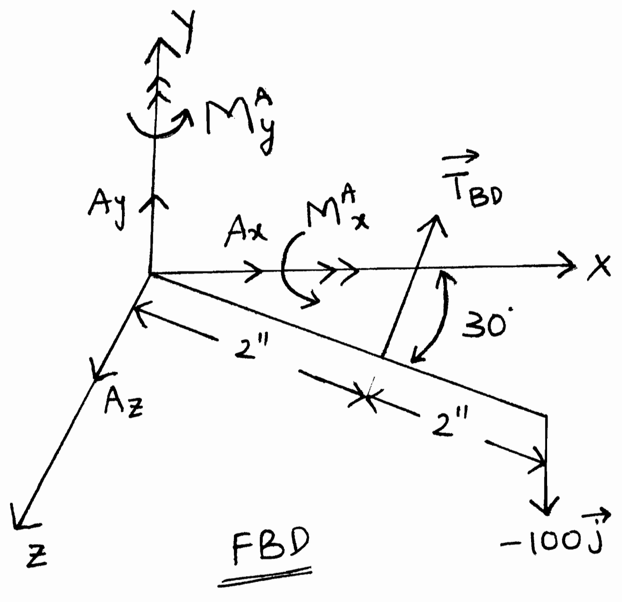
First, the FBD is drawn as shown.

Note that there are six unknowns (five at A and T_{BD}) and six equations in 3-D OK

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{M}_A = M_x^A \vec{i} + M_y^A \vec{j} + 0 \vec{k}$$

Note that there are five reactions at the hinge at A. (Why?!)



For $\vec{T}_{BD}$, we need $\vec{r}_{BD}$ first. (why?!)

A (0, 0, 0)

B (2 cos 30°, −2 sin 30°, 0)

C (4 cos 30°, −4 sin 30°, 0)

D (2, 2, −1)

$$\vec{r}_{BD} = (D) - (B)$$

$$= (2 - 2\cos 30°)\vec{i} + [2 - (-2\sin 30°)]\vec{j} + (-1 - 0)\vec{k}$$

$$= 0.26795\vec{i} + 3\vec{j} - 1\vec{k} \quad (ft)$$

$$|\vec{r}_{BD}| = 3.1736 \text{ ft}$$

$$\vec{T}_{BD} = \frac{\vec{r}_{BD}}{|\vec{r}_{BD}|} T_{BD} = (0.084431\vec{i} + 0.94530\vec{j} - 0.31510\vec{k}) T_{BD}$$

Start the statics equations by

$$\Sigma \vec{M}_A = \vec{0} \quad (\text{Why?!})$$

$$\Sigma \vec{M}_A = \vec{M}_A + \vec{M}_{T_{BD}} + \vec{M}_{-100\vec{j}}$$

$$\vec{M}_{T_{BD}} = \vec{r}_{AB} \times \vec{T}_{BD}$$

$$\vec{r}_{AB} = (B) - (A) = 1.7321\vec{i} - 1\vec{j} + 0\vec{k}$$

$$\vec{M}_{-100\vec{j}} = \vec{r}_{AC} \times (-100\vec{j})$$

$$\vec{r}_{AC} = (C) - (A) = 3.4641\vec{i} - 2\vec{j} + 0\vec{k}$$

CE 201 (091) Section 3 & 4
H.W. # 8 Solution

$$\Rightarrow \sum \vec{M}_A = \vec{0} = \left(M_x^A \vec{i} + M_y^A \vec{j}\right) + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.7321 & -1 & 0 \\ 0.084431 T_{BD} & 0.94530 T_{BD} & -0.31510 T_{BD} \end{vmatrix}$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.4641 & -2 & 0 \\ 0 & -100 & 0 \end{vmatrix}$$

$$\Rightarrow \left(M_x^A + 0.3151\, T_{BD}\right)\vec{i} + \left(M_y^A + 0.54578\, T_{BD}\right)\vec{j} + \left(1.7218 T_{BD} - 346.41\right)\vec{k} = \vec{0}$$

