Static CE 201
Sections 3&4 (091)
Solution H.W# 3

Problem #1:-

- Given:
  - The shown figure.
  - $\gamma = 6^\circ$, $D = 125$ kN, $L = 680$ kN.
  - the mass of the airplane $= 72$ Mg $= 72000$ kg.

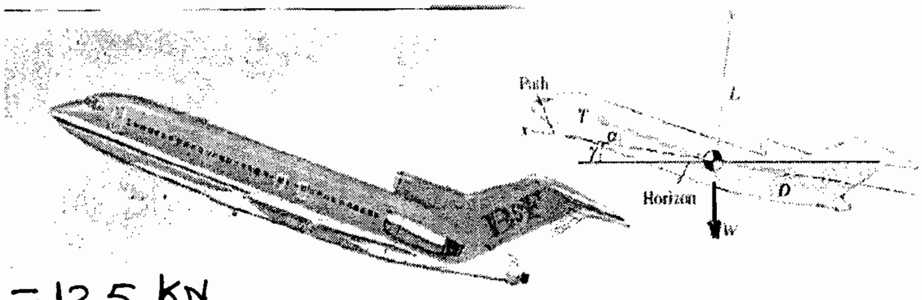
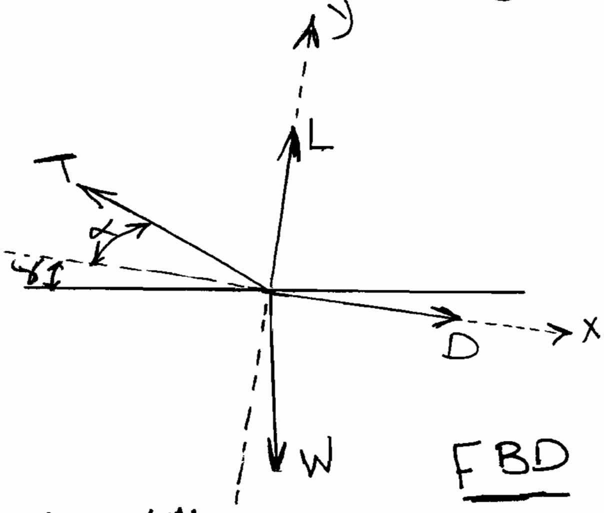


Fig. P1

- Required:-
  - the values of T and α to maintain steady flight.

- Solution:-

from the FBD



FBD

$\xrightarrow{+}\ \sum F_x = 0 \Rightarrow$

$D - T\cos\alpha + W\sin\gamma = 0 \quad \ldots (1)$

$+\uparrow \sum F_y = 0 \Rightarrow$

$T\sin\alpha + L - W\cos\gamma = 0 \quad \ldots (2)$

$\because W = M * g = (72000 * 9.81) = 706.32$ kN.

from equation (2), $\sin\alpha = \dfrac{W\cos\gamma - L}{T} \quad \ldots (3)$

from equation (1), $\cos\alpha = \dfrac{D + W\sin\gamma}{T} \quad \ldots (4)$

Divide equation (3) by (4).

$$\Rightarrow \tan\alpha = \frac{W\cos\gamma - L}{W\cos\gamma + D}$$

$$\Rightarrow \tan\alpha = \frac{706.32\cos 6^\circ - 680}{706.32\sin 6^\circ + 125} \Rightarrow \tan\alpha = 0.11291$$

$\Rightarrow \alpha = 6.442^\circ$ #

From equation (1). $\Rightarrow T = 200.1$ kN #

Note that the thrust necessary for steady flight is about 28% of the airplane's weight.

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Problem #2:-

- Given :-
  - the given figure Fig P2
  - $F_1 = 100\ lb$.

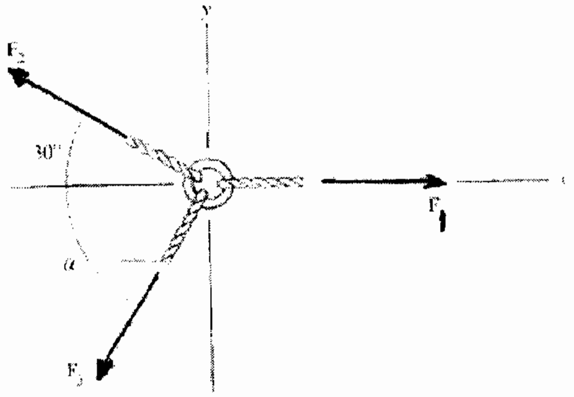


Fig. P2

- Required :-
  - F_3 and angle α

- Solution :-

* Method I, "the easy one".
  a & b.

In figure (I), to make F_3 the smallest (minimum), F_3 needs to be $\perp F_2$.

