Problem 1 :-

- Given :-
  - The figure shown Fig. P1.
  - The bar is allowed to hang freely.

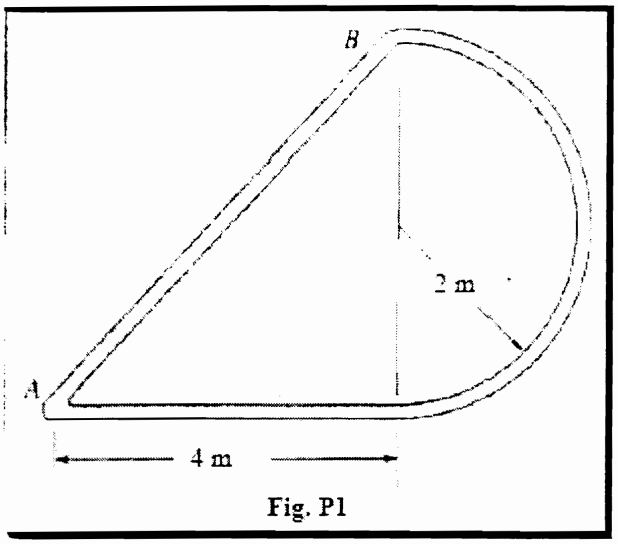


Fig. P1

- Required :-
  - The angle between AB and the vertical.

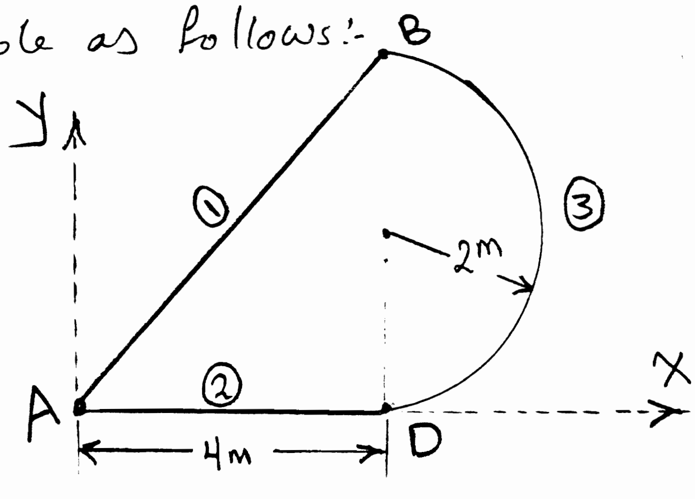
- Solution :-

Strategy (idea):

Since the bar is attached at A and is allowed to hang freely, the centroid of the bar and point A must lie on the same vertical line so that $\Sigma M = 0$; otherwise, equilibrium will not be maintained. [Note that the weight will equal the tension in the string so $\Sigma F_y = 0$ is satisfied. $\Sigma F_x = 0$ as there are no forces in the x-direction.]

Thus we need to locate the centroid of the bar. The best way is to use a table as follows:-

The bar is divided into 3 parts (segments) as shown.



Segment	L_i (m)	$\hat{x}_i$ (m)	$\hat{y}_i$ (m)	$\hat{x}_i L_i$ (m^2)	$\hat{y}_i L_i$ (m^2)
① AB	$\sqrt{4^2+4^2}=\sqrt{32}$ ≈ 5.65685	2	2	11.3137	11.3137
② AD	4	2	0	8	0
③ BD	$\pi r = 2\pi$ $\simeq 6.28319$	$4+\frac{2r}{\pi}$ ≈ 5.27324	2	33.1327	12.5664
Σ	15.9400			52.4464	23.8801

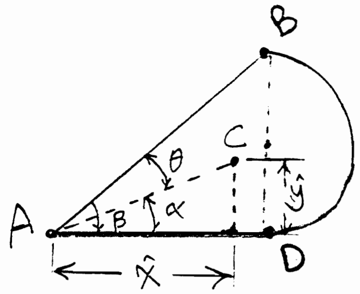
$$\hat{x} = \frac{\Sigma \bar{x}_i L_i}{\Sigma L_i} = \frac{52.4464}{15.9400} \Rightarrow \hat{x} = 3.2902\text{ m.}$$

$$\hat{y} = \frac{\Sigma \hat{y}_i L_i}{\Sigma L_i} = \frac{23.8801}{15.9400} \Rightarrow \hat{y} = 1.4981\text{ m.}$$