$$\sum M_z = 0 \Rightarrow 1.7218\, T_{BD} - 346.41 = 0$$

$$\Rightarrow \boxed{T_{BD} = 201.2 \text{ lb}}$$

$$\Rightarrow \sum M_x = 0 \Rightarrow M_x^A + 0.3151(201.2) = 0$$

$$\Rightarrow \boxed{M_x^A = -63.40 \text{ ft lb}}$$

$$\Rightarrow \sum M_y = 0 \Rightarrow M_y^A + 0.54578(201.2) = 0$$

$$\boxed{M_y^A = -109.8 \text{ ft lb}}$$

$$\sum \vec{F} = 0 \Rightarrow \vec{A} + \vec{T}_{BD} - 100\vec{j} = 0 \Rightarrow$$

$$\sum F_x = 0 \Rightarrow A_x + 0.084431(201.2) = 0 \Rightarrow \boxed{A_x = -16.99 \text{ lb}}$$

$$\sum F_y = 0 \Rightarrow A_y + 0.9453(201.2) - 100 = 0 \Rightarrow \boxed{A_y = -90.19 \text{ lb}}$$

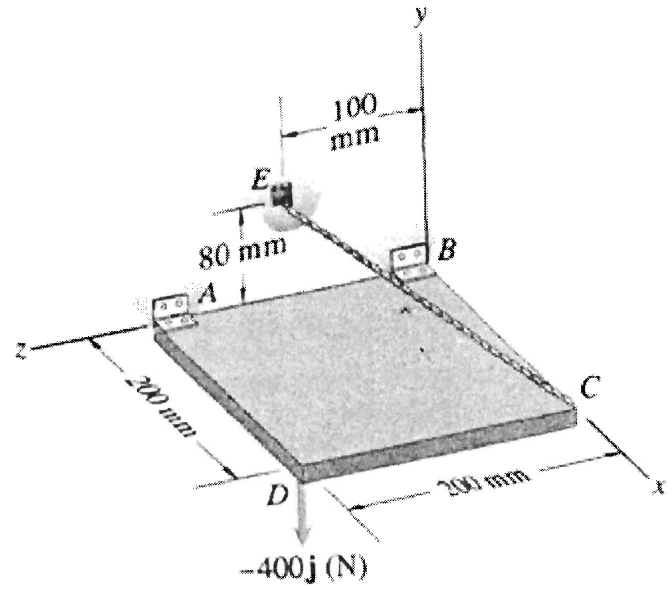
$$\sum F_z = 0 \Rightarrow A_z - 0.3151(201.2) = 0 \Rightarrow \boxed{A_z = 63.40 \text{ lb}}$$

CE 201 (091) Section 3 & 4
H.W. # 8 Solution

Problem 2:

Given:

The figure shown

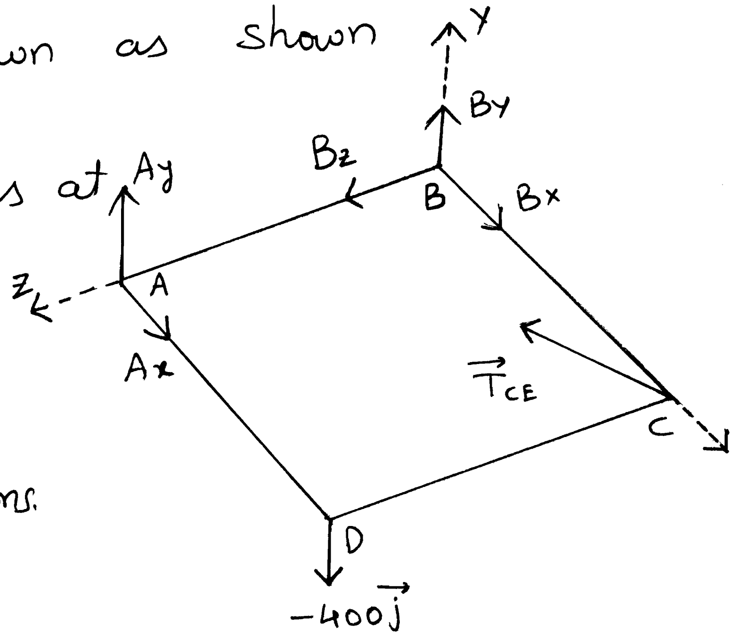


Required:

Reactions at the hinges and the tension in the cable.

