CE 201 STATICS (Sections 4 & 6) [081]

H.W. # 5 Solutions.

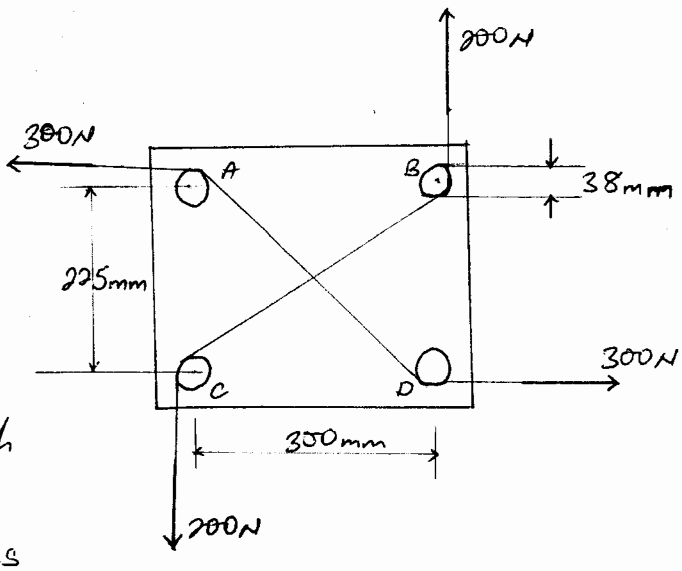
Problem 1

Given: The system shown.

Required: (a) The resultant couple acting on the board.

(b) if only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with minimum tension in the string?

(c) The value of that minimum tension.



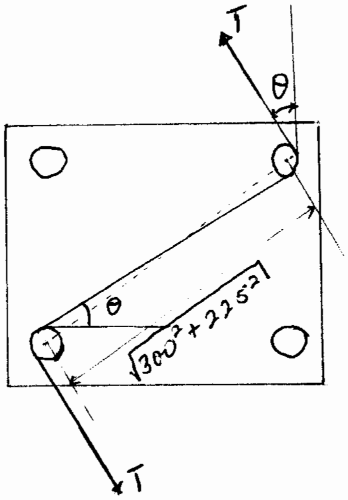
Solution

(a) $M = 200\left[0.3 + 2\,\frac{0.038}{2}\right] + 300\left[0.225 + 2\cdot\frac{0.038}{2}\right]$

$$\boxed{M = 146.5N}$$

(b) $\theta = \tan^{-1}\left(\frac{225}{300}\right)$

$$\boxed{\theta = 36.87}$$



(c) $d = \sqrt{300^2 + 225^2} + 2\cdot\frac{38}{2} = 413mm$

$\therefore T = \frac{M}{d} = \frac{146.5}{0.413} = 354.7N$

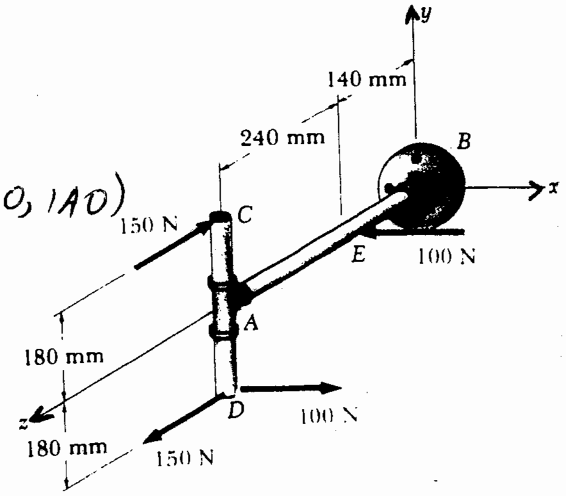
$$\boxed{T = 354.7N}$$

Note: The other string AD can be used.

Problem 2

Given: A system shown in figures P2 of the question sheets, with couples acting on the assembly.

Required: The Resultant Couple.



Solution

C(0, 180, 380); D(0, -180, 380); E(0, 0, 140)

$F_{D_1} = 100N$

$\vec{F}_{D_1} = 100\vec{i}$

$\vec{r}_{ED} = [0\vec{i} - 180\vec{j} + 240\vec{k}]$

$$\vec{M}_E = \vec{r}_{ED} \times \vec{F}_{D_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -180 & 240 \\ 100 & 0 & 0 \end{vmatrix} = 0\vec{i} - [-240 \times 100]\vec{j} + [180 \times 100]\vec{k}$$

$$\vec{M}_E = [24000\vec{j} + 18000\vec{k}]$$

$\vec{F}_{D_2} = 150\vec{k}$

$\vec{r}_{CD} = [0\vec{i} - 360\vec{j} + 0\vec{k}] = [-360\vec{j}]$

$$\vec{M}_c = \vec{V}_{CD} \times \vec{F}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -360 & 0 \\ 0 & 0 & 15 \end{vmatrix} = [-360 \times 150]\vec{i}$$

$\vec{M}_c = [-54000\vec{i}]$ (N·mm)

$\vec{M}_R = \vec{M}_E + \vec{M}_c$

$\vec{M}_R = [-54000\vec{i} + 24000\vec{j} + 18000\vec{k}]$ N.mm

Note: That you can do this easily by scalar.

