CE 201 (091) Section 3 & 4
H.W. # 6 Solution

Problem 1:

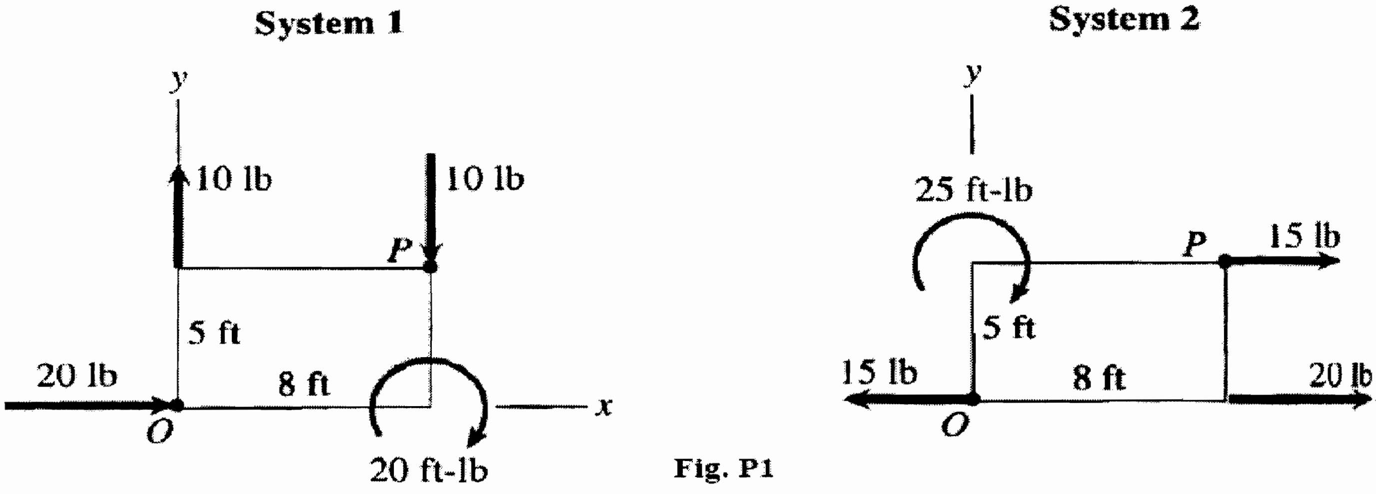

Fig. P1

Given: The figure shown

Required:

Two systems of forces and moments act on a rectangular plate are equivalent or not.

Solution:

For the two systems to be equivalent,

ΣF_1 must equal ΣF_2

and

ΣM_1 must equal ΣM_2

Check:

$$\Sigma \vec{F_1} = 20\vec{i} + 10\vec{j} - 10\vec{j} = 20\vec{i} \ (lb)$$

$$\left\{ \begin{array}{l} \text{OR, we can state it as:} \\ \Sigma F_x = 20 \qquad \Sigma F_y = +10 - 10 = 0 \end{array} \right\}$$

$$\sum \vec{F_2} = 20\vec{i} + 15\vec{i} - 15\vec{i} = 20\vec{i} \quad (lb)$$

$$\left\{ \begin{array}{l} \text{OR} \quad \sum F_x = 20 + 15 - 15 = 20\,(lb) \\ \qquad \sum F_y = 0 \end{array} \right\}$$

$$\Rightarrow \sum F_1 = \sum F_2 \Rightarrow \underline{\underline{OK}}$$

$$+\circlearrowleft \sum M_{O_1} = -20 - 10(8) = -100 \text{ ft}\cdot\text{lb}$$

[Note that the moment can be taken about <u>any</u> point; but it must be the same point for both systems.]

$$+\circlearrowleft \sum M_{O_2} = -25 - 15(5) = -100 \text{ ft}\cdot\text{lb}$$

$$\Rightarrow \sum M_1 = \sum M_2 \Rightarrow \underline{\underline{OK}}$$

$\Rightarrow$ $\boxed{\text{The two systems are equivalent.}}$

Problem 2:

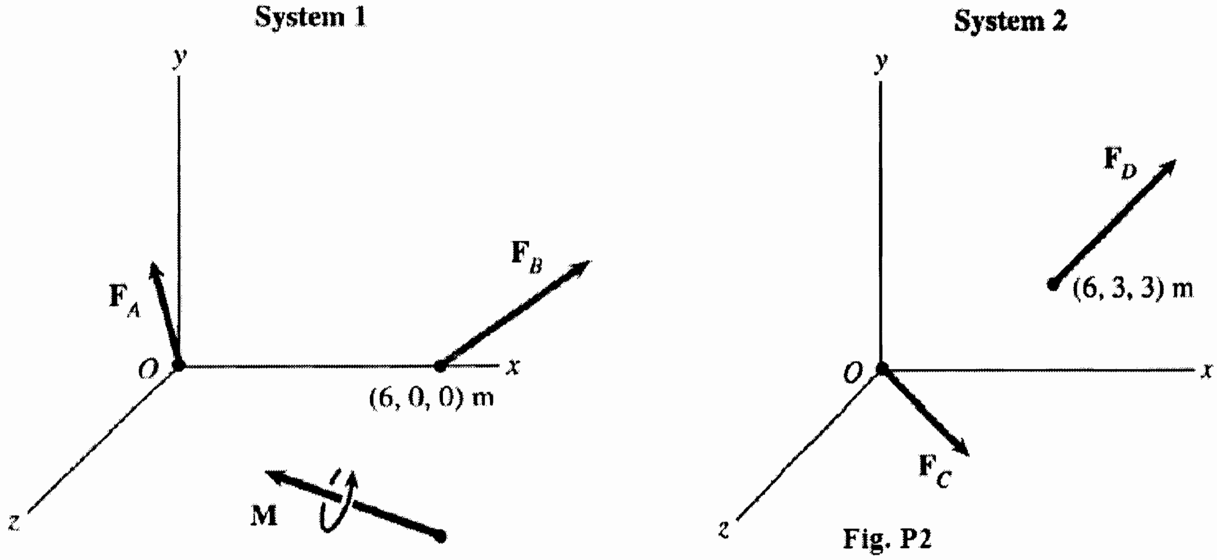

Fig. P2

Given :

The figure shown,

$F_A = -10\vec{i} + 10\vec{j} - 15\vec{k}$ (KN)

$F_B = 30\vec{i} + 5\vec{j} + 10\vec{k}$ (KN)

$M = -90\vec{i} + 150\vec{j} + 60\vec{k}$ (KN)

$F_c = 10\vec{i} - 5\vec{j} + 5\vec{k}$ (KN)

$F_D = 10\vec{i} + 20\vec{j} - 10\vec{k}$ (KN)

Required:

Two system of forces and moments are equivalent or not.

Solution:

As in Problem 1, check:

$$\Sigma \vec{F_1} \stackrel{?}{=} \Sigma \vec{F_2}$$

$$\Sigma \vec{M_1} \stackrel{?}{=} \Sigma \vec{M_2}$$

$$\Sigma \vec{F_1} = \vec{F_A} + \vec{F_B}$$

$$= (-10+30)\vec{i} + (10+5)\vec{j} + (-15+10)\vec{k}$$

$$= 20\vec{i} + 15\vec{j} - 5\vec{k} \quad \text{(kN)}$$

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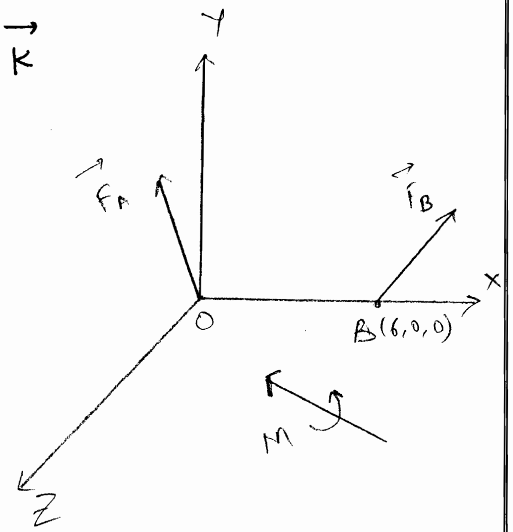
$$\sum \vec{F}_2 = \vec{F}_c + \vec{F}_D$$

$$= (10+10)\vec{i} + (-5+20)\vec{j} + (5-10)\vec{k}$$

$$= 20\vec{i} + 15\vec{j} - 5\vec{k}$$

$$= \sum \vec{F}_1 \Longrightarrow \underline{\underline{OK}}.$$



$$\sum \vec{M}_{O_1} = \vec{M} + \vec{r}_{OB} \times \vec{F}_B$$

[We chose point 'O'. Why ?!]