From the figure shown,

$$\tan\alpha = \frac{\hat{y}}{\hat{x}} = \frac{1.4981}{3.2902}$$

$\Rightarrow \alpha = 24.481°$

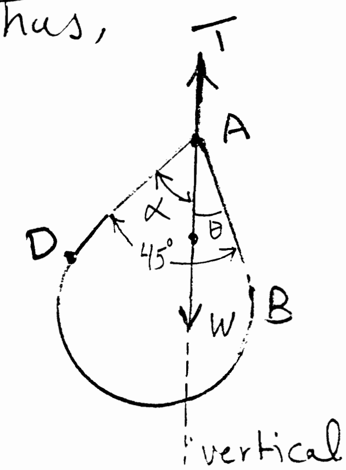


Since AC must be vertical (as explained at the beggining), then, the angle between AC and AB is the required angle as shown below. Thus,

$$\theta = \beta - \alpha = \tan^{-1}\frac{4}{4} - 24.481$$

$$= 45 - 24.481 \Rightarrow$$

$$\boxed{\theta = 20.52°}\ [CW]$$



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Problem-2 :-

- Given :-
  - The figure Shown Fig. P2.

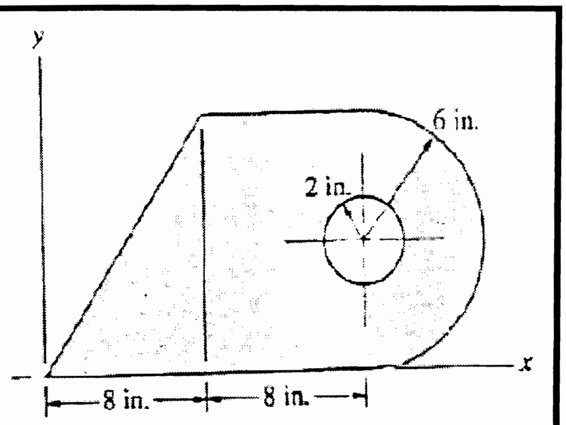


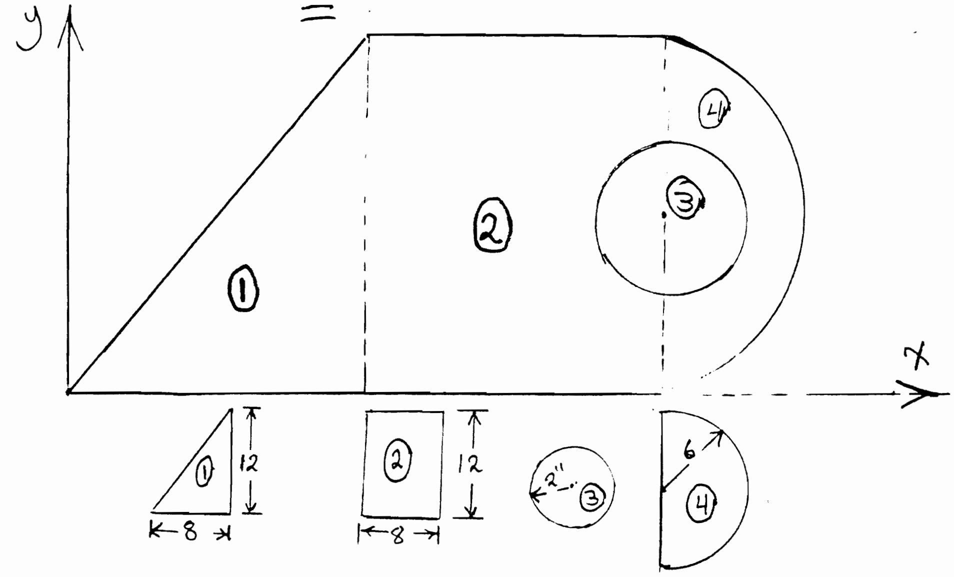
Fig. P2

- Required :-
  - The centroid of the shaded area.

