CE 201 Statics
Sections 3&4 (091)
Solution of H.W# 11

Problem # 1.

- Given:-
  - The figure shown. Fig. P1
  -

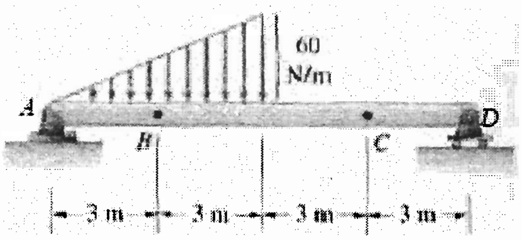


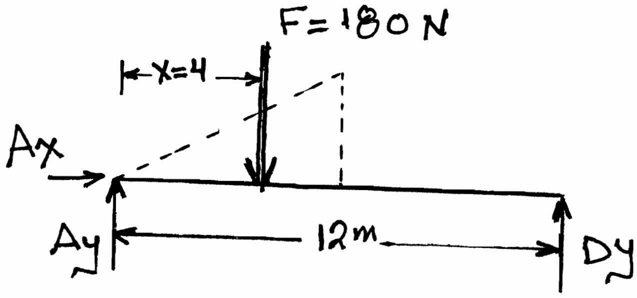
Fig. P1

- Required:-
  - Internal forces at B and C

- Solution:-

First, we need to determine the reactions. (Why?!)

In FBD ①



FBD①

$F = \frac{1}{2}(6)(60)$

$= 180\ N.$

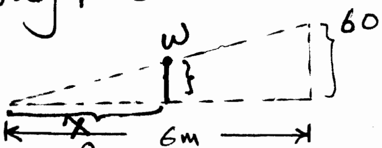
$x = \frac{2}{3}(3+3) = 4m$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow A_x = 0$

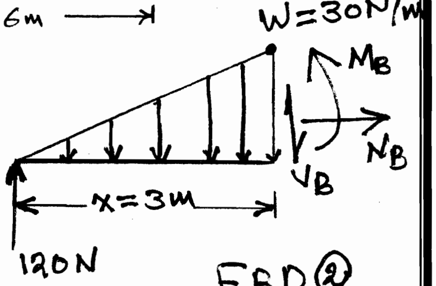
$+\circlearrowleft \Sigma M_A = 0 \Rightarrow 12\,D_y - 180(4) = 0 \Rightarrow D_y = 60\ N$

$+\uparrow \Sigma F_y = 0 \Rightarrow 60 - 180 + A_y = 0 \Rightarrow A_y = 120\ N.$

Now, we make a cut (section) through B and take FBD of the left part. (Why?!)



Using similar traingle, the function for the load w can be obtained:

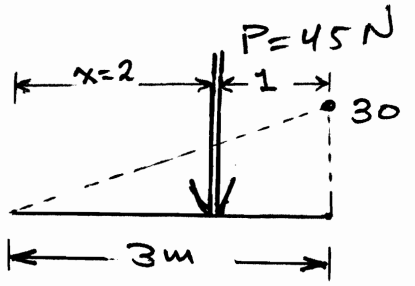


FBD②

$\frac{w}{60} = \frac{x}{6} \Rightarrow \frac{60}{6}x = w$

$\Rightarrow w = 10x$; @ $x=3$, $w = 10(3) = 30\ N/m.$

$P = \text{Area} = \frac{1}{2}(3)(30) = 45\ N$ @ $x = 2\ m$



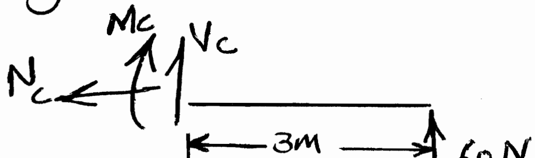
In FBD ②,

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow$ $\boxed{N_B = 0}$

$+\uparrow \sum F_y = 0 \Rightarrow 120 - 45 - V_B = 0 \Rightarrow$ $\boxed{V_B = 75\ N}$ as shown

$+\circlearrowleft \sum M_B = 0 \Rightarrow M_B + 45(1) - 120(3) = 0 \Rightarrow$ $\boxed{M_B = 315\ N\cdot m}$ as shown

Now, we make a cut (section) through C and take the right part. (why ?!)



FBD ③

FBD ③ is drawn as shown.

Note the directions of the internal forces at C; all are positive, compared with FBD ② @ B. (why ?!)

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow$ $\boxed{N_C = 0}$

$+\uparrow \sum F_y = 0 \Rightarrow 60 + V_C = 0 \Rightarrow$ $\boxed{V_C = -60\ N}$ " opposite direction "

$+\circlearrowleft \sum M_C = 0 \Rightarrow -M_C + 60(3) = 0 \Rightarrow$

$\boxed{M_C = 180\ N\cdot m}$ as shown.