Solution:

First, the FBD is drawn as shown



Note how many reactions at A and at B. (Why ?!)

Also note that there are 6 unknowns and 6 equations.
⇒ ok

Start the solution by taking $\Sigma \vec{M}_B = \vec{0}$ (Why ?!)

$$\Sigma \vec{M}_B = (\vec{r}_{BA} \times \vec{A}) + (\vec{r}_{BC} \times \vec{T}_{CE}) + (\vec{r}_{BD} \times (-400\vec{j})) = \vec{0}$$

coordinates : ((mm))

$A(0,0,200)$; $B(0,0,0)$; $C(200,0,0)$; $D(200,0,200)$;

$E(0,80,100)$

$$\vec{r}_{BA} = (A)-(B) = 0\vec{i} + 0\vec{j} + 200\vec{k} \quad (mm)$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + 0\vec{k}$$

CE 201 (091) Section 3 & 4
H.W. # 8 Solution

$$\vec{r}_{BC} = (C) - (B) = 200\vec{i} + 0\vec{j} + 0\vec{k}$$

$$\vec{T}_{CE} = \frac{\vec{CE}}{CE}\, T_{CE}$$

$$\vec{CE} = (E) - (C) = -200\vec{i} + 80\vec{j} + 100\vec{k} \Rightarrow CE = 237.487 \text{ mm}$$

$$\Rightarrow \vec{T}_{CE} = (-0.8421521\vec{i} + 0.336861\vec{j} + 0.421076\vec{k})\, T_{CE}$$

$$\vec{r}_{BD} = (D) - (B) = 200\vec{i} + 0\vec{j} + 200\vec{k}$$

$$\sum \vec{M}_B = \vec{0} = \begin{vmatrix} i & j & k \\ 0 & 0 & 200 \\ A_x & A_y & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 200 & 0 & 0 \\ -0.842152\,T_{CE} & 0.336861\,T_{CE} & 0.421076\,T_{CE} \end{vmatrix}$$

$$+ \begin{vmatrix} i & \vec{j} & \vec{k} \\ 200 & 0 & 200 \\ 0 & -400 & 0 \end{vmatrix}$$

$$\Rightarrow (-200A_y + 80{,}000)\vec{i} + (200A_x - 84.2152\,T_{CE})\vec{j} + (67.372\,T_{CE} - 80{,}000)\vec{k} = \vec{0}$$

$$\sum M_x = 0 \Rightarrow -200A_y + 80{,}000 = 0 \Rightarrow \boxed{A_y = 400 \text{ N}}$$

$$\sum M_z = 0 \Rightarrow 67.372\,T_{CE} - 80{,}000 = 0 \Rightarrow \boxed{T_{CE} = 1187 \text{ N}}$$

$$\sum M_y = 0 \Rightarrow 200A_x - 84.2152\,T_{CE} = 0 \Rightarrow \boxed{A_x = 500 \text{ N}}$$

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0 = A_x + B_x + T_{CE}^x \Rightarrow$$

$$500 + B_x + (-0.842152 \times 1187) = 0 \Rightarrow \boxed{B_x = 500 \text{ N}}$$

$$\sum F_y = 0 = A_y + B_y + T_{CE}^y - 400 \Rightarrow \boxed{B_y = -400 \text{ N}}$$

$$\sum F_z = 0 \Rightarrow B_z + T_{CE}^z = 0 \Rightarrow \boxed{B_z = -500 \text{ N}}$$

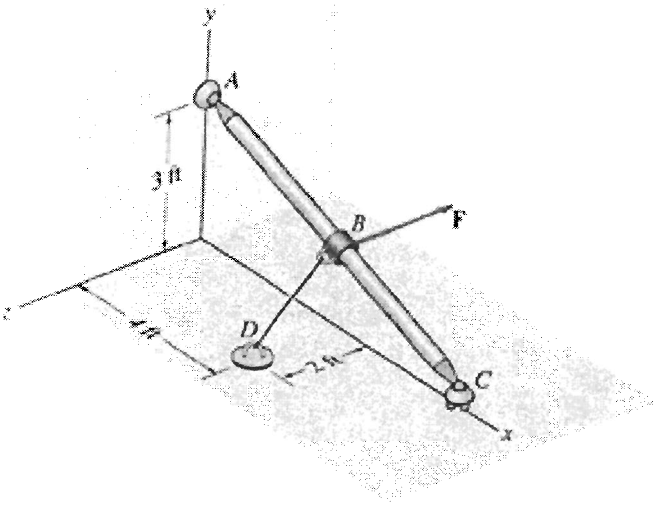
CE 201 (091) Section 3 & 4
H.W. # 8 Solution

