Problem 1:

GIVEN :

Force F = 50lb
The figure shown

REQUIRED:
Moment about x-axis using
a) Scalar analysis b) Vector analysis
compare answers & comment

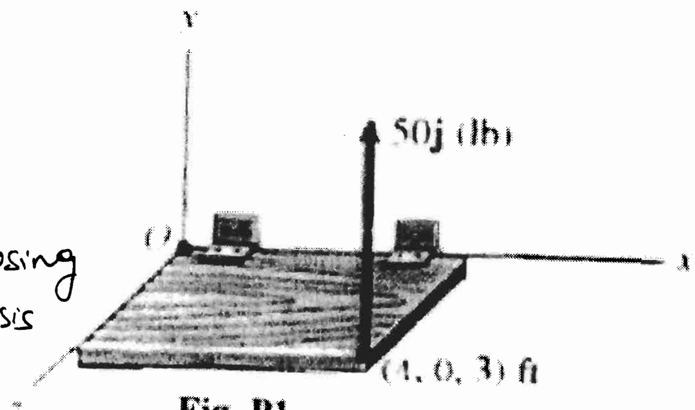


Fig. P1

SOLUTION :

a) Scalar Analysis:

F = 50 lb and it is in the Y- direction

$M = Fd \Rightarrow$

$M_x = F_y d_z$

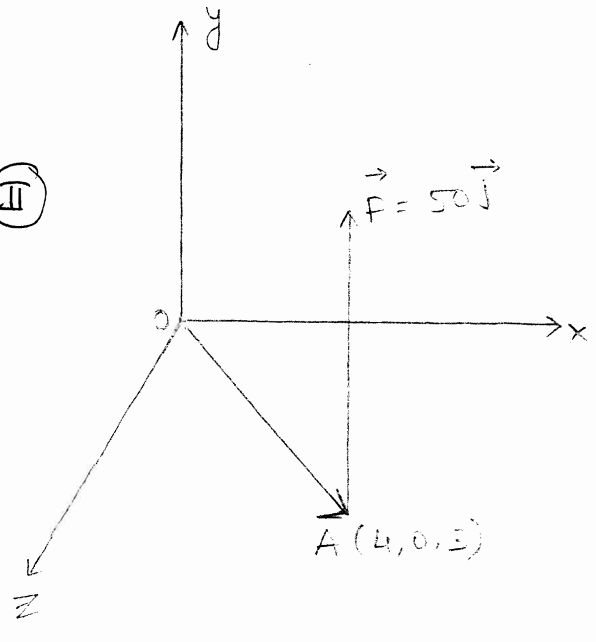
$= 50(3) \Rightarrow$

negative

$$\boxed{\begin{aligned} M_x &= 150 \text{ ft}\cdot\text{lb } \underline{cw} \\ &= -150\vec{i} \text{ ft}\cdot\text{lb} \end{aligned}}$$

b) Vector Analysis: Method II

$\vec{M} = \vec{r} \times \vec{F}$




$\vec{r}$ as $\vec{OA}$

$\vec{OA} = 4\vec{i} + 0\vec{j} + 3\vec{k}$

$\vec{F} = 0\vec{i} + 50\vec{j} + 0\vec{k}$

$$\Rightarrow \vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 3 \\ 0 & 50 & 0 \end{vmatrix}$$

$\Rightarrow \vec{M}_x = [(0) - 3(50)]\vec{i}$

$\Rightarrow \boxed{\vec{M}_x = -150\vec{i} \text{ ft. lb}}$

The two answers are the same; but in this particular problem Scalar Analysis is easier.

Vector Analysis : Method (II)

$M_{axis} = \vec{M}_{point} \cdot \vec{u}_{axis}$

$= (\vec{r} \times \vec{F}) \cdot \vec{u}$

$= \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

$\vec{u} = 1\vec{i} + 0\vec{j} + 0\vec{k}$ (Because need M about the x-axis)

$\vec{r} = \vec{OA} = 4\vec{i} + 0\vec{j} + 3\vec{k}$

$\Rightarrow M_x = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 0 & 3 \\ 0 & 50 & 0 \end{vmatrix}$

$= 1[(0) - (3)(50)] + 0 + 0$

$\Rightarrow \boxed{M_x = -150 \text{ ft. lb}}$

Problem 2:

Given:
The figure Shown

Required:
Moment of force about bar BC using
a) Vector BA b) Vector CA
Compare Answers & Comment

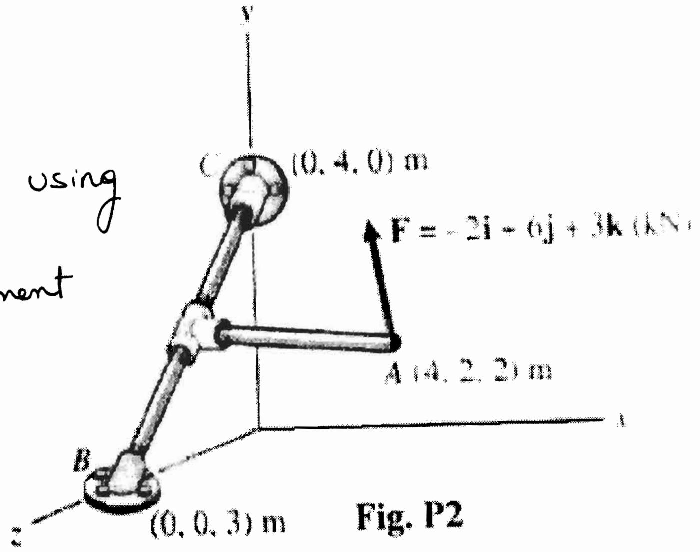



Fig. P2

Solution:

$$M_{axis} = \vec{M}_{point} \cdot \vec{U}_{axis}$$

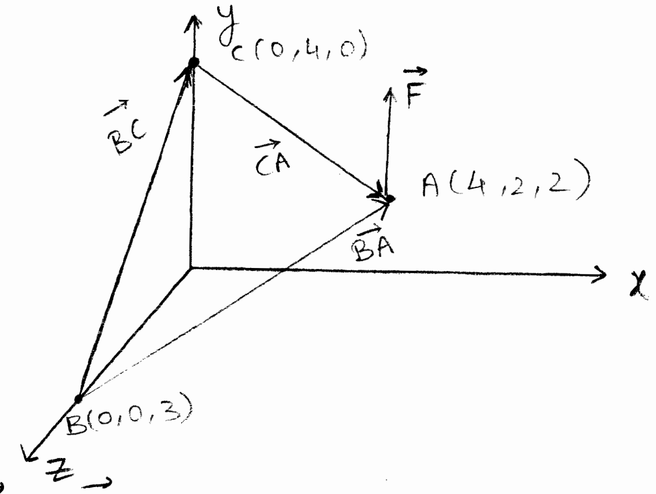
$$\vec{M} = \vec{r} \times \vec{F} \Rightarrow$$

$$M_{axis} = (\vec{r} \times \vec{F}) \cdot \vec{u}$$

$$\vec{F} = -2\vec{i} + 6\vec{j} + 3\vec{k} \ (kN)$$

$\vec{r}$ could be $\vec{BA}$ or $\vec{CA}$

$$\vec{u} = \vec{u}_{BC}$$




a) $\vec{r} = \vec{BA}$

$$\vec{BA} = (A)-(B) = 4\vec{i} + 2\vec{j} - 1\vec{k}$$

$$\vec{BC} = (C)-(B) = 0\vec{i} + 4\vec{j} - 3\vec{k} \Rightarrow BC = 5\ m$$

$$\Rightarrow \vec{u}_{BC} = \frac{\vec{BC}}{BC} = 0\vec{i} + 0.8\vec{j} - 0.6\vec{k}$$

$$\Rightarrow M_{BC} = \begin{vmatrix} u_x^{BC} & u_y^{BC} & u_z^{BC} \\ BA_x & BA_y & BA_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\Rightarrow M_{BC} = \begin{vmatrix} 0 & 0.8 & -0.6 \\ 4 & 2 & -1 \\ -2 & 6 & 3 \end{vmatrix}$$

$$= 0 - (0.8)[(4)(3) - (-1)(-2)] + (-0.6)[(4)(6) - (2)(-2)]$$

$$= 0 - 8 - 16.8$$

$$\Rightarrow \boxed{M_{BC} = -24.8 \text{ kN}\cdot\text{m}}$$

b) $\vec{r} = \vec{CA}$

$$\vec{CA} = (A)\cdot(C) = 4\vec{i} - 2\vec{j} + 2\vec{k}$$

$$M_{BC} = \begin{vmatrix} 0 & 0.8 & -0.6 \\ 4 & -2 & 2 \\ -2 & 6 & 3 \end{vmatrix}$$