$$\vec{r}_{OB} = (B) - (O)$$

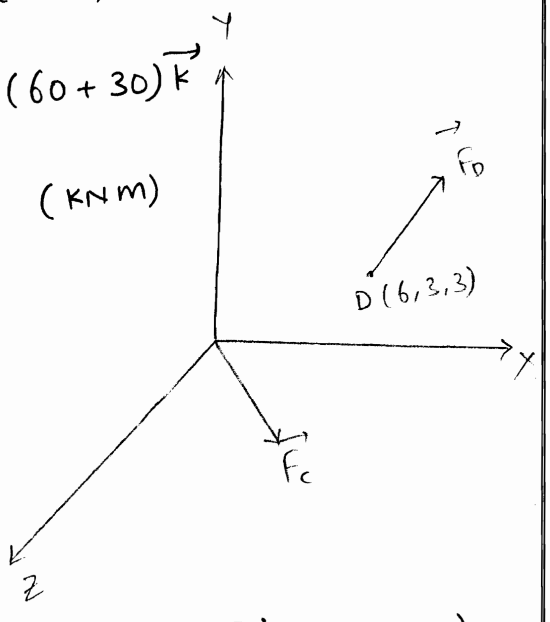
$$= 6\vec{i} + 0\vec{j} + 0\vec{k}$$

$$\vec{r}_{OB} \times \vec{F}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 0 \\ 30 & 5 & 10 \end{vmatrix}$$

$$= 0\vec{i} - 60\vec{j} + 30\vec{k} \qquad (\text{kNm})$$

$$\sum \vec{M}_{O_1} = (-90+0)\vec{i} + (150-60)\vec{j} + (60+30)\vec{k}$$

$$= -90\vec{i} + 90\vec{j} + 90\vec{k} \qquad (\text{kNm})$$



$$\sum \vec{M}_{O_2} = \vec{r}_{OD} \times \vec{F}_D$$

$$\vec{r}_{OD} = (D) - (O)$$

$$= 6\vec{i} + 3\vec{j} + 3\vec{k}$$

$$\sum \vec{M}_{O_2} = \begin{vmatrix} i & j & k \\ 6 & 3 & 3 \\ 10 & 20 & -10 \end{vmatrix} = -90\vec{i} + 90\vec{j} + 90\vec{k} \qquad (\text{kNm})$$

$$= \sum \vec{M}_{O_1}.$$

Since $\sum \vec{F}_1 = \sum \vec{F}_2$ and $\sum \vec{M}_1 = \sum \vec{M}_2$

$\boxed{\text{The two systems are equivalent}}$

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Problem 3:

Given:

The figure shown

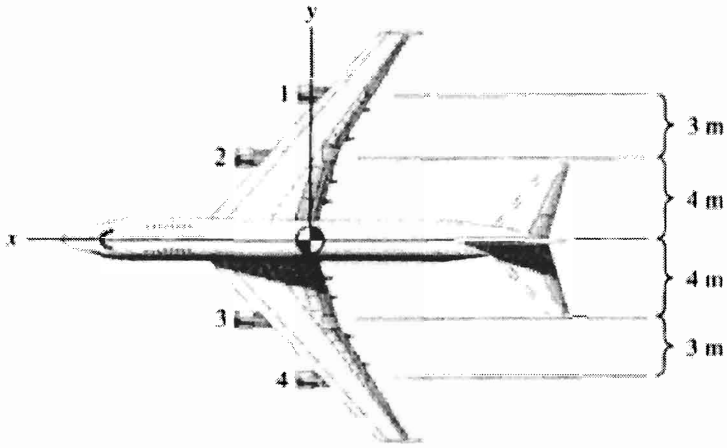


Fig. P3

Magnitudes:

engine 1 : 160 kN

engine 2 : 175 kN

engine 3 : 185 kN

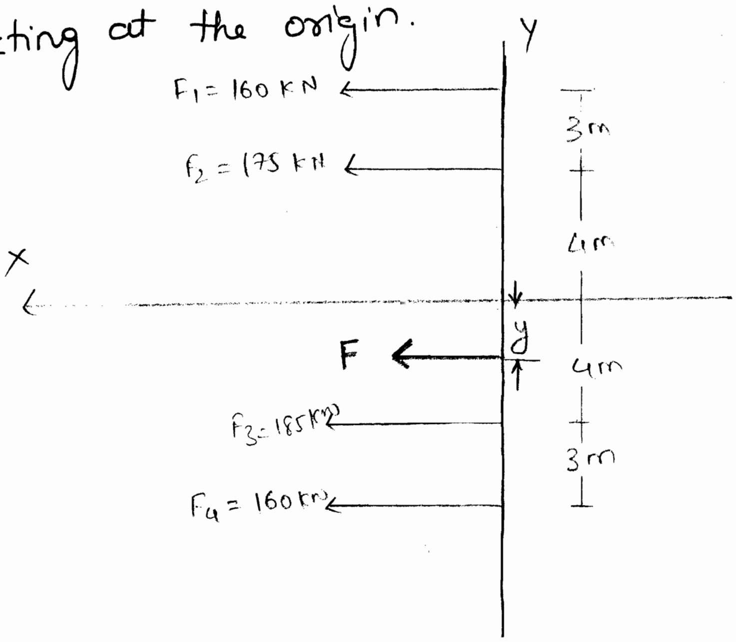
engine 4 : 160 kN

Required:

a) Force F = ? , Line of action intersect the Y-axis.

b) Necessary thrust engine 1 for four thrust forces can be represented by a force acting at the origin.

Solution:



$F = \Sigma F$

$= 160 + 175 + 185 + 160$

$\Rightarrow$ $\boxed{F = 680 \text{ kN in the x-direction}}$

For this force to be equivalent to the original system, it must have the same moment (about any point).

⇒ Let's find it, location ($\bar{y}$) to produce the Same moment.

⇒ $\overset{\curvearrowleft}{+} \sum M_o = 160(7) + 175(4) - 185(4) - 160(7)$

$= -40 \text{ kN m}$

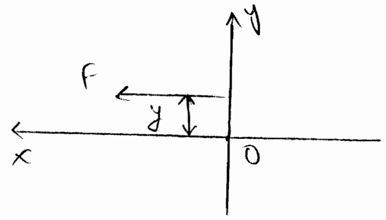
$= -F\bar{y} = -680\,\vec{y}$

⇒ $\boxed{\vec{y} = 0.0588 \text{ m} \ \underline{\text{below}} \text{ the } x\text{-axis}}$

Note: Since the location of F is not known we can assume it to be above the x-axis as shown here.

Now M_o if F is ⟲ [CCW]

Thus $-40 = F\bar{y} = 680\vec{y}$



⇒ $\vec{y} = -0.0588$ m which means it is below the x-axis

b) If the force is acting at the origin, then $\sum M_o$ of $F = 0$.

Since the two systems must be equivalent, then $\sum M_o$ of the original system $= 0$

⇒ $F_1(7) + 175(4) - 185(4) - 160(7) = 0$

⇒ $\boxed{F_1 = 165.7 \text{ kN}}$

Another Method:

We know from part (a) that $\Sigma M_o = -40$ kNm (CW)

thus, F_1 needs to increase [as it gives +(CCW) M about O] by ΔF_1 which gives this moment. $\Rightarrow$

$$\Delta F_1 (7) = 40 \Rightarrow \Delta F_1 = 5.714 \text{ kN} \Rightarrow$$

$$F_1 = F_{origin} + \Delta F_1 \Rightarrow \boxed{F_1 = 165.7 \text{ kN}}$$

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Problem 4:

Given:

The figure shown,

$w = -300\sqrt{1-0.04x^2}$ N/m

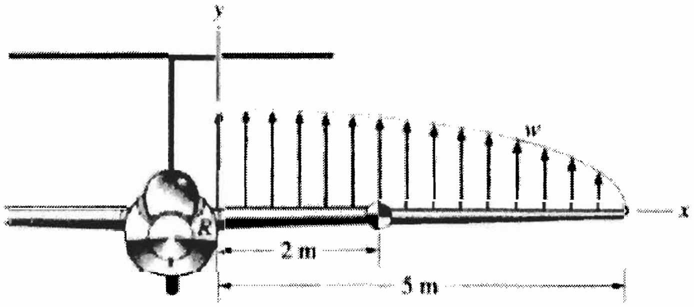
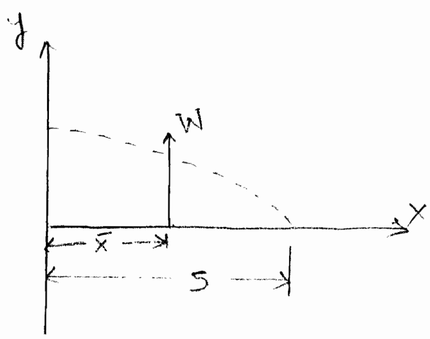


Fig. P4

Required:

Magnitude and location of the resultant of this distributed load.



Solution:

$$w = -300\sqrt{1-0.04x^2}$$

$$R = \int w = W \Rightarrow$$

$$W = \int_0^5 -300\sqrt{1-0.04x^2}\, dx$$

$$= \int_0^5 -300\sqrt{\frac{25(1-0.04x^2)}{25}}\, dx$$

$$= -60\int_0^5 \sqrt{25-x^2}\, dx$$

$$= -60\left[\frac{x\sqrt{25-x^2}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= -375\pi$$

$$\Rightarrow \boxed{W = -1178 \text{ N}}$$

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Note: We may use ⊕ sign for ω and W if we consider forces "up" as ⊕.

$$m_W \equiv m_\omega$$

$$\bar{x} W = \int_0^5 (300\sqrt{1-0.04x^2})(x)\, dx$$

$$= \int_0^5 \frac{300}{25}\sqrt{25-x^2}\; x\, dx$$

$$= -60\left[\frac{\sqrt{(25-x^2)^3}}{3}\right]_0^5$$

$$= \frac{60}{3}\sqrt{(25)^3} = 2500$$

$$\Rightarrow \bar{x} = \frac{2500}{W} = \frac{2500}{1178.1}$$

$$\Rightarrow \boxed{\bar{x} = 2.122 \text{ m}}$$

Reasonable answer?!

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Problem 5:

Given:

The figure shown

Required:

Equivalent Force, couple moment acting at point B.

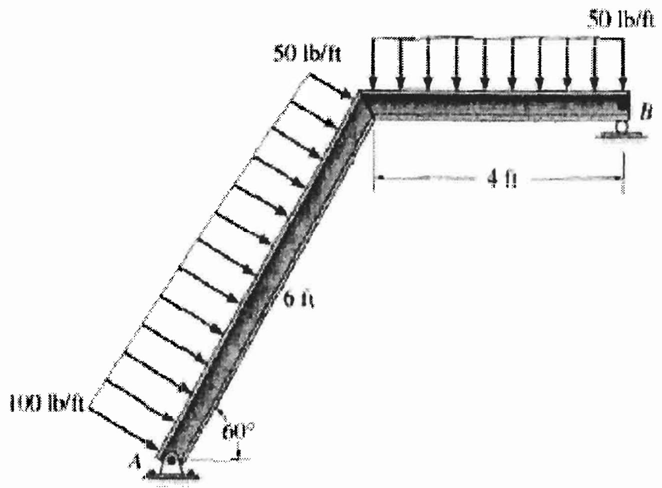
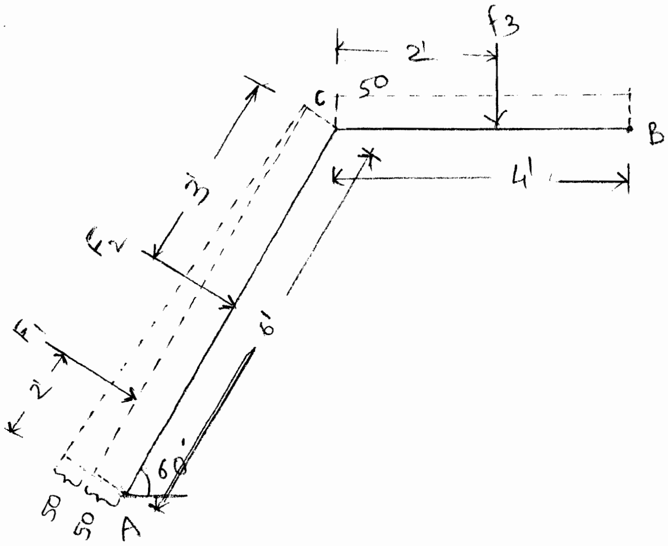