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Problem #2 :-

- Given :-
  - The figure shown Fig. P_2
  - $F_1 = \{+350\,i - 400\,j\}$ lb.
  - $F_2 = \{-300\,j + 150\,k\}$ lb.

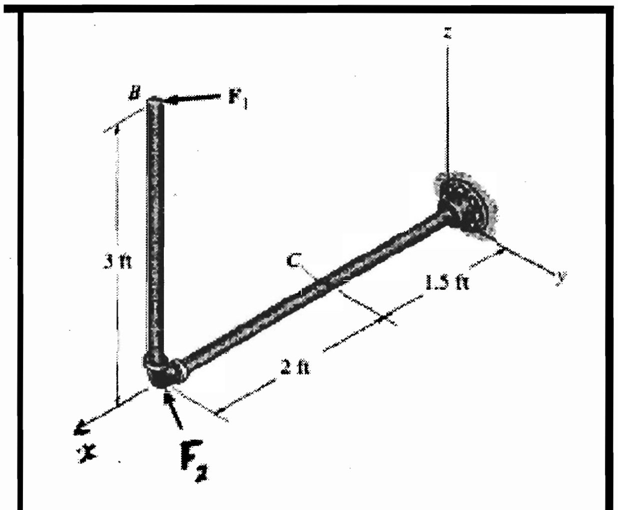


Fig. P2

- Required :-
  - The internal forces at C.

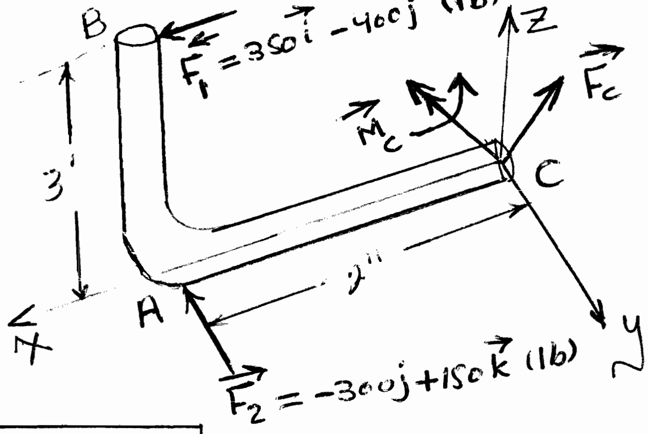
- Solution :-

A section/cut is taken through "C" and the FBD to the left is taken. (why ?!) No need to calculate the reactions. (why ?!).

$\sum \vec{F} = 0 \Rightarrow$

$\vec{F_c} + (350\vec{i} - 400\vec{j})$

$+ (-300\vec{j} + 150\vec{k}) = 0$



FBD

$\Rightarrow \boxed{\vec{F_c} = -350\vec{i} + 700\vec{j} - 150\vec{k} \text{ (lb)}}$

OR $\boxed{F_c^x = -350 \text{ lb} = N_c = 350 \text{ lb (T)}}$

$\boxed{F_c^y = V_c^y = 700 \text{ lb}}$

$\boxed{F_c^z = V_c^z = -150}$

sign not that important (why ?!)

$\sum \vec{M}_C = \vec{0} \Rightarrow$

$\vec{M}_C + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = \vec{0}$

$\vec{r}_1 = (B) - (C) = 2\vec{i} + 0\vec{j} + 3\vec{k} \Leftarrow \vec{CB}$

$\vec{r}_2 = (A) - (C) = 2\vec{i} - 0\vec{j} + 0\vec{k} \Leftarrow \vec{CA}$

$$\vec{M}_C + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 3 \\ 350 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 0 \\ 0 & -300 & 150 \end{vmatrix} = \vec{0}$$

$$\vec{M}_C + (1200\vec{i} + 1050\vec{j} - 800\vec{k}) + (0\vec{i} - 300\vec{j} - 600\vec{k}) = \vec{0}$$

$$\Rightarrow \boxed{\vec{M}_C = -1200\vec{i} - 750\vec{j} + 1400\vec{k} \quad (\text{ft.lb})}$$

Note that M_C^x is <u>twisting</u> moment, while M_C^y and M_C^z are <u>bending</u> moment.

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Problem 3:-

- Given:
  - The figure shown Fig. P3.

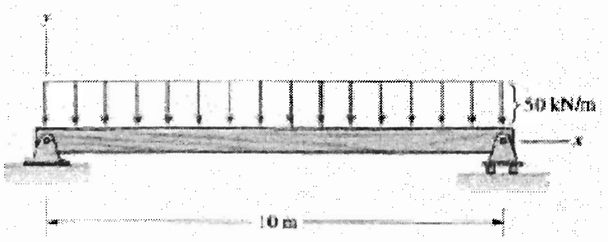


Fig. P3

- Required :-
  - Equations of the shear force and Bending moment and their diagrams.