- Solution :-

The area is divided into 4 parts as shown. (why ?!)

A table will be used. Note that no symmetry.



Part	A_i (in^2)	$\bar{x}_i$ (in)	$\bar{y}_i$ (in)	$\bar{x}_i A_i$ (in^3)	$\bar{y}_i A_i$ (in^3)
1	$\frac{12*8}{2} = 48$	$\frac{2(8)}{3} \simeq 5.33333$	$\frac{12}{3} = 4$	256	192
2	$8*12 = 96$	$8+4 = 12$	$\frac{12}{2} = 6$	1152	576
3	$\ominus 4\pi = -12.5664$	$8+8 = 16$	6	$\ominus$ 201.062	$\ominus$ 75.398
4	$\frac{\pi}{2} 36 = 18\pi \doteq 56.5487$	$8+8+\frac{4(6)}{3\pi} \simeq 18.5465$	6	1048.79	339.292
Σ	187.982			2255.73	1031.89

$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = \frac{2255.73}{187.982} \Rightarrow \boxed{\bar{x} = 12 \text{ in}} \quad \#$$

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{1031.89}{187.982} \Rightarrow \boxed{\bar{y} = 5.49 \text{ in}} \quad \#$$

Problem. 3 :-

- Given :-
  - The figure shown Fig. P3

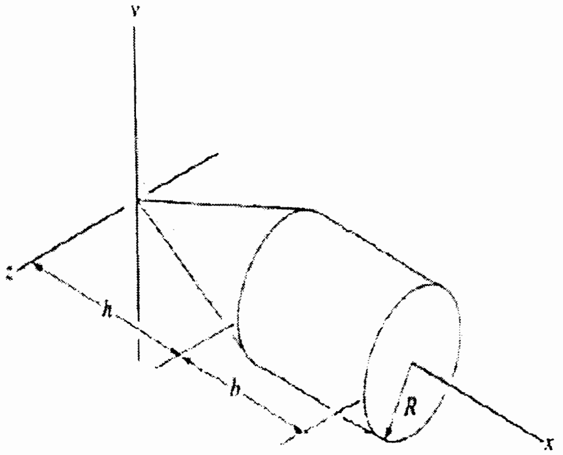


Fig. P3

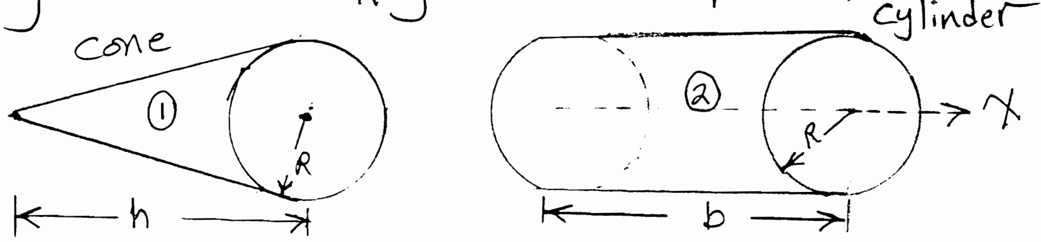
- Required :-
  - Derivation a formula for the centroid of the shaded Volume.

- Solution :-

The volume is divided into 2 parts as shown.
From symmetry about the x-y and x-z planes,

$\boxed{\bar{y} = 0}$

$\boxed{\bar{z} = 0}$



[How ?! see the origin of the coordinate axes.]

