# CE 201 STATICS (Sections 4 & 6) [081]

## H. W. # 4 SOLUTIONS



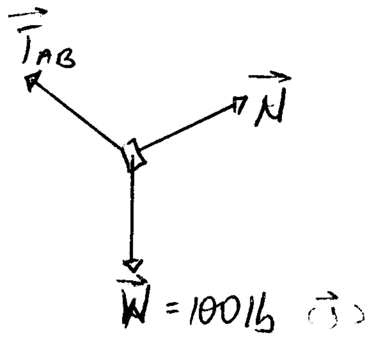
### Problem 1

**Given:** The figure shown, 100 lb slider held in place on a smooth circular bar by cable AB.

**Required:** (a) Tension in the cable.
(b) Normal force exerted on the weight by the bar.

**Solution**

① The FBD is drawn first.




FBD

② Coordinates

$A(4\cos 20°, 4\sin 20°, 0)$

$A(3.7588, 1.3681, 0)$

$B(0, 4, 3)$.

③ Position Vectors

$\vec{AB} = (B) - (A)$

$= -3.7588\vec{i} + 2.6319\vec{j} + 3\vec{k}$

④ Force Vector

$\vec{T}_{AB} = T_{AB}\vec{u}_T$

$= T_{AB}\vec{u}_{AB}$

$|\vec{AB}| = [(-3.7588)^2 + (2.6319)^2 + 3^2]^{1/2}$

$AB = 5.4823$ ft

$\vec{u}_{AB} = \frac{\vec{AB}}{AB} = (-0.68563\vec{i} + 0.48007\vec{j} + 0.54722\vec{k})$

$$\vec{T}_{AB} = T_{AB}(\vec{U}_{AB})$$
$$= T_{AB}(-0.68563\vec{i} + 0.48007\vec{j} + 0.54722\vec{k})$$

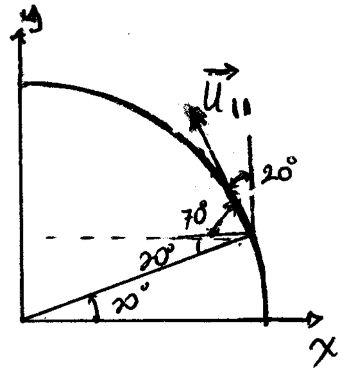
$$\vec{N} = N_x\vec{i} + N_y\vec{j} + N_z\vec{k}$$

We know that $\vec{N}$ is normal to the tangents at A

$A(\vec{U}_{11}) \Rightarrow$

$\vec{N}\cdot\vec{U}_{11} = 0$ (Since they are orthogonal)

$$\vec{U}_{11} = \cos(20° + 90°)\vec{i} + \sin(20° + 90°)\vec{j} + 0\vec{k}$$




or

$$U_{11} = -\cos 70\vec{i} + \sin 70\vec{j} + 0\vec{k}$$

$$\Rightarrow -3.4202 N_x + 0.93969 N_y = 0$$

$$\Rightarrow N_y = 0.36397 N_x \quad (1)$$

(5) $\Sigma\vec{F} = 0 = \vec{T} + \vec{N} + \vec{W} = 0$

$$\Sigma F_x = -0.68563\,T_{AB} + N_x = 0 \quad (2)$$
$$\Sigma F_y = 0.48007\,T_{AB} + N_y - 100 = 0 \quad (3)$$
$$\Sigma F_z = 0.54722\,T_{AB} + N_z = 0 \quad (4)$$

(6) Solvings the system of equations (1) to (4) yields,

$$\boxed{T_{AB} = 137.1 \text{ lb}}$$

$$\Rightarrow \boxed{\vec{T}_{AB} = -93.97\vec{i} + 65.80\vec{j} + 75.0\vec{k} \text{ (lb)}}$$

$\Rightarrow$

$N_x = 93.97 \text{ lb}$

$N_y = 34.20 \text{ lb}$

$N_z = -75.0 \text{ lb}$

$$\Rightarrow \boxed{\vec{N} = 93.97\vec{i} + 34.20\vec{j} - 75.0\vec{k} \text{ (lb)}}$$