$$= 0 - (0.8)[(4)(3) - (2)(-2)] + (-0.6)[(4)(6) - (-2)(-2)]$$

$$= 0 - 12.8 - 12$$

$$\Rightarrow \boxed{M_{BC} = -24.8 \text{ kN}\cdot\text{m}}$$

The two answers of (a) & (b) are the same, and no difference in the effort as no one is easier than the other.

Problem 3:

Given:

Tension in cable AB = 2 KN

Required:

magnitude of moment about the shaft CD due to force exerted by cable at A

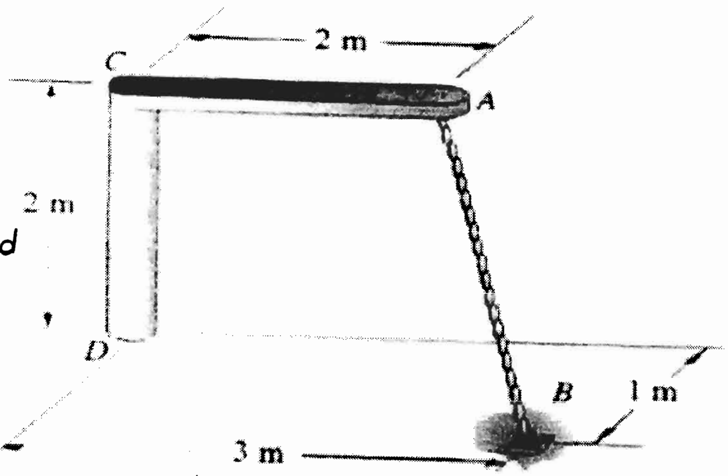



Fig. P3

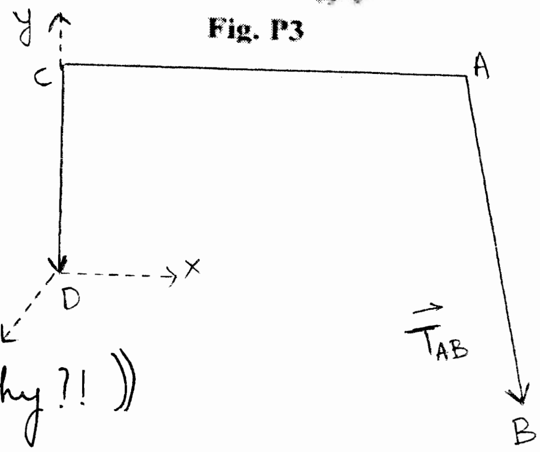
Solution:

$M_{axis} = \vec{u}_{CD} \cdot (\vec{r} \times \vec{F})$

coordinates:

$A(2,2,0)$ ; $B(3,0,1)$

$C(0,2,0)$ ; $D(0,0,0)$




$\vec{u}_{CD} = -\vec{j}$

(( By inspection! How & why?! ))

$\vec{r}$ could be $\vec{CA}$, $\vec{CB}$, $\vec{DA}$ or $\vec{DB}$ (Why?!)

$\vec{CA}$ will be chosen. (Why?!)

$\vec{CA} = (A)-(C) = 2\vec{i} + 0\vec{j} + 0\vec{k}$

$\vec{AB} = (\vec{B})-(A) = 1\vec{i} - 2\vec{j} + 1\vec{k}$

$\Rightarrow AB = \sqrt{6} \approx 2.44949 \text{ m}$

$\vec{T}_{AB} = T_{AB}\,\vec{u}_{AB} = T_{AB}\,\dfrac{\vec{AB}}{AB}$

$= 0.81650\,\vec{i} - 1.63300\,\vec{j} + 0.81650\,\vec{k}$ (KN)

$$M_{CD} = \vec{U}_{CD} \cdot (\vec{CA} \times T_{AB})$$

$$= \begin{vmatrix} U_{CD}^{x} & U_{CD}^{y} & U_{CD}^{z} \\ CA_x & CA_y & CA_z \\ T_{AB}^{x} & T_{AB}^{y} & T_{AB}^{z} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -1 & 0 \\ 2 & 0 & 0 \\ 0.81650 & -1.6330 & 0.81650 \end{vmatrix}$$