$M = -360(150) = -54,000$ N mm $= 54$ N-m)+

Problem 3

Given: Figure P3 [Question sheet]

Required: Equivalent Force-Couple System at A.

Solution

A (0, 100, 0); B(75, 100, 50); C(75, 100, -50);

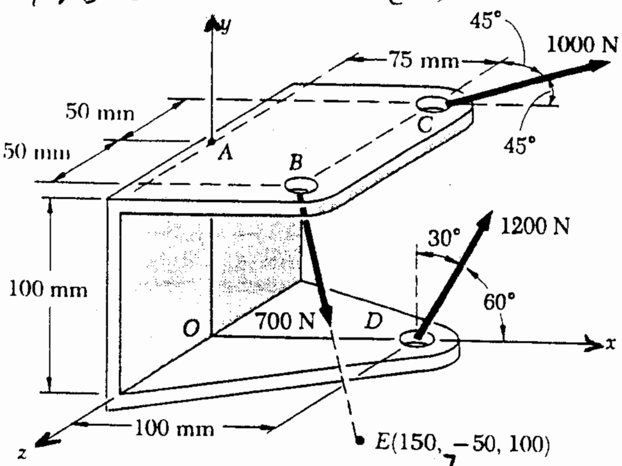
D(100, 0, 0); E(150, -50, 100)

$F_1 = 1000N$; $\vec{F_1} = 1000\cos 45\vec{i} - 1000\sin 45\vec{k} = [707.107\vec{i} - 707.107\vec{k}]$

$F_2 = 1200N$; $\vec{F_2} = 1200\cos 60\vec{i} + 1200\cos 30\vec{j} = [600\vec{i} + 1039.28\vec{j}]$

$F_3 = 700N$; $\vec{F_3} = 700\left[\frac{75\vec{i} - 150\vec{j} + 50\vec{k}}{175}\right] = [300\vec{i} - 600\vec{j} + 200\vec{k}]$

$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 1607.11\vec{i} + 439.230\vec{j} - 507.107\vec{k}$ (N)



Now; $\vec{r}_{AC} = 75\vec{i} + 0\vec{j} - 50\vec{k}$

$\vec{r}_{AC} = 75\vec{i} + 0\vec{j} + 50\vec{k}$

$\vec{r}_{AD} = 100\vec{i} - 100\vec{j} + 0\vec{k}$

$\circlearrowleft \vec{M}_A = \vec{r} \times \vec{F}$

$$[\vec{r}_{AC} \times \vec{F_1}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 75 & 0 & -50 \\ 707.107 & 0 & -707.107 \end{vmatrix} = \vec{j}[75 \times (-707.107) + 50 \times 707.107]$$

$$\Rightarrow [17677.7\vec{j}]$$

$$[\vec{r}_{AD} \times \vec{F_2}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 100 & -100 & 0 \\ 600 & 1039.23 & 0 \end{vmatrix} = \vec{k}[100 \times 1039.23 + 100 \times 600]$$

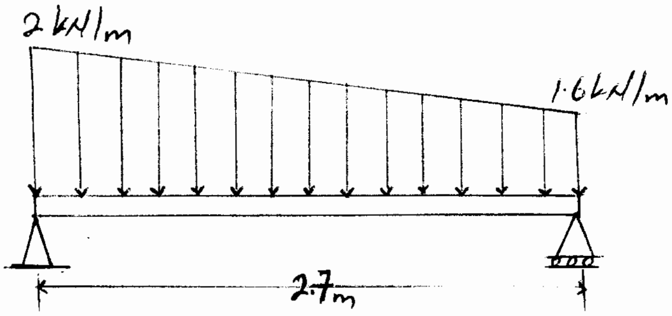
$$\Rightarrow [163923\vec{k}]$$

$$\left[\vec{r}_{AB} \times \vec{F}_3\right] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 75 & 0 & 50 \\ 300 & -600 & 200 \end{vmatrix} = \left[30000\,\vec{i} - 45000\,\vec{k}\right]$$

$$\vec{M}_A = \left[30000\,\vec{i} + 17677.7\,\vec{j} + 118923\,\vec{k}\right] \text{ N-mm}$$

$$= \underline{\underline{\left[30\,\vec{i} + 17.68\,\vec{j} + 118.9\,\vec{k}\right]}} \text{ N.m.}$$