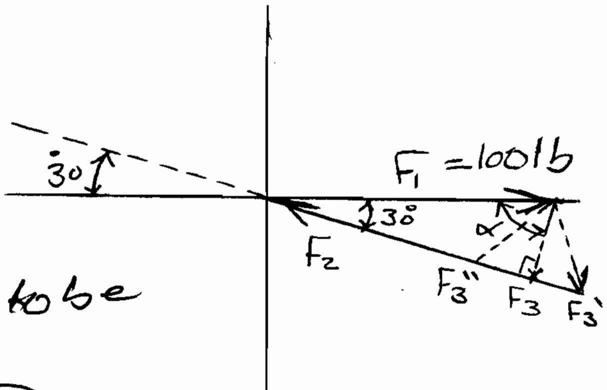


Fig. (I)

Thus, $\alpha = 180 - 90 - 30 = 60°$ #

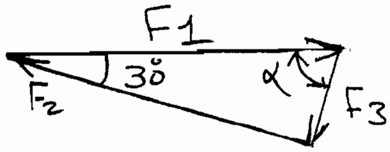
$\Rightarrow F_3 = F_1 \cos\alpha = 100 \cos 60$

$\Rightarrow F_3 = 50\ lb.$ #

Note:- that the traingle closes because we have "equilibirium" Why?!

* Method (II).
  (mathematical, more difficult")

a). Using the Cos law,



$$F_3^2 = F_1^2 + F_2^2 - 2F_1F_2\cos 30° \quad \dots (1)$$

$\Rightarrow$

$\Rightarrow$ To minimize F_3, we set

$$\frac{\partial F_3^2}{\partial F_2} = 0 \quad \Rightarrow 2F_2 - 2F_1 \cos 30^\circ = 0$$

(Note that F_1 = constant = 100 lb)

$\Rightarrow F_2 = F_1 \cos 30 = 86.603 \text{ lb} \quad \ldots (2)$

From of (2). in eq (1)

$$\Rightarrow F_3^2 = (100)^2 + (86.603)^2 - 2 * 100 * 86.603 * \cos 30$$

$\Rightarrow$ $F_3 = 50$ lb #

b). Using the sine law,

$$\frac{86.603}{\sin\alpha} = \frac{50}{\sin 30} \Rightarrow \alpha = 60^\circ$$ #

Which method is easier ?!

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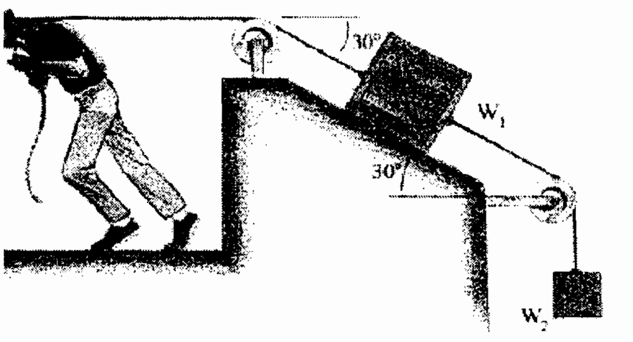


Fig. P3

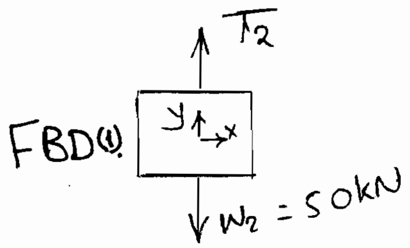
Problem # 3.

- Given :-
  - the figure shown.
  - $W_1 = 200$ kN, $W_2 = 50$ kN.

  friction = zero
- Required:
  - The force the man exert to hold the blocks in place.

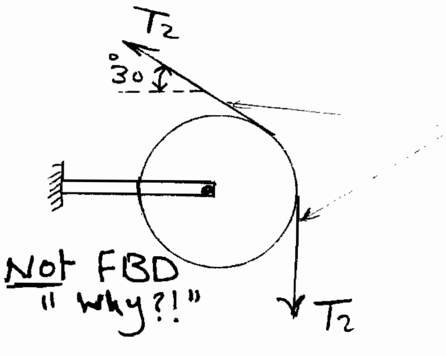
- solution:-

Note that two FBD's are needed, one for W_1 and one for W_2.



In FBD (1), $+\uparrow \sum F_y = 0 \Rightarrow$

$\Rightarrow T_2 = W_2 = 50$ kN



the same why?!

