CE 201 (091)
Sections 3&4
Solution of H.W # 2

Problem # 1 :-

- Given :- • the shown Figure.
  • $F_x = 100$ N

- Required :- • the magnitude of F
  • The angles θ_x, θ_y and θ_z between F and the positive coordinate axes.

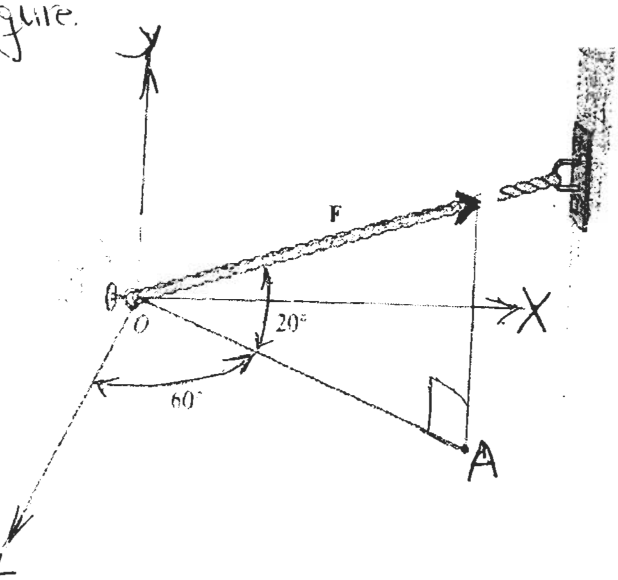

Fig. P1

- Solution :-

$F_X = F \cos 20° \sin 60°$

$\Rightarrow 100 = F \cos 20° \sin 60° \Rightarrow \boxed{F = 122.88 \simeq 122.9 \text{ N}}$ #

* from the drawing, $\theta_y = 90 - 20 \Rightarrow \boxed{\theta_y = 70°}$ #

$F_x = F \cos\theta_x = F(\cos 20° \sin 60°)$

$\Rightarrow \cos\theta_x = \cos 20° \sin 60°$

$\Rightarrow \boxed{\theta_x = 35.53°}$ #

$F_z = F\cos\theta_z = F(\cos 20° \cos 60°)$

$\Rightarrow \cos\theta_z = \cos 20° \cos 60°$

$\Rightarrow \boxed{\theta_z = 61.98°}$ #

Check, $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z \overset{?}{=} 1$

Yes. $\Rightarrow$ ok!

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Problem #2:-

- Given:- the Figure shown.
  - $F_{AB} = 200$ Ib along AB.
  - $F_{AC} = 100$ Ib along AC
- Required:-
  - the magnitude of the total force exerted at point A by the two cables.

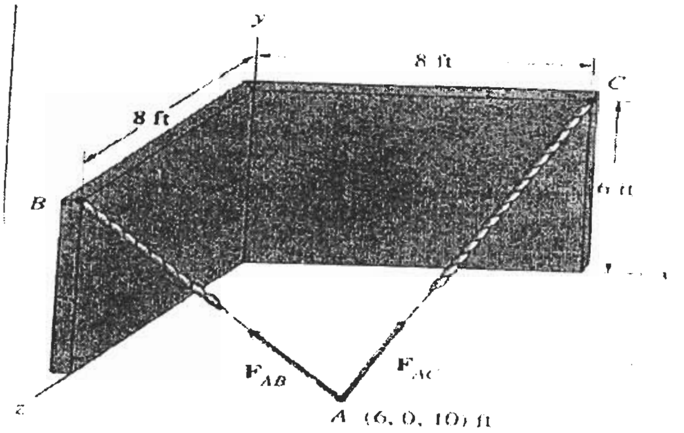

Fig. P2

- Solution:-

Coordinates: $A(6,0,10)$, $B(0,6,8)$, $C(8,6,0)$ ft

$$\vec{AB} = (B) - (A) = (0-6)\vec{i} + (6-0)\vec{j} + (8-10)\vec{k}$$

$$= -6\vec{i} + 6\vec{j} - 2\vec{k} \Rightarrow AB = \sqrt{(-6)^2+(6)^2+(-2)^2} = \sqrt{76} \text{ ft}$$

$$\Rightarrow \vec{F}_{AB} = F_{AB}\,\vec{u}_{AB} = \frac{200}{\sqrt{76}}\left(-6\vec{i} + 6\vec{j} - 2\vec{k}\right) \quad (\text{Ib})$$