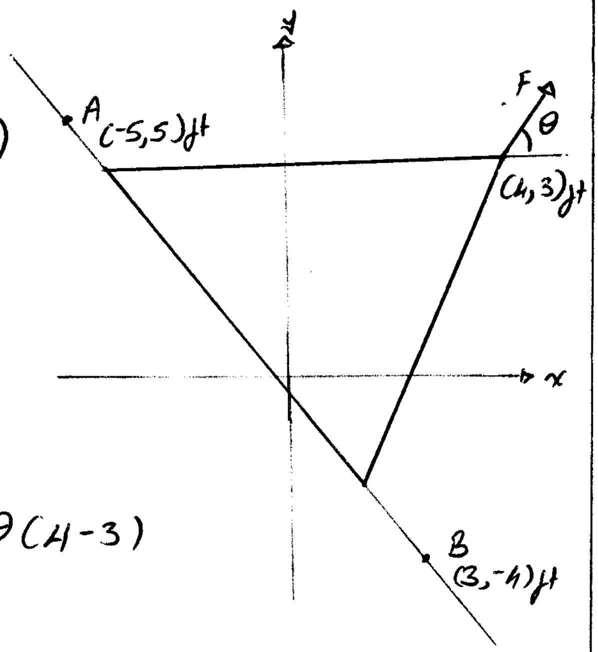
$$\Rightarrow \boxed{N = 125 \text{ lb}}$$

## Problem 2

Given: The figure shown with force, F exerting a 200 ft-lb clockwise moment about A and 100 ft-lb clockwise moment about B.

Required: F and $\theta$

## Solution




M of F about A is $M_A$

$M_A = F\cos\theta(5-3) + F\sin\theta(4-(-5))$

$\Rightarrow 2F\cos\theta + 9F\sin\theta = 200$

(Note that ccw (+) and cw (−))

M of F about B is $M_B$

$M_B = -F\cos\theta[3-(-4)] + F\sin\theta(4-3)$

$\Rightarrow -7F\cos\theta + F\sin\theta = -100$

Since $F\cos\theta = F_x$

$F\sin\theta = F_y$

Then $2F_x + 9F_y = 200$ ①

$-7F_x + F_y = -100$ ②

Solving equations ① and ② yields

$F_x = 16.92\text{ lb}$

$F_y = 18.46\text{ lb}$

$F = \sqrt{F_x^2 + F_y^2} \Rightarrow \boxed{F = 25.04\text{ lb}}$

$\tan\theta = F_y/F_x$, $\boxed{\theta = 47.49^\circ}$ ⇐ We can also use $\cos\theta = F_x/F$ or $\sin\theta = F_y/F$

## Problem 3

Given: The figure shown with an 80-N Force.

Required: (i) Moment of the force about origin O by using
a) The Scalar Method
b) Cross product, letting r be the vector from O to A
c) Cross product, letting r be the vector from O to B
(ii) The two dimensional description of the moment.

## Solution

(i)
(a) Scala Method

$M = Fd$

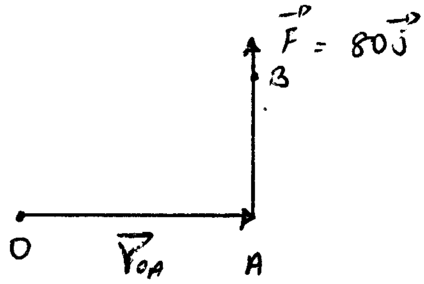
$= 80(6)$

$$\boxed{M = 480\ N.m \circlearrowright \quad \text{It is } M_z}$$

(b)
Cross product letting r be the vector from O to A.