Problem 3:

Given:

The figure shown

Force $F = -50\,k\,(lb)$

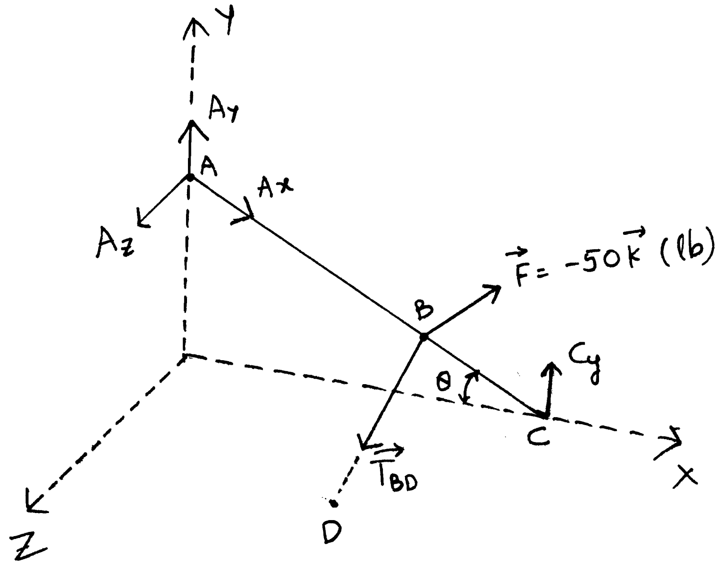


Required:

Tension in cable BD and the reactions at A and C.

Solution:

First, the FBD is drawn.



coordinates

$A(0, 3, 0)$, $B\left(\frac{8\cos\theta}{2}, \frac{3}{2}, 0\right)$

$C(8\cos\theta, 0, 0)$, $D(4, 0, 2)$

$\theta = \sin^{-1}(3/8)$

$\theta = 22.0243^\circ$

Note that $8\cos\theta = X_c = \sqrt{8^2 - 3^2}$

$= \sqrt{55} \approx 7.4162$

Thus, $B(3.7081, 1.5, 0)$; $C(7.4162, 0, 0)$

Note that there are 5 unknowns and 6 equations. We should select one equation to be "trivial" (i.e., non usable; $0 = 0$). However, we need to check and verify this "guessing".

Let's start by taking $\sum \vec{M}_A = \vec{O}$ (Why ?!)

$$\sum \vec{M}_A = 0 = \underbrace{\vec{r}_{AB} \times \vec{F} + \vec{r}_{AB} \times \vec{T}_{BD}}_{\text{we can combine them}} + \vec{r}_{AC} \times \vec{C} \Rightarrow$$

(Why & How?!)

$$\vec{r}_{AB} = (B)-(A) = 3.7081\vec{i} - 1.5\vec{j} + 0\vec{k} \quad (ft)$$

$$\vec{F} = -50\vec{k} \quad (lb)$$

$$\vec{T}_{BD} = T_{BD}\frac{\vec{r}_{BD}}{r_{BD}}$$

$$\vec{r}_{BD} = (D)-(B) = 0.2919\vec{i} - 1.5\vec{j} + 2\vec{k} \Rightarrow r_{BD} = 2.5170 \text{ ft}$$

$$\Rightarrow \vec{T}_{BD} = (0.1160\vec{i} - 0.5960\vec{j} + 0.7946\vec{k})\,T_{BD}$$

$$\vec{r}_{AC} = (C)-(A) = 7.4162\vec{i} - 3\vec{j} + 0\vec{k}$$

$$\vec{C} = 0\vec{i} + C_y\vec{j} + 0\vec{k} \Rightarrow$$

$$\sum \vec{M}_A = \vec{O} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.7081 & -1.5 & 0 \\ 0 & 0 & -50 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.7081 & -1.5 & 0 \\ 0.1160T_{BD} & -0.5960T_{BD} & 0.7946T_{BD} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7.4162 & -3 & 0 \\ 0 & C_y & 0 \end{vmatrix}$$