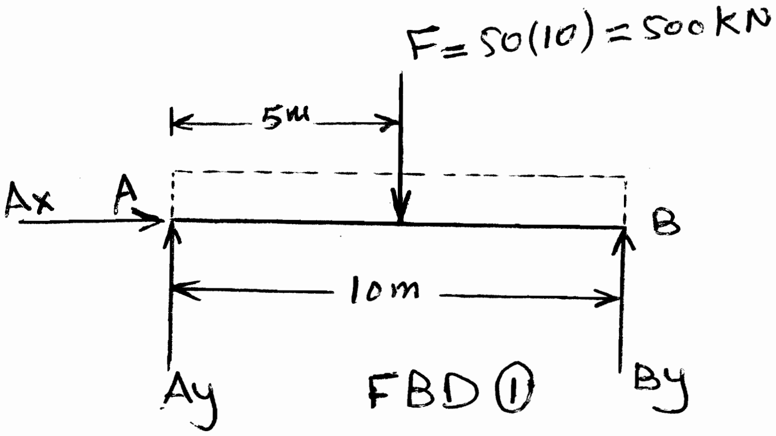
- Solution :-

The reactions are first calculated from FBD ①. (why ?!)

$+\circlearrowleft \sum M|_B = 0 \Rightarrow$

$500(5) - A_y(10) = 0$

$\Rightarrow A_y = 250 \text{ kN} \uparrow$



$+\uparrow \sum F_y = 0 \Rightarrow$

$250 - 500 - B_y = 0 \Rightarrow$

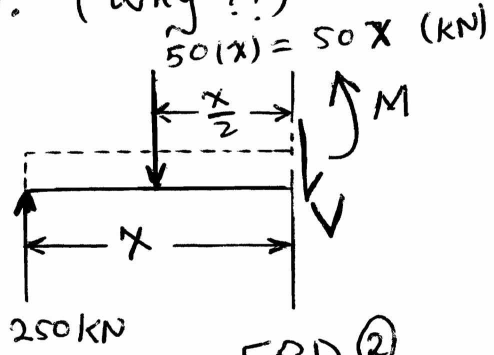
$B_y = 250 \text{kN} \uparrow$

Note that you can get their values directly (From symmetry). $\Rightarrow A_y = B_y = F/2 = 250 \text{ kN}$.

Note that only one section is needed. (why?!)

AB: $0 \le x \le 10\,m$

FBD ② is drawn (the left part). (why?!)



FBD ②

$+\uparrow \sum F_y = 0 \Rightarrow$

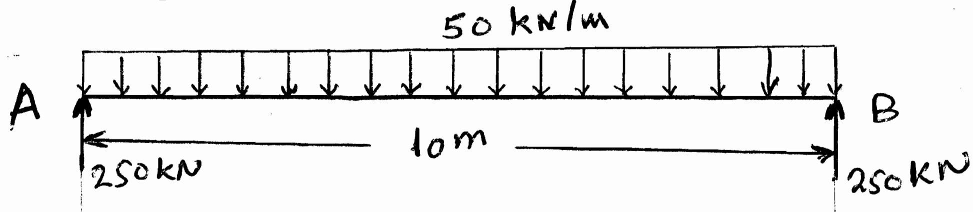
$250 - 50x - V = 0 \Rightarrow$

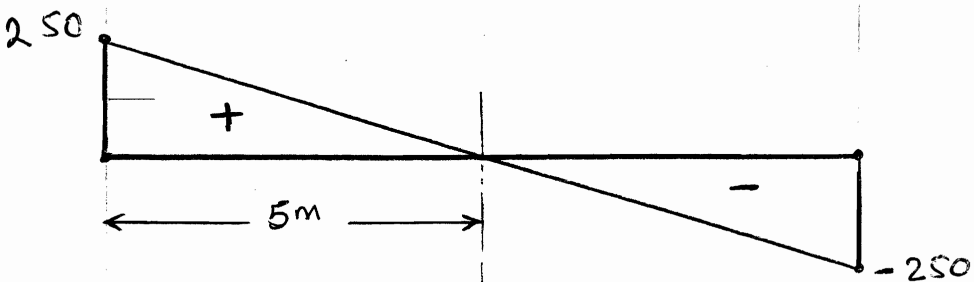
$$\boxed{V = 250 - 50x \quad (kN)}$$

$+\circlearrowleft \sum M = 0 \Rightarrow \quad M - 250x + 50x\left(\frac{x}{2}\right) = 0$

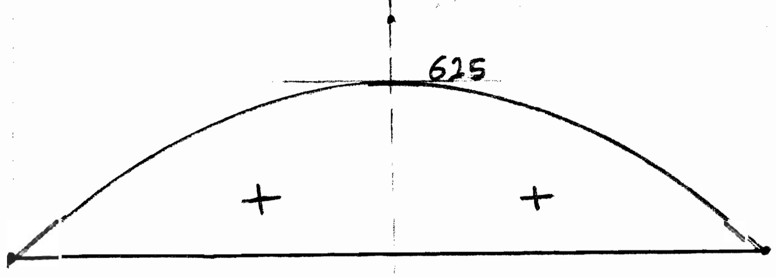
$$\Rightarrow \boxed{M = 250x - 25x^2 \quad (kN.m)}$$

The SFD & BMD are drawn below.