$\vec{F} = 0\vec{i} + 8\vec{j} + 0\vec{k}$

$\vec{r}_{OA} = 6\vec{i} + 0\vec{j} + 0\vec{k}$




$M_o = \vec{r}_{OA} \times \vec{F}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 0 \\ 0 & 80 & 0 \end{vmatrix}$$

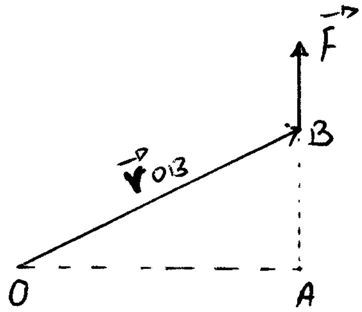
$= 0i - 0j + (6)(80)\vec{k}$

$$\boxed{\vec{M}_o = 480\vec{k}\ N.m}$$ as before

(c) Cross product letting r be vector from O to B

$\vec{r}_{OB} = 6\vec{i} + 4\vec{j} + 0\vec{k}$

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 4 & 0 \\ 0 & 80 & 0 \end{vmatrix}$$




$$= 0i - 0j + (6)(80)\vec{k}$$

$\boxed{\vec{M}_O = 480\vec{k} \text{ N·m}}$ as before.

(ii) The 2-D moment is $\pm dF$ when $d$ is the perpendicular distance from the point of interest to the line of action of F
ccw is ⊕, while cw is ⊖

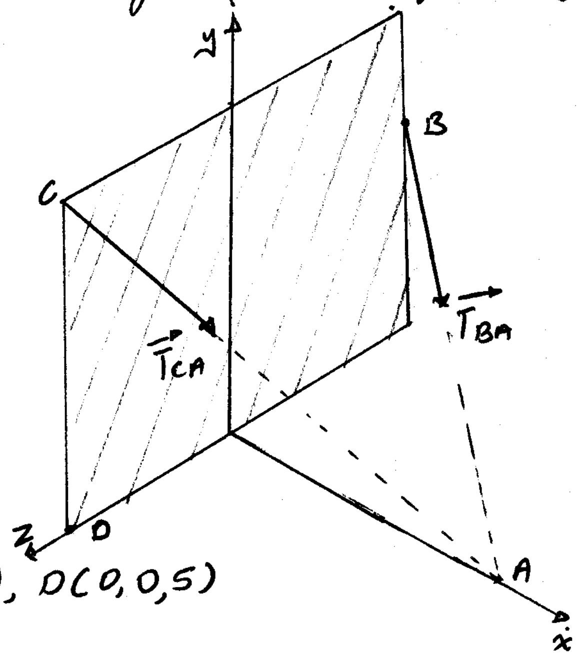
# Problem 4



Given: The figure shown, with tension in cable AB = 150N and tension in cable AC = 100N.

Required: Sum of moments about D due to forces exerted on the wall by the cables by
a) using vector DA for both forces
b) using vector DB for $T_{AB}$ and the vector DC for $T_{AC}$
c) first finding the resultant force, R, then finding M of R

Comparison of Results.




## Solution

(a)

$$\vec{M}_D = \vec{M}_{T_{BA}} + \vec{M}_{T_{CA}}$$
$$= \vec{V}_{DA} \times \vec{T}_{BA} + \vec{V}_{DA} \times \vec{T}_{CA}$$

A(8,0,0), B(0,4,-5), C(0,8,5), D(0,0,5)

$$\vec{V}_{DA} = (A-D) = 8\vec{i} + 0\vec{j} - 5\vec{k}$$

$$\vec{T}_{BA} = T_{BA}\vec{U}_{T_{BA}}$$ ((Note BA <u>not</u> AB. Why?!))

$$= T_{BA}\vec{U}_{BA}$$

$$\vec{BA} = (A)-(B) = 8\vec{i} - 4\vec{j} + 5\vec{k}$$

$$BA = \sqrt{105} \approx 10.247\,m$$

$$\vec{U}_{BA} = \frac{1}{\sqrt{105}}(8\vec{i} - 4\vec{j} + 5\vec{k})$$

$$= 0.78072\vec{i} - 0.39036\vec{j} + 0.48795\vec{k}$$

$$\vec{T}_{BA} = T_{BA}\vec{U}_{BA} = 117.11\vec{i} - 58.554\vec{j} + 73.192\vec{k} \text{ (N)}$$

$$\vec{T}_{CA} = T_{CA}\,\vec{U}_{T_{CA}} \quad \text{((Note CA not AC. Why?!))}$$
$$= T_{CA}\,\vec{U}_{CA}$$