$$= 0 - (-1)[2(0.81650) - 0] + 0$$

$$\Rightarrow \boxed{M_{CD} = 1.633 \text{ kN m}}$$

We can express it in a vector form as

$$\vec{M}_{CD} = M_{CD} \vec{U}_{CD}$$

$$= 1.633(0\vec{i} - 1\vec{j} + 0\vec{k})$$

$$= -1.633\vec{j} \quad (\text{kNm})$$

Problem 4:

Given:
The figure shown

Required:
Sum of the moments exerted on object

Solution:

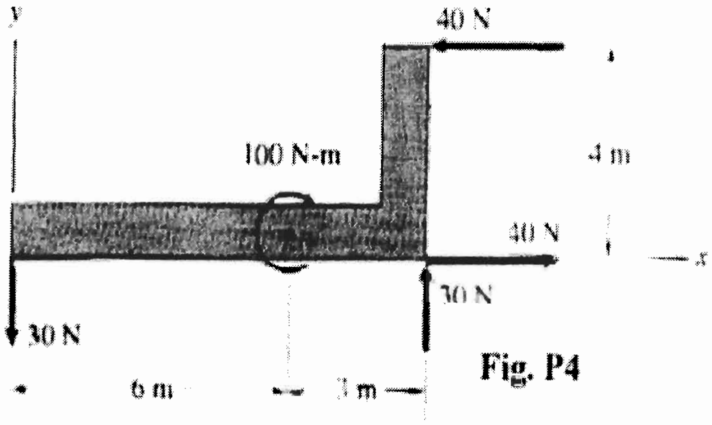
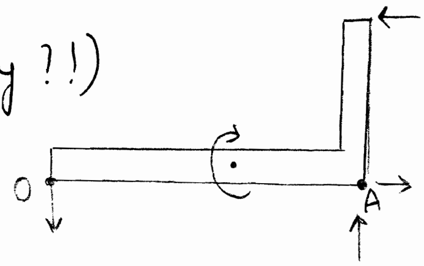



Fig. P4

Note that we may have more than one couple.

Since Moments/couples only are applied, then the moment about any point. (Why ?!)

Lets choose point A. (Why ?!)




$$\circlearrowleft + \quad \Sigma M = -100 + 30(6+3) + 40(4)$$

$$\boxed{\Sigma M = 330 \text{ KNm} \quad (\text{positive} = \quad)}$$

Problem 5:

Given:

Tension in cable AB & CD = 500 N

Required:

a) Two forces exerted by the cables on rectangular hatch at B & C form a couple.

b) Moment on plate by cables.