SFD (kN)



BMD (kN.m)

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Problem #4:-

- Given:-
  - The figure shown Fig. P4.
  -

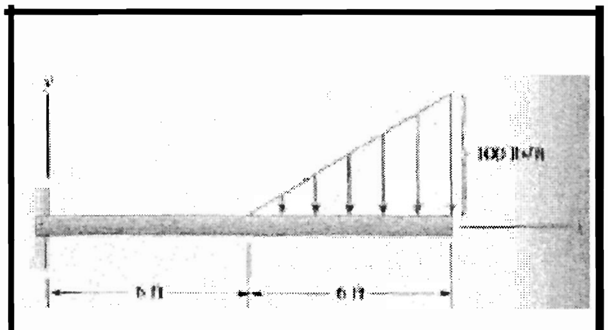


Fig. P4

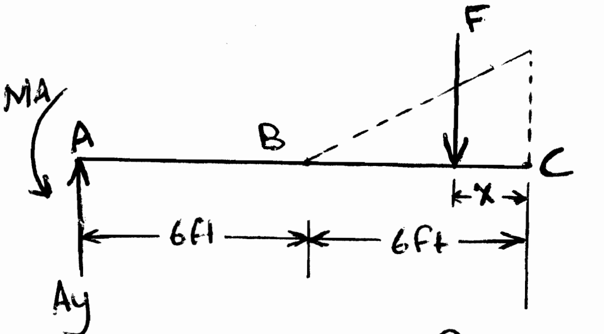
- Required:-
  - The equations of shear force and Bending Moment and their diagrams.

- Solution:-

The reactions are first determined from FBD ①

$F = 100(6)/2 = 300\,lb.$

$x = \frac{1}{3}(6) = 2\ ft$ (from the right)



FBD ①

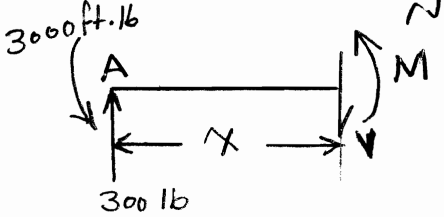
$+\uparrow \sum F_y = 0$

$\Rightarrow A_y - 300 = 0 \Rightarrow A_y = 300\,lb\ (\uparrow)$

$+\circlearrowleft \sum M_A = 0 \Rightarrow M_A - 300(12-2) = 0 \Rightarrow M_A = 3000\ ft.lb\ (\circlearrowleft)$

Two sections (segments) are needed. (why ?!)

AB : $0 \leq x \leq 6\ ft$



FBD ②

In FBD ②,

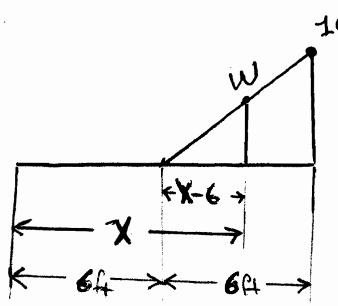
$+\uparrow \sum F_y = 0 \Rightarrow 300 - V = 0 \Rightarrow \boxed{V = 300\,lb}$

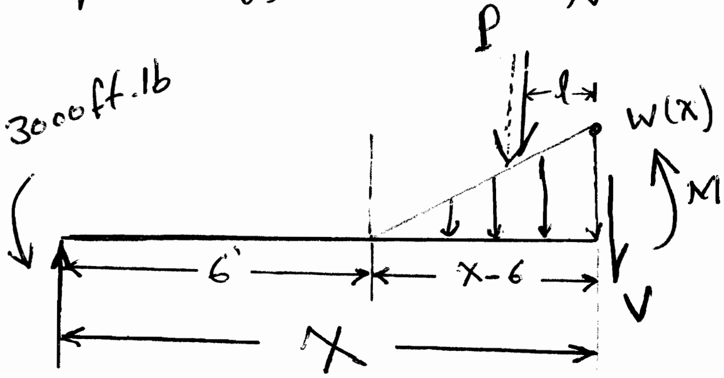
$+\circlearrowleft \sum M = 0 \Rightarrow$

$M + 3000 - 300x = 0 \Rightarrow \boxed{M = -3000 + 300x \quad (\text{ft.lb})}$

BC : $6' \leq x \leq 12'$

In FBD 3, the left part is taken (why ?!).