$$\Rightarrow (75\vec{i} + 185.4\vec{j}) + (-1.192T_{BD}\vec{i} - 2.9466T_{BD}\vec{j} - 2.036T_{BD}\vec{k}) + (7.4162C_y\vec{k}) = 0$$

$$\Rightarrow \sum M_x = 0 = 75 - 1.192T_{BD} \Rightarrow \boxed{T_{BD} = 62.92 \text{ lb}}$$

$$\sum M_y = 0 = 185.4 - 2.9466(62.92) \Rightarrow 0 = 0 \quad ((\text{trivial!}))$$

Thus we have only <u>5</u> usable equations and <u>5</u> unknowns. $\Rightarrow$ <u>OK</u>

$$\sum M_z = 0 = -2.036(62.92) + 7.4162\, C_y$$

$$\Rightarrow \boxed{C_y = 17.27 \text{ lb}}$$

Now, we take $\sum \vec{F} = \vec{0} \Rightarrow$

$$\sum F_x = 0 = A_x + 0.1160\, T_{BD} \Rightarrow \boxed{A_x = -7.29 \text{ lb}}$$

opposite of that shown in FBD

$$\sum F_y = 0 = A_y - 0.5960\, T_{BD} + C_y \Rightarrow \boxed{A_y = 20.23 \text{ lb}}$$

$$\sum F_z = 0 = A_z - 50 + 0.7946\, T_{BD} \Rightarrow \boxed{A_z = 0}$$

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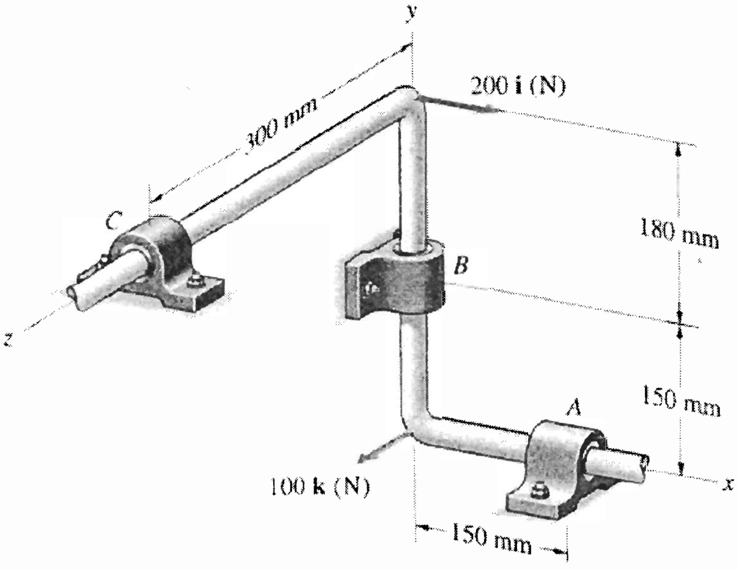
H.W. # 8 Solution

Problem 4:

Given:

The figure shown



Required:

Reactions at the bearings

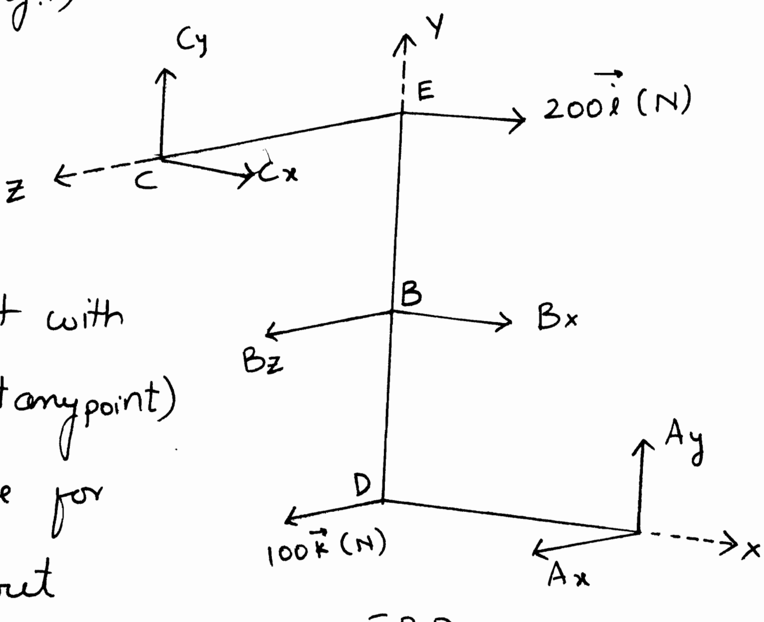
Solution:

First, the FBD is drawn.

Note that each of the bearings has 2 reactions. (Why!!)

Also note that there are 6 equations and 6 unknowns.

$\Rightarrow$ ok.



FBD

Note that we can not start with any equation (ΣF or ΣM about any point) which will enable us to solve for the unknowns directly without the need to solve a system simultaneous equations. (Try yourself!)

Lets take $\Sigma \vec{M}_A = 0$ (we can also use $\Sigma \vec{M}_B = 0$ or $\Sigma \vec{M}_C = 0$; they are of the same difficulty.) (Why?!)

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H.W. # 8 Solution

$$\sum \vec{M}_A = \vec{0} = \vec{r}_{AD} \times (100\vec{k}) + \vec{r}_{AB} \times \vec{B} + \vec{r}_{AE} \times (200\vec{i}) + \vec{r}_{AC} \times \vec{C} \Rightarrow$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0 & 0 \\ 0 & 0 & 100 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0.15 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0.33 & 0 \\ 200 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0.33 & 0.3 \\ C_x & C_y & 0 \end{vmatrix} = \vec{0}$$

$$\Rightarrow (0.15B_z - 0.3C_y)\vec{i} + (15 + 0.15B_z + 0.3C_x)\vec{j} + (-0.15B_x - 66 - 0.15C_y - 0.33C_x)\vec{k} = \vec{0}$$

Note that there are $\underline{\underline{3}}$ equations ($\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$) and $\underline{\underline{4}}$ unknowns (B_x, B_z, C_x, C_y). So we can not solve. Thus we need to supplement it by another equation.

Let's take $\Sigma F_x = B_x + C_x + 200 = 0$. Why not ΣF_y or ΣF_z?!

Solving the system of equations. (4 eqns & 4 unknowns) $\Rightarrow$

$\boxed{B_x = 750\,N}$ $\boxed{B_z = 1800\,N}$ $\boxed{C_x = -950\,N}$ opposite direction $\boxed{C_y = 900\,N}$

Now, we take $\Sigma F_y = 0 = A_y + C_y \Rightarrow \boxed{A_y = -900\,N}$ opposite direction

$\Sigma F_z = 0 \Rightarrow A_z + B_z + 100 = 0 \Rightarrow \boxed{A_z = -1900\,N}$ opposite direction

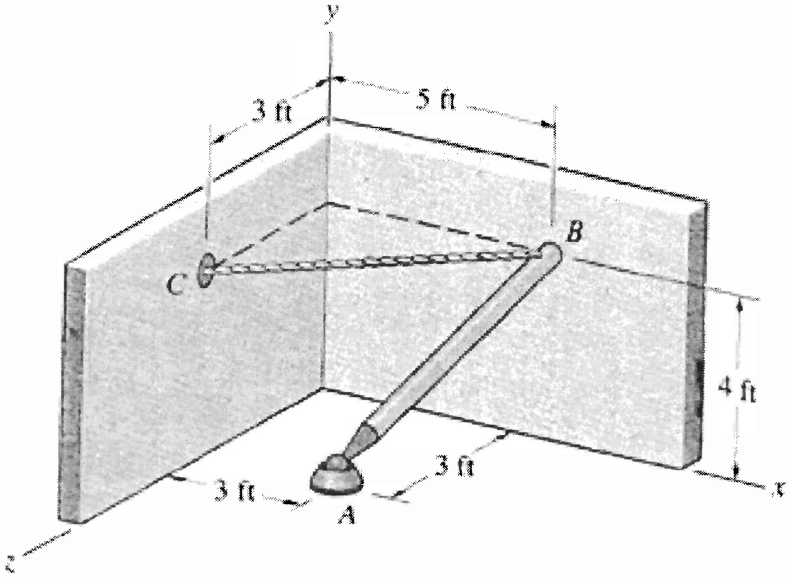
CE 201 (091) Section 3 & 4
H.W. # 8 Solution