In FBD (2),

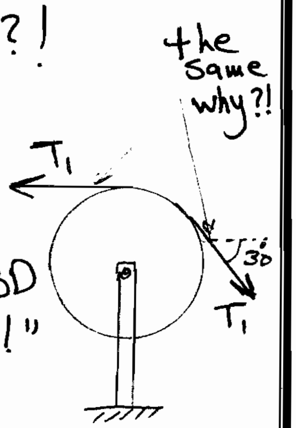
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow$

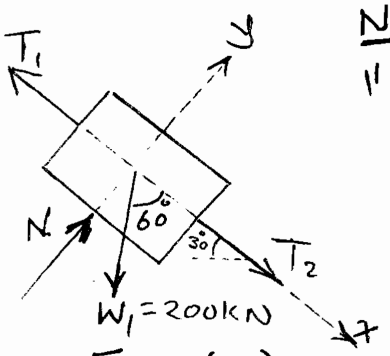
$T_2 + W_1 \cos 60° - T_1 = 0$

$\Rightarrow 50 + 200 \cos 60° - T_1 = 0$

$\Rightarrow T_1 = 150$ kN #

which is the force that the man must exert. #





FBD(2)

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Problem # 4.

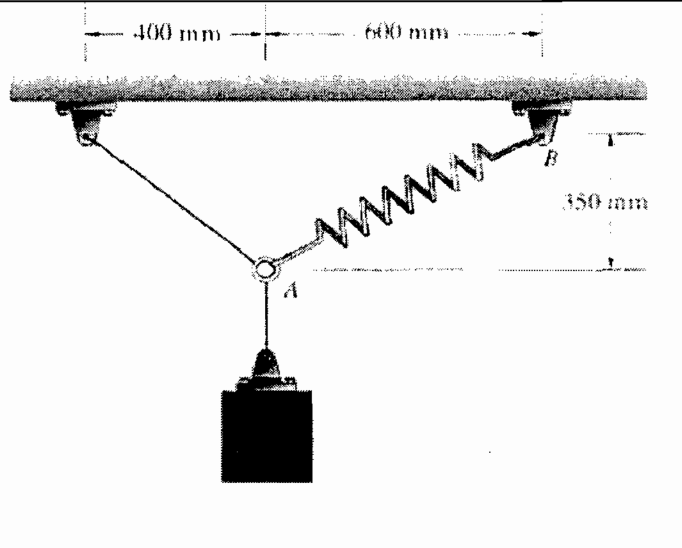


Fig. P4

- Given :-
  - the shown figure.
  - unstretched length of AB is 660 mm
  - $K_{AB} = 1000$ N/m.
- Required :-
  - the mass of the suspended object.
- Solution :-

the equilibirium Length for spring AB is

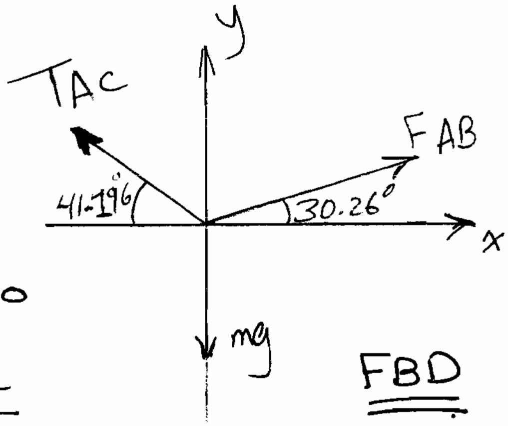
$$l = \sqrt{(600)^2 + (350)^2} = 694.622 \text{ mm} \Rightarrow$$

$$S = 694.622 - 660 = 34.6222 \text{ mm} = 0.0346222 \text{ m}.$$

$$\therefore F_{AB} = SK = 0.0346222 * 1000 = 34.6222 \text{ N}.$$

$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow T_{AC} \cos 41.196 = F_{AB} \cos 30.26$$

$$\Rightarrow T_{AC} = 39.743$$



FBD

$$+\uparrow \sum F_y = 0 \Rightarrow$$

$$F_{AC} \sin 41.196 + F_{AB} \sin 30.26 - mg = 0$$

$$\Rightarrow mg = 43.62 \text{ N} \Rightarrow m = \frac{43.62}{9.81}$$

$$\Rightarrow m = 4.45 \text{ Kg}$$

#

Problem # 5 :-

- Given :-
  - The given figure Fig P_5.
  - $F = 2i$ (kip) at A
- Required:-
  - T_{AB}, T_{AC}, T_{AD}.
- Solution:-