$$= -137.65\vec{i} + 137.65\vec{j} - 45.88\vec{k} \quad (\text{lb})$$

$$\vec{AC} = (C) - (A) = (8-6)\vec{i} + (6-0)\vec{j} + (0-10)\vec{k}$$

$$= 2\vec{i} + 6\vec{j} - 10\vec{k} \Rightarrow AC = \sqrt{140} \text{ ft}$$

$$\Rightarrow \vec{F}_{AC} = F_{AC}\,\vec{u}_{AC} = \frac{100}{\sqrt{140}}\left(2\vec{i} + 6\vec{j} - 10\vec{k}\right)$$

$$= 16.903\vec{i} + 50.709\vec{j} - 84.515\vec{k} \quad (\text{lb})$$

$$\vec{R} = \Sigma\vec{F} = \vec{F}_{AB} + \vec{F}_{AC}$$

$$= (-137.65 + 16.903)\vec{i} + (137.65 + 50.709)\vec{j} + (-45.88 - 84.515)\vec{k}$$

$$\Rightarrow \vec{R} = -120.75\vec{i} + 188.36\vec{j} - 130.40\vec{k}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \Rightarrow R = 258.97 \cong 259 \text{ lb}$$

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Problem #3:-

- Given :-
  - The figure shown.
  - $F_{AB} = 2\,kN$
  - x and z components of the vector sum of the forces = 0.0
  - Height of the tower = 70 ms.
- Required:-
  - F_{AC} & F_{AD}.

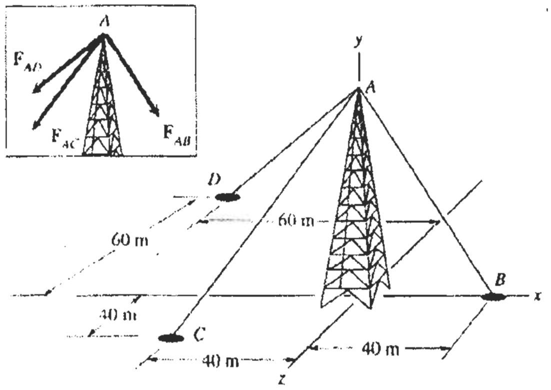

Fig. P3

- Solution :-

Coordinates:

A(0, 70, 0) , B(40, 0, 0) , C(−40, 0, 40) , D(−60, 0, −60) ms.

since $\sum \vec{F}_x$ is needed, the force vectors are found through the position vectors.

$\vec{AB} = (B) - (A) = 40\vec{i} - 70\vec{j} + 0\vec{k} \Rightarrow AB = \sqrt{6500}$ m.

$\Rightarrow \vec{F_{AB}} = F_{AB}\,\vec{u}_{AB} = \frac{2}{\sqrt{6500}}(40\vec{i} - 70\vec{j} + 0\vec{k})$

$= 0.992228\vec{i} - 1.73649\vec{j} + 0\vec{k}$

$\vec{AC} = (C) - (A) = -40\vec{i} - 70\vec{j} + 40\vec{k} \Rightarrow AC = 90$ ms.

$\Rightarrow \vec{F_{AC}} = F_{AC}\,\vec{u}_{AC} = \frac{F_{AC}}{90}(-40\vec{i} - 70\vec{j} + 40\vec{k})$

$= (-0.444444\vec{i} - 0.777778\vec{j} + 0.444444\vec{k})\,F_{AC}$

$\vec{AD} = (D) - (A) = -60\vec{i} - 70\vec{j} - 60\vec{k} \Rightarrow AD = 110$ ms.

$\Rightarrow \vec{F_{AD}} = F_{AD}\,\vec{u}_{AD} = \frac{F_{AD}}{110}(-60\vec{i} - 70\vec{j} - 60\vec{k})$

$= (-0.545455\vec{i} - 0.636364\vec{j} - 0.545455\vec{k})\,F_{AD}$.

$\because \sum F_x = 0 \Rightarrow F_{ABx} + F_{ACx} + F_{ADx} = 0 \Rightarrow$

$\longrightarrow$

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$\Rightarrow 0.992278 - 0.444444 F_{AC} - 0.545455 F_{AD} = 0$ ---- (1)

$\because \sum F_z = 0 \Rightarrow$

$0.444444 F_{AC} - 0.545455 F_{AD} = 0$ ---- (2)

From equation (2), $F_{AD} = 0.814813 F_{AC}$ --- (3)

Using equation (3) into (1),

$\Rightarrow F_{AC} = 1.1163 KN$ #

By subist. into eq. (3)

$\Rightarrow F_{AD} = 0.814813 * (1.1163)$

$\Rightarrow F_{AD} = 0.9096 KN$

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Problem #4 :-

- Given :-
  - the figure shown.