FBD 3

$$\frac{w}{100} = \frac{x-6}{6} \Rightarrow w = \frac{100}{6}(x-6)$$

$$P = \text{Area } \Delta = \frac{(w)}{2}(x-6) = \frac{100}{12}(x-6)(x-6)$$

$$= \frac{100}{12}(x-6)^2$$

$$l = \frac{1}{3}(x-6)$$

$$+\uparrow \sum F_y = 0 \Rightarrow 300 - \frac{100}{12}(x-6)^2 - V = 0$$

$$\Rightarrow \boxed{V = 300 - \frac{100}{12}(x-6)^2 \quad (\text{lb})}$$

OR, if expanded: $V = 300 - \frac{100}{12}x^2 + 100x - 300$

$$\Rightarrow \boxed{V = 100\left(x - \frac{x^2}{12}\right) \quad (\text{lb})}$$

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$$\overset{+}{\circlearrowleft}\ \sum M = 0 \Rightarrow M + 3000 - 300x + P.\ell = 0$$

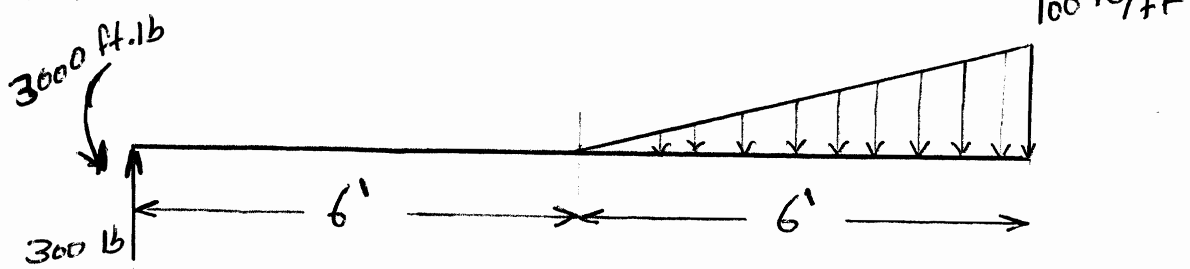
$$P\ell = \frac{100}{12}(x-6)^2\left[\frac{1}{3}(x-6)\right] = \frac{100}{36}(x-6)^3$$

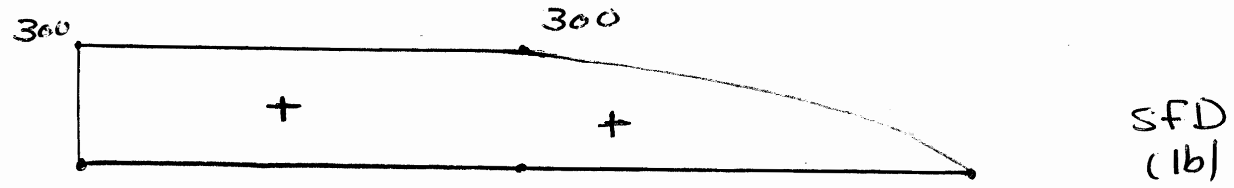
$$\Rightarrow \boxed{M = -3000 + 300x - \frac{100}{36}(x-6)^3 \quad (\text{ft.lb})}$$

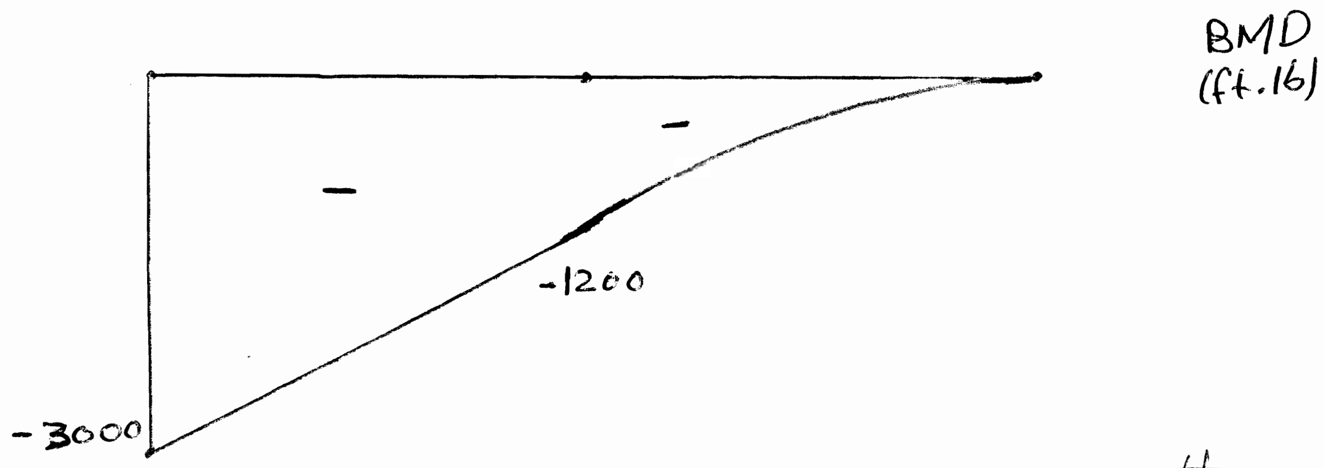
OR, if expanded,

$$\boxed{M = -2400 + 50x^2 - \frac{25}{9}x^3 \quad (\text{ft. lb})}$$

The SFD and BMD are drawn below.