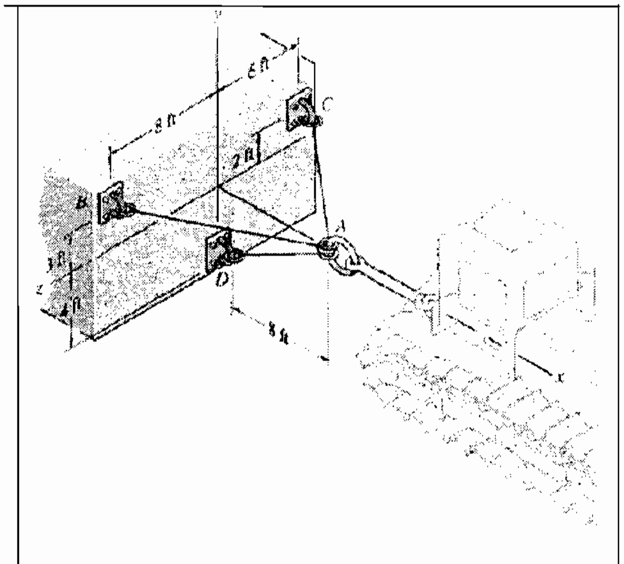
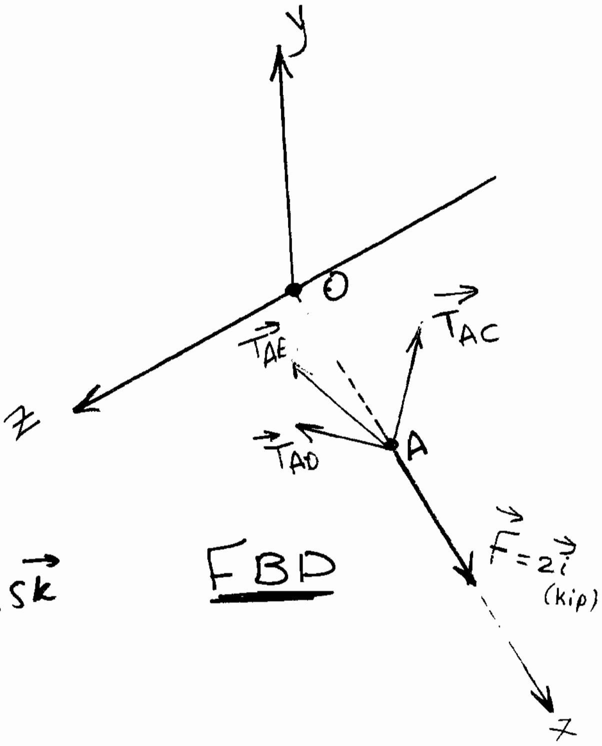


Fig. P5

The FBD is drawn as shown.



FBD

The coordinates are:-

$A(8,0,0)$, $B(0,3,8)$

$C(0,2,-6)$, $D(0,-4,0)$

$$\vec{T}_{AB} = T_{AB}\,\vec{U}_{AB}$$

$$= T_{AB}\,\frac{\vec{AB}}{AB}$$

$$\vec{U}_{AB} = \frac{(B)-(A)}{AB} = \frac{-8\vec{i}+3\vec{j}+8\vec{k}}{\sqrt{(-8)^2+(3)^2+(8)^2}}$$

$$\Rightarrow \vec{U}_{AB} = -0.6835\vec{i} + 0.2563\vec{j} + 0.6835\vec{k}$$

Similarly for $\vec{T}_{AC}$ and $\vec{T}_{AD}$,

$$\vec{U}_{AC} = -0.7845\,\vec{i} + 0.1961\,\vec{j} - 0.5883\,\vec{k}$$

$$\vec{U}_{AD} = -0.8944\,\vec{i} - 0.4472\vec{j} + 0\vec{k}$$

$$\vec{F} = 2\vec{i} + 0\vec{j} + 0\vec{k}$$

$$\vec{T}_{AB} = T_{AB}\,\vec{U}_{AB} \;;\; \vec{T}_{AC} = T_{AC}\,\vec{U}_{AC} \;;\; \vec{T}_{AD} = T_{AD}\,\vec{U}_{AD}$$

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$$\Rightarrow \vec{R} = \sum \vec{F} = 0 \Rightarrow \vec{F} + \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} = 0$$

$$\Rightarrow \sum F_x = 0 \Rightarrow -0.6835\,T_{AB} - 0.7845\,T_{AC} - 0.8944\,T_{AB} + 2 = 0 \quad \text{---- (1)}$$

$$\sum F_y = 0 \Rightarrow 0.2563\,T_{AB} + 0.1961\,T_{AC} - 0.4472\,T_{AD} = 0 \quad \text{---- (2)}$$

$$\sum F_z = 0 \Rightarrow 0.6835\,T_{AB} - 0.5883\,T_{AC} + 0\,T_{AD} = 0 \quad \text{---- (3)}$$

solving equations (1), (2) and (3) yields the following:-

$T_{AB} = 0.7803$ kip #

$T_{AC} = 0.9065$ kip #

$T_{AD} = 0.8447$ kip #

#