Problem 5:

Given :

The figure shown

Weight of bar = 80 lb



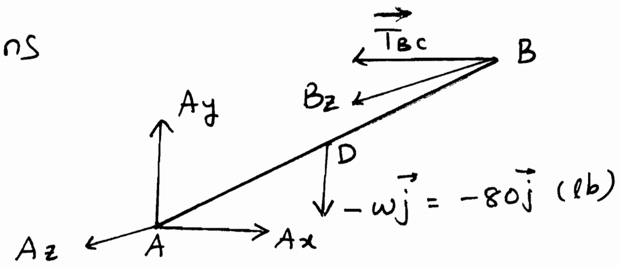
Required:

All reactions

Solution:

First, the FBD is drawn as shown.

Note that there are 5 unknowns only (and of course 6 equations). So expect one equation to be "useless".



Note B_z only. (Why ?!)

coordinates

$A(3, 0, 3)$; $B(5, 4, 0)$; $C(0, 4, 3)$; $D\left(\frac{3+5}{2}, \frac{0+4}{2}, \frac{3+0}{2}\right)$

$\Downarrow$ $D(4, 2, 1.5)$

Lets take $\Sigma \vec{M}_A = \vec{0}$ (Why ?!)

$$\Sigma \vec{M}_A = \vec{0} = \vec{r}_{AD} \times \vec{W} + \underbrace{\vec{r}_{AB} \times \vec{B} + \vec{r}_{AB} \times \vec{T}_{BC}}_{\text{we can combine them (Why?!)}}$$

CE 201 (091) Section 3 & 4
H.W. # 8 Solution

$$\vec{T}_{BC} = \frac{\vec{r}_{BC}}{r_{BC}} T_{BC}$$

$$\vec{r}_{BC} = (C) - (B)$$

$$= -5\vec{i} + 0\vec{j} + 3\vec{k} \Rightarrow r_{BC} = \sqrt{25+9} = 5.831 \text{ ft}$$

$$\Rightarrow \vec{T}_{BC} = (-0.8575\,\vec{i} + 0\vec{j} + 0.5145\,\vec{k})\,T_{BC} \Rightarrow$$

$$\sum \vec{M}_A = \vec{0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1.5 \\ 0 & -80 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & -3 \\ 0 & 0 & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & -3 \\ -0.8575T_{BC} & 0 & 0.5145\,T_{BC} \end{vmatrix}$$

$$\Rightarrow (-120 + 4B_z + 2.058\,T_{BC})\vec{i} + (-2B_z + 1.5435\,T_{BC})\vec{j} + (3.43\,T_{BC} - 80)\vec{k} = \vec{0}$$

start with $\Sigma M_z = 0$ (why ?!)

$$3.43\,T_{BC} - 80 = 0 \Rightarrow \boxed{T_{BC} = 23.32 \text{ lb}}$$

$$\Sigma M_y = 0 \Rightarrow -2B_z + 1.5435(23.32) = 0 \Rightarrow \boxed{B_z = 18.0 \text{ lb}}$$

$$\Sigma M_x = 0 \Rightarrow -120 + 4(18) + 2.058(23.32) \Rightarrow 0 = 0 \quad ((\text{trivial}))$$

$\Rightarrow \Sigma M_x = 0$ is useless $\Rightarrow$ 5 "usable" equations and 5 unknowns $\Rightarrow$ <u>ok</u>

Now, $\Sigma F_x = 0 = A_x - 0.8575(23.32) \Rightarrow \boxed{A_x = 20.0 \text{ lb}}$

$\Sigma F_y = 0 = A_y - 80 \Rightarrow A_y = 80.0 \text{ lb}$

$\Sigma F_z = 0 \Rightarrow A_z + 18 + 0.5145(23.32) = 0$

$\Rightarrow \boxed{A_z = -30.0 \text{ lb}}$ In the opposite direction.