Fig. P5

Solution:

The distributed loads are replaced by three equivalent, concentrated forces at the locations shown.



$F_1 = \frac{1}{2}(6)(100-50) = 150$ lb

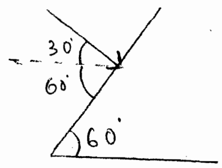
@ 2ft from A (why?!)

$F_2 = 6(50) = 300$ lb

@ 3ft from A (or C) (Why?!)

$F_3 = 4(50) = 200$ lb

@ 2ft from B (or C) (Why?!)



$F_x = F_1 \cos 30° + F_2 \cos 30°$

$= 150 \overset{\text{or } \sin 60}{\overline{\cos 30°}} + 300 \cos 30°$

$= 389.711$ lb $(\rightarrow)$

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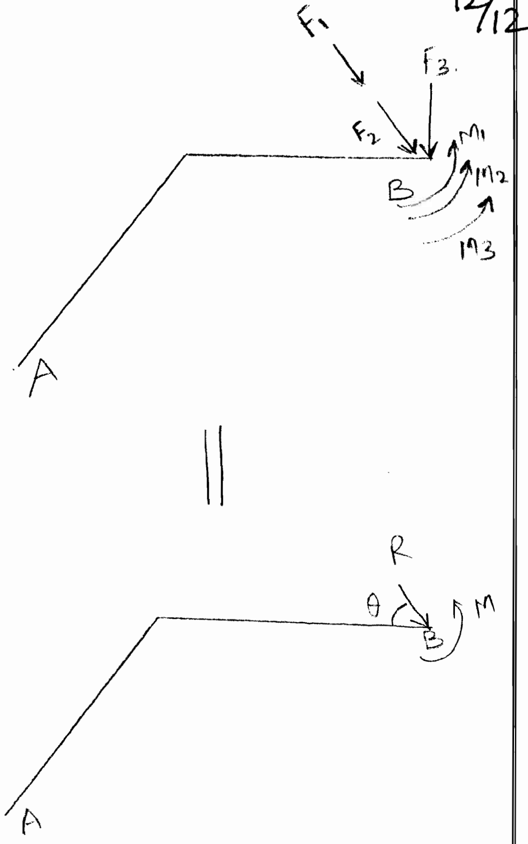

$$\Sigma F_y = 150 \underbrace{\sin 30}_{\text{or } \cos 60} + 300 \sin 30 + 200$$

$$= 425 \text{ lb } (\downarrow)$$

$$R = \sqrt{(\Sigma F_y)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(389.711)^2 + (425)^2}$$

$$\Rightarrow \boxed{R = 576.6 \text{ lb}} \searrow \text{As shown}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{R_y}{R_x} = \frac{425}{389.711}$$

$$\Rightarrow \boxed{\theta = 47.48^\circ} \text{ As shown}$$

$$+\circlearrowleft \Sigma M_B = (150 \cos 30)(6-2)\sin 60$$
$$+ (150 \sin 30)(4 \cos 60 + 4)$$
$$+ (300 \cos 30)(3 \sin 60)$$
$$+ (300 \sin 30)(3 \cos 60 + 4)$$
$$+ 200(2)$$

$$\Rightarrow \boxed{M = 2{,}800 \text{ ft} \cdot \text{lb}} \circlearrowleft \text{ as shown}$$