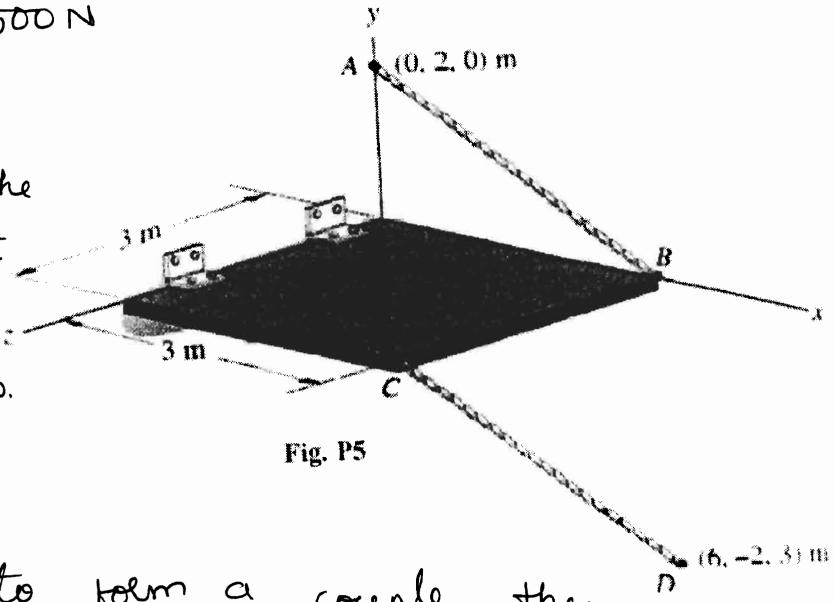


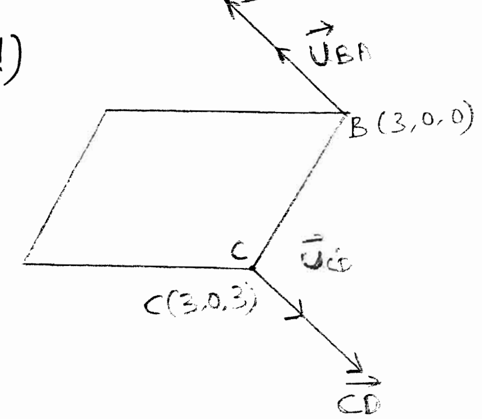

Fig. P5

Solution:

a) For two forces to form a couple, they have to be ① equal in magnitude and ② opposite in direction. Condition ① is satisfied as T = 500N in both. For condition ② to be true, the following equation must be satisfied:

$$\vec{u}_{BA} = -\vec{u}_{CD} \quad (\text{Why?!})$$

Note $\vec{u}_{BA}$ not $\vec{u}_{AB}$ (why?!)




$$\vec{u}_{BA} = \frac{\vec{BA}}{BA}$$

$$\vec{BA} = (A) - (B)$$

$$= -3\vec{i} + 2\vec{j} + 0\vec{k}$$

$$\Rightarrow BA = \sqrt{13} \text{ m}$$

$$\Rightarrow \vec{u}_{BA} = \frac{-3}{\sqrt{13}}\vec{i} + \frac{2}{\sqrt{13}}\vec{j} + 0 \cdot \vec{k}$$

$$\vec{u}_{CD} = \frac{\vec{CD}}{CD}$$

$$\vec{CD} = (D) - (C)$$

$$= 3\vec{i} - 2\vec{j} + 0\vec{k} \Rightarrow CD = \sqrt{13} \text{ m}$$

$$\Rightarrow \vec{u}_{CD} = \frac{3}{\sqrt{13}}\vec{i} - \frac{2}{\sqrt{13}}\vec{j} + 0\vec{k}$$

Thus $\vec{u}_{CD} = -\vec{u}_{BA}$

Therefore, $\boxed{\vec{T}_{BA} \text{ and } \vec{T}_{CD} \text{ form a couple}}$

b) Since the forces in the cables form a couple, then we can take the moment about any point. (why ?!)

The easiest is to take the moment about either point B or C. (why ?!)

Lets take $\vec{M}_B$ due to the cables

$$\vec{M}_B \text{ (due to cables)} = \vec{M}_{T\,BA} + \vec{M}_{T\,CD}$$

$$\vec{M}_{T_{BA}} = 0 \quad \text{(why ?!)}$$

$$\vec{M}_{T_{CD}} = \vec{r} \times \vec{T}_{CD}$$

$\vec{r}$ could be $\vec{BC}$ or $\vec{BD}$.

Lets take $\vec{BC}$. (why?!)

Note $\vec{BC}$ not $\vec{CD}$; why?!

$$\vec{BC} = (C) - (B)$$
$$= 0\vec{i} + 0\vec{j} + 3\vec{k}$$

$$\vec{T}_{CD} = T_{CD}\,\vec{u}_{CD} = 500\left(\frac{3}{\sqrt{13}}\vec{i} - \frac{2}{\sqrt{13}}\vec{j} + 0\vec{k}\right)$$

$$\vec{T}_{CD} \approx 416.03\,\vec{i} - 277.35\,\vec{j} + 0\vec{k}$$

$$\vec{M}^{B}_{T_{CD}} = \vec{BC} \times \vec{T}_{CD}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 3 \\ 416.03 & -277.35 & 0 \end{vmatrix}$$

$$= \left[0 - (3)(-277.35)\right]\vec{i} - \left[0 - (3)(416.033)\right]\vec{j} + \left[0 - 0\right]\vec{k}$$

$$\Rightarrow \boxed{\vec{M}^{B}_{T_{CD}} = 832.1\,\vec{i} + 1248\,\vec{j} + 0\vec{k} \ (N \cdot m)}$$