$$\vec{CA} = (A)-(C) = 8\vec{i} - 8\vec{j} - 5\vec{k}$$
$$CA = \sqrt{153}\ m \approx 12.369 m$$
$$\vec{U}_{CA} = \frac{1}{\sqrt{153}}(8\vec{i} - 8\vec{j} - 5\vec{k})$$
$$= 0.64676\vec{i} - 0.64676\vec{j} - 0.40422\vec{k}$$
$$\vec{T}_{CA} = T_{CA}\vec{U}_{CA} = 64.676\vec{i} - 64.676\vec{j} - 40.422\vec{k}\ (N)$$

$$\vec{M}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & 0 & -5 \\ 117.11 & -58.554 & 73.192 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & 0 & -5 \\ -64.676 & -64.676 & -40.422 \end{vmatrix}$$

$$= (-292.77\vec{i} - 1171.1\vec{j} - 468.43\vec{k}) + (-323.38\vec{i} + 0\vec{j} - 517.41\vec{k})$$

$$\boxed{\vec{M}_D = -616.2\vec{i} - 1171\vec{j} - 985.8\vec{k}\ (N.m)}$$

(b) Using $\vec{DB}$ and $\vec{DC}$

$$\vec{V}_{DB} = 0\vec{i} + 4\vec{j} - 10\vec{k}$$
$$\vec{V}_{DC} = 0\vec{i} + 8\vec{j} + 0\vec{k}$$

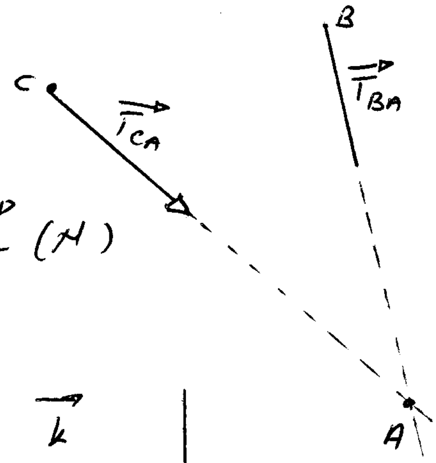
$$\Rightarrow \vec{M}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & -10 \\ 117.11 & -58.554 & 73.192 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 8 & 0 \\ -64.676 & -64.676 & -40.422 \end{vmatrix}$$

$$= (-292.77\vec{i} - 1171.1\vec{j} - 468.44\vec{k}) + (-323.38\vec{i} + 0\vec{j} - 517.41\vec{k})$$

$$\boxed{\vec{M}_D = -616.2\vec{i} - 1171\vec{j} - 985.8\vec{k}\ (N.m)}$$

(c)

Extend the two tensions to (A)




$$\sum \vec{F} = \vec{R} = \vec{T}_{BA} + \vec{T}_{CA}$$

$$= 181.79\vec{i} - 123.23\vec{j} + 32.77\vec{k} \ (N)$$

$\vec{V}$ is chosen on $\vec{V}_{DA}$ (why?!)

$$\Rightarrow \vec{M}_D = \vec{V}_{DA} \times \vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & 0 & -5 \\ 181.79 & -123.23 & 32.77 \end{vmatrix}$$

$$\boxed{\vec{M}_D = -616.2\vec{i} - 1171\vec{j} - 985.8\vec{k} \ (N.m)}$$

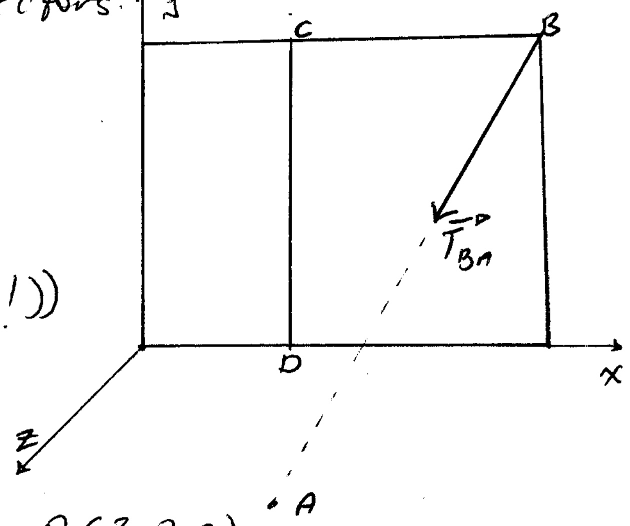
Note that all the results of the three parts (a), (b) and (c) are the same.