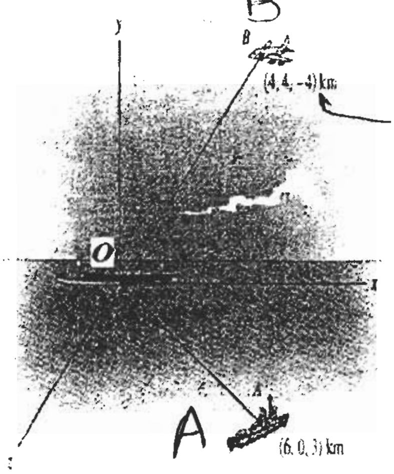

Fig. P4 (6,0,3) km

- Required :-
  - the angle θ between the lines OA and OB.

- Solution :-

Coordinates :

$O(0,0,0)$, $A(6,0,3)$, $B(4,4,-4)$ km.

$$\cos\theta = \frac{\vec{OA} \cdot \vec{OB}}{(OA)(OB)}$$

$$\vec{OA} = (A) - (O) = 6\vec{i} + 0\vec{j} + 3\vec{k} \Rightarrow OA = \sqrt{45} \text{ km.}$$

$$\vec{OB} = (B) - (O) = 4\vec{i} + 4\vec{j} - 4\vec{k} \Rightarrow OB = \sqrt{48} \text{ km.}$$

$$\therefore \cos\theta = \frac{(6)(4) + (0)(4) + (3)(-4)}{(\sqrt{45})(\sqrt{48})}$$

$$= 0.25820$$

$$\Rightarrow \theta = \cos^{-1}(0.25820)$$

$$\Rightarrow \theta = \pm 75.04^\circ$$

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Problem # 5 :-

- Given :-
  - $F_{OA} = 50$ N.
  - The Figure shown.

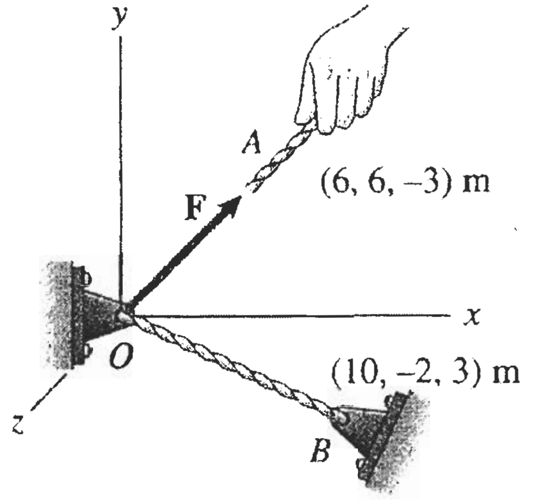

Fig. P5

- Required :-
  - the components of F parallel and normal to the cable OB.

- Solution :-

$F_{OB} = F\cos$ (angle between OA and OB)

Since we do not have that angle, the dot product will be utilized.

$\therefore\ F_{OB} = \vec{F}\cdot\vec{u}_{OB}$

$\vec{F} = F\vec{u}_F = F\,u_{OA} = F\,\dfrac{\vec{OA}}{OA}$

$\vec{OA} = (A)-(O) = 6\vec{i}+6\vec{j}-3\vec{k} \Rightarrow OA = 9$ m.

$\vec{F} = \dfrac{50}{9}(6\vec{i}+6\vec{j}-3\vec{k}) = 33.3333\vec{i}+33.3333\vec{j}-16.6667\vec{k}$.

$\vec{u}_{OB} = \dfrac{\vec{OB}}{OB}$

$\vec{OB} = (B)-(O) = 10\vec{i}-2\vec{j}+3\vec{k} \Rightarrow OB = \sqrt{113}$ ms.

$\Rightarrow \vec{u}_{OB} = \dfrac{10}{\sqrt{113}}\vec{i} - \dfrac{2}{\sqrt{113}}\vec{j} + \dfrac{3}{\sqrt{113}}\vec{k}$

$\Rightarrow F_{OB} = (33.3333)\left(\dfrac{10}{\sqrt{113}}\right) + (33.3333)\left(-\dfrac{2}{\sqrt{113}}\right) + (-16.6667)\left(\dfrac{3}{\sqrt{113}}\right)$

$\Rightarrow F_{OB} = 20.38$ N $= F_{\text{parallel to OB}}$ #

& $F_{\perp OB} = \sqrt{F^2 - F_{OB}^2} = \sqrt{(50)^2-(20.38)^2} \Rightarrow F_{\perp OB} = 45.66$ N #