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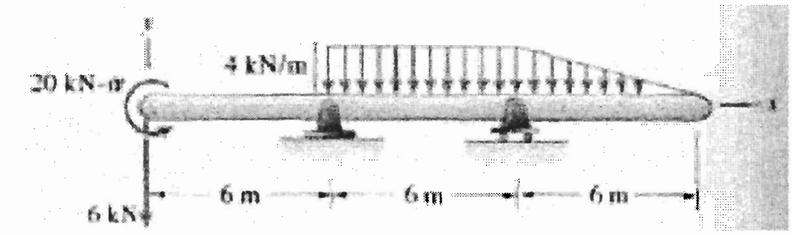


Fig. P5

Problem #5:-

- Given :- The figure shown above Fig. P5.

- Required:- The equations of the shear force and Bending moment and their diagrams.

- solution :-

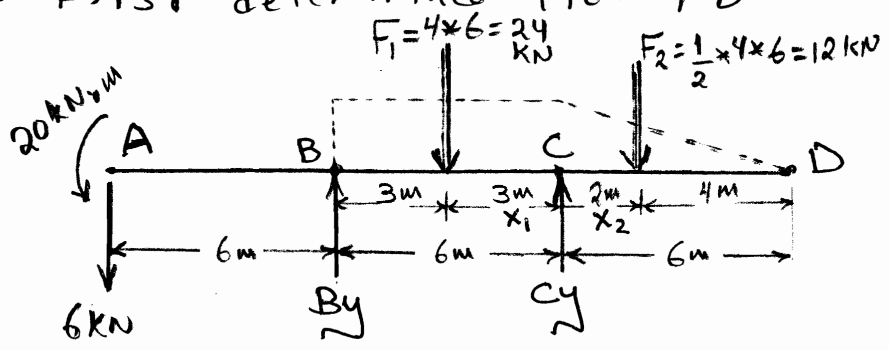
The reactions are first determined from FBD ①,

$F_1 = 4*6 = 24\,kN$

$x_1 = \frac{1}{2}*6 = 3\,m$

$F_2 = \frac{1}{2}*4*6 = 12\,kN$

$x_2 = \frac{1}{3}*6 = 2\,m$ from C (the left)



FBD ①

$$+\circlearrowleft \sum M_B = 0 \Rightarrow +20 + 6(6) - 24(3) - 12(8) + C_y(6) = 0$$

$$\Rightarrow C_y = 18.6667\text{ kN}.$$

$$+\circlearrowleft \sum M_C = 0 \Rightarrow +20 + 6(12) + 24(3) - 12(2) - B_y(6) = 0$$

$$\Rightarrow B_y = 23.3333\text{ kN}.$$

Note that three sections (segments) are needed. (Why?!)

→

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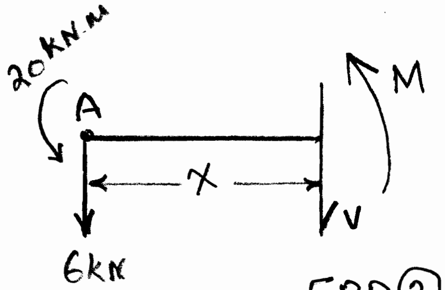
AB: $0 \le x \le 6$ m :-

In FBD ②,



FBD ②

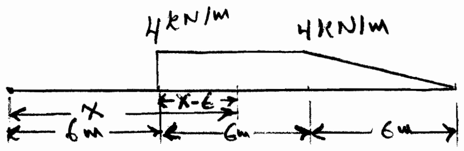
$+\uparrow \sum F_y = 0 \Rightarrow$

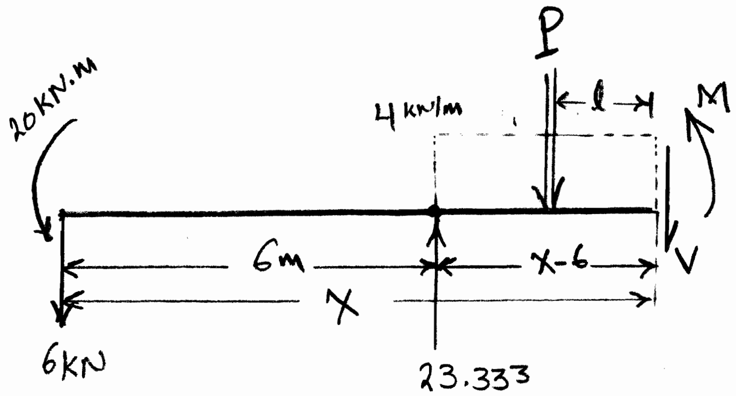
$$-6 - V = 0 \Rightarrow \boxed{V = -6 \text{ kN}}$$

$$+\circlearrowleft \sum M = 0 \Rightarrow M + 20 + 6(x) = 0 \Rightarrow \boxed{M = -20 - 6x \quad (\text{kN.m})}$$