Thus, in general, choose the easiest one/method if you have the choice.

## Problem 5

Given: The figures shown with tension in cable $AB = 80\,lb$

Required: Moment about line CD due to force exerted by the cable on the wall at B, using four different position vectors.




## Solution

$$M = U\cdot(\vec{V}\times\vec{F})$$

$\vec{U}_{CD} = -\vec{j}$ ((why and how??!!))

$$\vec{T}_{BA} = T_{BA}\,\vec{U}_T = T_{BA}\,\vec{U}_{BA}$$

$A(6,0,10)$, $B(8,6,0)$, $C(3,6,0)$, $D(3,0,0)$

$$\vec{BA} = (A)-(B) = -2\vec{i} - 6\vec{j} + 10\vec{k}$$

$$BA = \sqrt{140} \approx 11.8322\,ft$$

$$\vec{T}_{BA} = \frac{80}{\sqrt{140}}(-2\vec{i} - 6\vec{j} + 10\vec{k})$$

$$= -13.522\vec{i} - 40.567\vec{j} + 67.612\vec{k}\ (lb)$$

First, Take $\vec{r}$ as CB

$$\Rightarrow \vec{CB} = 5\vec{i} + 0\vec{j} + 0\vec{k}$$

$$\Rightarrow M_{CD} = \begin{vmatrix} 0 & -1 & 0 \\ 5 & 0 & 0 \\ -13.522 & -40.567 & 67.612 \end{vmatrix}$$

$$= -(-1)(5)(67.612)$$


$$\Rightarrow \boxed{M_{CD} = 338.1\ ft\cdot lb}\ (+)$$

## Second

Take $\vec{r}$ as $\vec{DB}$

$\vec{DB} = 5\vec{i} + 6\vec{j} + 0\vec{k}$

$$M_{CD} = \begin{vmatrix} 0 & -1 & 0 \\ 5 & 6 & 0 \\ -13.522 & -40.567 & 67.616 \end{vmatrix}$$

$$= -(-1)(5)(67.612)$$

$$\boxed{M_{CD} = 338.1 \text{ ft.lb}}$$

## Third

Take $\vec{r}$ as $\vec{CA}$

$\vec{CA} = 3\vec{i} - 6\vec{j} + 10\vec{k}$

$$M_{CD} = \begin{vmatrix} 0 & -1 & 0 \\ 3 & -6 & 10 \\ -13.522 & -40.567 & 67.612 \end{vmatrix}$$

$$= 0 - (-1)[(3)(67.612) - 10(-13.522)] + 0$$

$$\Rightarrow \boxed{M_{CD} = 338.1 \text{ ft.lb}}$$

Fourth

Take F as $\vec{OA}$

$\vec{OA} = 3\vec{i} + 0\vec{j} + 10\vec{k}$

$$M_{CD} = \begin{vmatrix} 0 & -1 & 0 \\ 3 & 0 & 10 \\ -13.522 & -40.567 & 67.612 \end{vmatrix}$$

$$= 0 - (-1)[(3)(67.612) - 10(-13.522] + 0$$

$$\boxed{M_{CD} = 338.1 \text{ ft.lb}}$$

$\vec{M}_{CD}$ as a vector in $M_{CD}\vec{U}_{CD}$

$$\boxed{\vec{M}_{CD} = 0\vec{i} - 338.1\vec{j} + 0\vec{k} \text{ (ft.lb)}}$$

which means it is directed down in the negative y direction, which is the same as from C to D (CD).

Note that all answers are the same.
Thus in general, choose the easiest, which in this case is the first one ($\vec{CB}$).