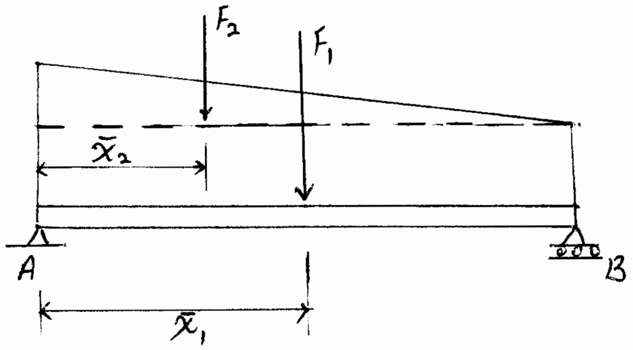
Problem 4



Given: The system shown in the figure.

Required:
Magnitude and locations of the resultant of the distributed load shown in figures.

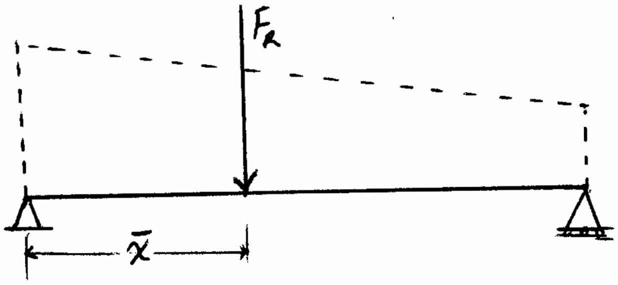
Solution



$F_1 = 2.7 \times 1.6 = 4.32 \text{ kN}$

$F_2 = \frac{1}{2}(0.4) \times 2.7 = 0.54 \text{ kN}$

$\therefore F_R = \Sigma F = F_1 + F_2 = 4.86 \text{ kN}$



$\Sigma M = \bar{x} F_R$

$\bar{x}_1 F_1 + \bar{x}_2 F_2 = \bar{x} F_R$

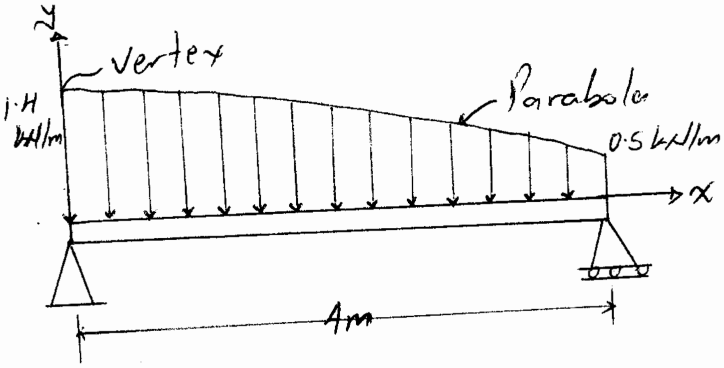
$1.35 \times 4.32 + 0.9 \times 0.54 = \bar{x} \times 4.86$

$\bar{x} = 1.3 \text{m}$

$F_R = 4.86 \text{ kN}$
$\bar{x} = 1.3 \text{m}$

Problem 5

Given: The system shown in figure 5



Required:
The magnitude and location of the resultants of the distributed load shown.

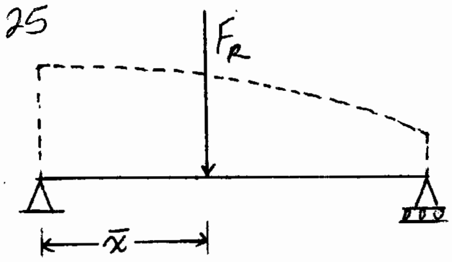
Solution

The equation of the distributed load is of the form;

$$w - a = bx^2$$

at $x = 0 \Rightarrow w = 1.4\ kN/m \Rightarrow a = 1.4$

at $x = 4m \Rightarrow w = 0.5\ kN/m \Rightarrow b = 0.05625$



$\therefore w = 1.4 - 0.05625x^2$

$\therefore F_R = \Sigma F = \int_0^4 (1.4 - 0.05625x^2)\, dx$

$\therefore F_R = \left[1.4x - \dfrac{0.05625}{3} x^3\right]_0^4 = 4.4\ kN$

The location of the resultant

$$\bar{x} = \frac{\int_0^4 (1.4 - 0.05625x^2)\, x\, dx}{\int_0^4 (1.4 - 0.05x^2)\, dx} = \left[\frac{1.4\frac{x^2}{2} - \frac{1}{4}0.05625x^4}{4.4}\right]_0^4$$

$\bar{x} = 1.727m$
$F_R = 4.4\ kN$