BC: $6 \le x \le 12$ m :-

In FBD ③,





$$P = \text{Area } \square = 4 * (x-6) = 4x - 24$$

$$l = \frac{x-6}{2}$$

$$+\uparrow \sum F_y = 0 \Rightarrow -6 + 23.333 - [4x - 24] - V = 0$$

$$\Rightarrow \boxed{V = 41.33 - 4x \quad (\text{kN})}$$

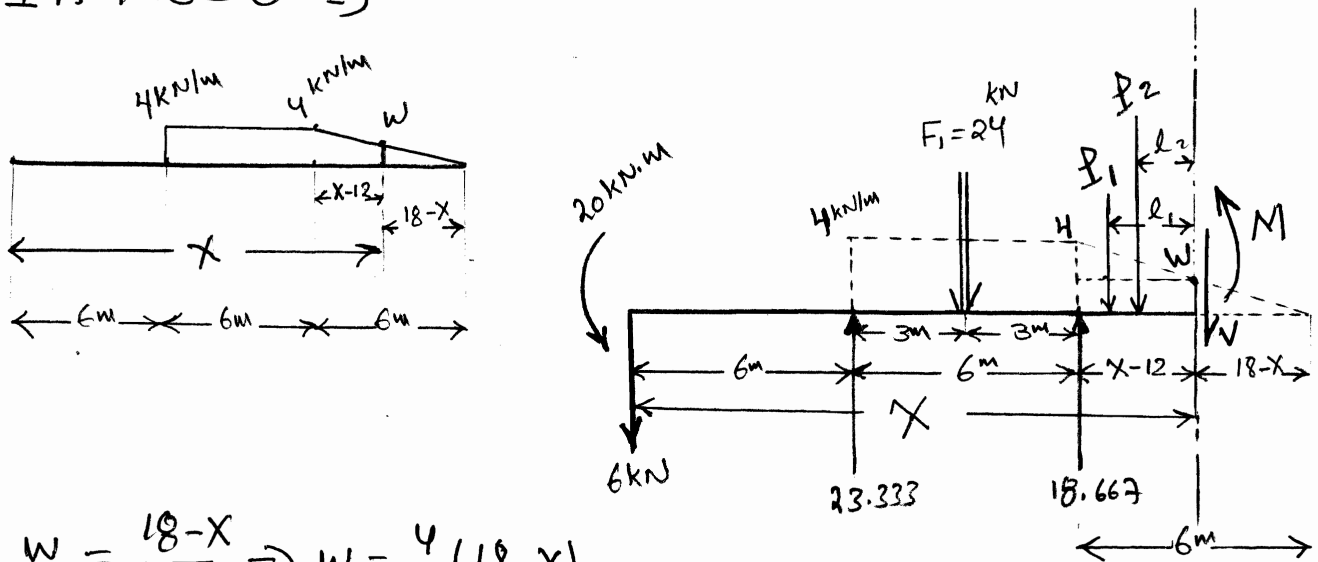
$$+\circlearrowleft \sum M = 0 \Rightarrow M + 20 + 6(x) + [4x - 24)\frac{(x-6)}{2} - 23.333(x-6) = 0$$

$$\Rightarrow M + 20 + 6x + (2x^2 - 24x + 72) - 23.333x + 139.998 = 0$$

$$\Rightarrow \boxed{M = -231.99 + 41.33x - 2x^2 \quad (\text{kN.m})}$$

CD: $12 \le x \le 18$:-

In FRD(4) $\Rightarrow$



$$\frac{W}{4} = \frac{18-X}{6} \Rightarrow W = \frac{4}{6}(18-X)$$

$$P_1 = \frac{1}{2}\left[(4-W)(X-12)\right] = \frac{1}{2}\left[\left(4-\frac{4}{6}(18-X)\right)(X-12)\right]$$

$$\Rightarrow P_1 = \frac{1}{2}\left[4X-48+\frac{4}{6}X^2-20X+144\right] = \frac{X^2}{3} - 8X + 48$$

$$\therefore P_1 = \frac{X^2}{3} - 8X + 48 \quad (KN)$$

$$P_2 = W(X-12) = \frac{4}{6}(18-X)(X-12) = -\frac{4}{6}X^2 + 20X - 144$$

$$\therefore P_2 = -\frac{2}{3}X^2 + 20X - 144 \quad (KN)$$

$$l_1 = \frac{2}{3}(X-12)\ (m)\ ;\ l_2 = \frac{1}{2}(X-12)\quad (m).$$

$$+\uparrow \sum F_y = 0 \Rightarrow -6 + 23.333 - 24 + 18.667 - \left(\frac{X^2}{3} - 8X + 48\right) -$$
$$\left(-\frac{2}{3}X^2 + 20X - 144\right) - V = 0$$