Part	V_i (*) (u^3)	$\bar{X}_i$ (*) (unit)	$\bar{X}_i V_i$ (u^4)
1	$\frac{1}{3}\pi R^2 h$	$\frac{3}{4}h$	$\frac{\pi}{4}R^2h^2$
2	$\pi R^2 b$	$h + b/2$	$(h+\frac{b}{2})\pi R^2 b$
Σ	$\pi R^2(\frac{h}{3}+b)$		$\pi R^2[\frac{h^2}{4}+b(h+\frac{b}{2})]$

(*) see the table on the inside backcover of your textbook.

$$\bar{X} = \frac{\Sigma \bar{x}_i V_i}{\Sigma V_i} = \frac{\pi R^2[\frac{h^2}{4}+b(h+\frac{b}{2})]}{\pi R^2(\frac{h}{3}+b)}$$

$$\Rightarrow \boxed{\bar{X} = \frac{\frac{h^2}{4}+b(h+\frac{b}{2})}{\frac{h}{3}+b}}$$

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Problem. 4 :-

- Given :-
  - The figure shown Fig. P4.

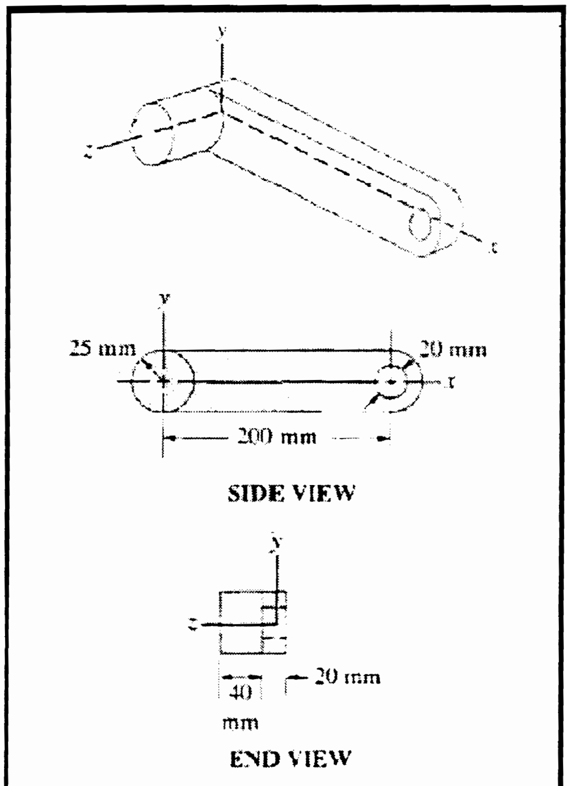


Fig. P4

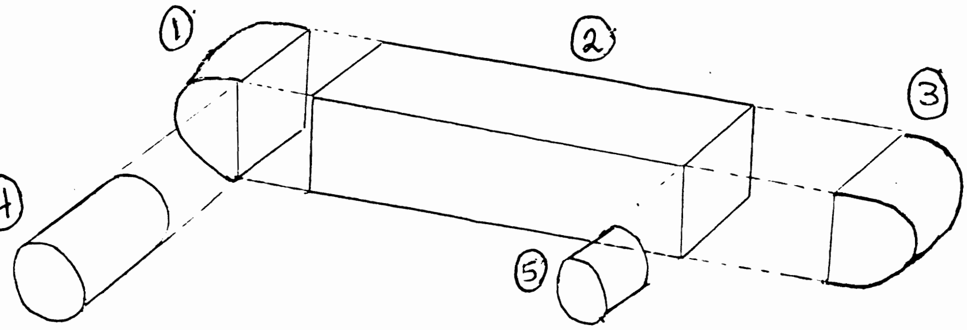
- Required :-
  - The centroid of the shaded volume.

- Solution :-

The volume is divided into 5 parts (why?!). From symmetry about the x-z plane,

$\boxed{\bar{y} = 0}$

Note that volume 5 is negative.



Part	V_i (mm^3)	$\bar{x}_i$ (mm)	$\bar{z}_i$ (mm)	$\bar{x}_i V_i$ (mm^4)	$\bar{z}_i V_i$ (mm^4)
1	$\frac{\pi}{2}(25)^2(20) \approx 19,635$	$-\frac{4(25)}{3\pi} \approx -10.610$	0	-208,334	0
2	$200(50)(20) = 200,000$	100	0	20,000,000	0
3	$\frac{\pi}{2}(25)^2(20) \cong 19,635$	$200 + \frac{4(25)}{3\pi} \approx 210.61$	0	4,135,334	0
4	$\pi(25)^2(40) \approx 78,540$	0	$\frac{20}{2} + \frac{40}{2} = 30$	0	2,356,200
5 neg.	$\ominus \pi(\frac{20}{2})^2(20) \approx \ominus 6,283$	200	0	-1,256,600	0
Σ	$\approx 311,527$			22,670,400	2,356,200

$$\bar{X} = \frac{\Sigma \bar{x}_i v_i}{\Sigma v_i} \approx \frac{22,670,400}{311,527} \Rightarrow \boxed{\bar{x} \approx 72.77 \text{ mm}}$$

$$\bar{z} = \frac{\Sigma \bar{z}_i v_i}{\Sigma v_i} \approx \frac{2,356,200}{311,527} \Rightarrow \boxed{\bar{z} \approx 7.563 \text{ mm}}$$

Reasonable answer ?!

Problem. 5 :-

- Given :-
  - The figure shown Fig. P5.

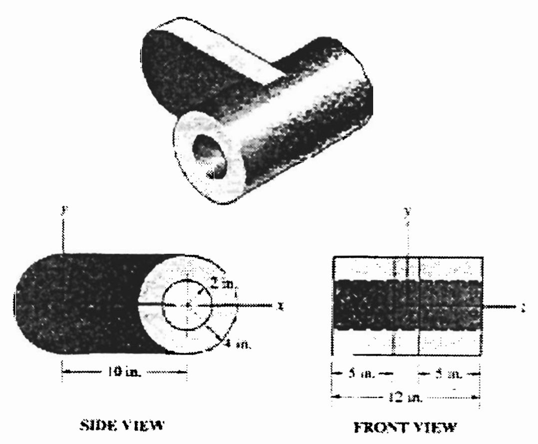


Fig. P5

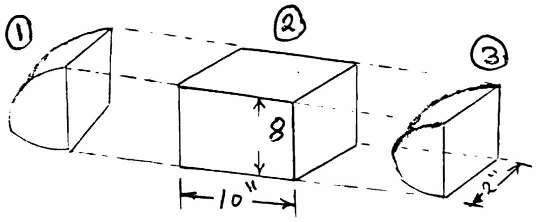
- Required :-
  - The centroid of the shaded volume.

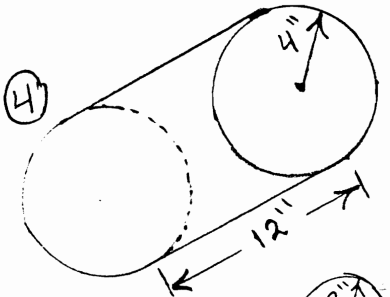
- Solution :-

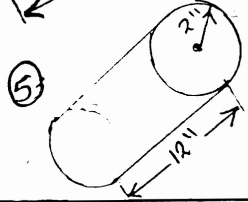
The volume is divided into 5 parts. (why ?!) From symmetry about the x-y plane and x-z planes. (How ?!)

$\boxed{\bar{z}=0}$ and $\boxed{\bar{y}=0}$

Note that volumes ③ and ⑤ are negative.







Part	V_i (in^3)	$\bar{x}_i$ (in)	$\bar{x}_i V_i$
①	$\frac{\pi}{2}(4)^2(2) \approx 50.2655$	$-\frac{4(4)}{3\pi} \approx 1.69765$	-85.333
②	$10(2)(8) = 160$	5	800
③	$\ominus \frac{\pi}{2}(4)^2(2) \approx \ominus 50.2655$	$10 - \frac{4(4)}{3\pi} \approx 8.30235$	-417.322
④	$\pi (4)^2(12) \approx 603.186$	10	6031.86
⑤	$\ominus \pi(2)^2(12) \approx \ominus 150.796$	10	-1507.96
Σ	612.39		4821.25

$$\bar{X} = \frac{\Sigma \bar{x}_i V_i}{\Sigma V_i} = \frac{4821.25}{612.39} \Rightarrow \boxed{\bar{X} = 7.873 \text{ in}} \quad \#$$

seems "ok" ?!