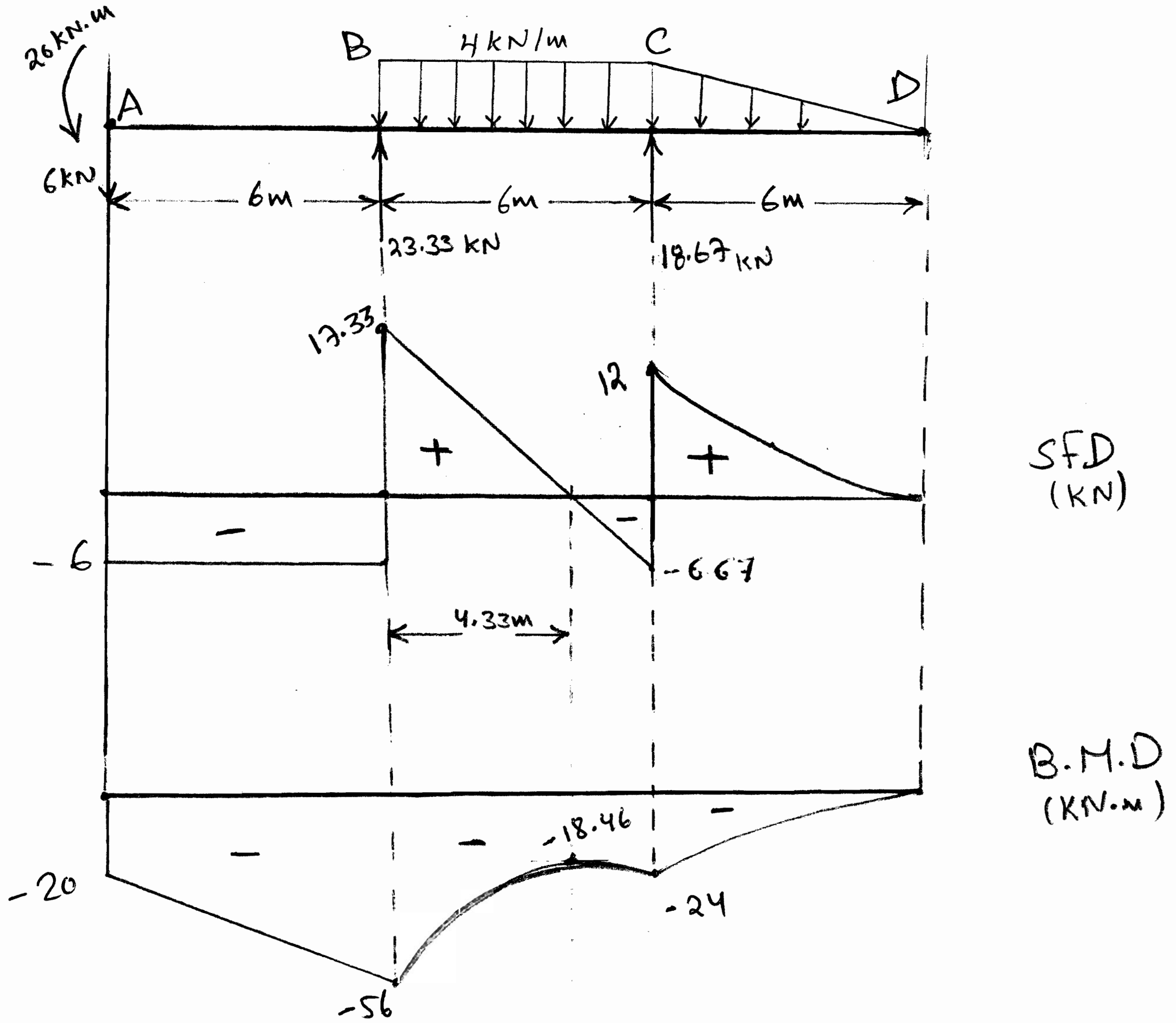
$$\Rightarrow \boxed{V = 108 - 12X + \frac{X^2}{3} \quad (KN)}$$

$$+\circlearrowleft \sum M = 0 \Rightarrow 20 + 6(x) - 23.333(x-6) + 24(x-9) - 18.667(x-12)$$
$$+ \left[\frac{x^2}{3} - 8x + 48\right]\left(\frac{2}{3}(x-12)\right) + \left(-\frac{2}{3}x^2 + 20x - 144\right)\left(\frac{1}{2}(x-12)\right) + M = 0$$

By expanding the above equation;

$$\Rightarrow \boxed{M = -648 + 108x - 6x^2 + x^3/9 \quad (\text{kN.m})}$$

The SFD and BMD are drawn below.



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