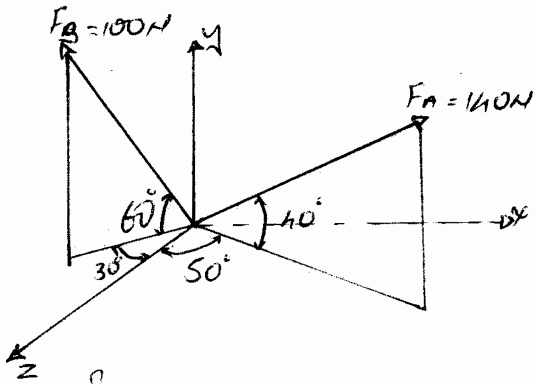
CE 201 STATICS (Sections 4 & 6)

FIRST SEMESTER - 081, H.W. #2 Solution

PROBLEM 1

Given: The figure shown

Required: Value and direction of the resultant.



Solution:

Obtaining the forces in cartesian vectors.

$F_{Ay} = 140 \sin 40° = 89.9903 j$

$F_{Ax} = (140 \cos 40°) \sin 50° = 82.1554$

$F_{Az} = (140 \cos 40°) \cos 50° = 68.9365 z$

$\vec{F}_A = 82.1554 \vec{i} + 89.9903 \vec{j} + 68.9365 \vec{z}$ (N)

$F_{Bx} = (100 \cos 60°) \sin 30° = -25 i$

$F_{By} = 100 \sin 60° = 86.6025 j$

$F_{Bz} = (100 \cos 60°) \cos 30° = 43.3013 z$

$\vec{F}_B = -25 \vec{i} + 86.6025 \vec{j} + 43.3013 \vec{z}$ (N)

Resultant $\vec{F}_R = \vec{F}_A + \vec{F}_B$

$F_R = 57.1554 i + 176.5928 j + 112.2378 z$

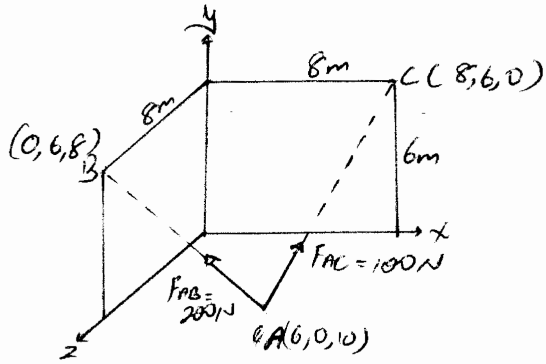
$F_R = [(57.1554)^2 + (176.5928)^2 + (112.2378)^2]^{1/2}$, $\boxed{F_R = 216.91 N}$

$\cos\theta_x = \frac{F_x}{F_R} = \frac{57.1554}{216.91}$, $\cos\theta_y = \frac{F_y}{F_R} = \frac{176.5928}{216.91}$, $\cos\theta_z = \frac{F_z}{F_R} = \frac{112.2378}{216.91}$

$\boxed{\theta_x = 74.72°}$ $\boxed{\theta_y = 35.50°}$ $\boxed{\theta_z = 58.84}$

CE 201 STATICS (Sections 4 & 6)
FIRST SEMESTER - 081, H.W. # 2

PROBLEM 2



Given: The figure shown
$F_{AB} = 200N$, $F_{AC} = 100N$

Required: Value and direction of the resultant.

Solution

Obtaining the cartesian vectors and length of the two forces

$\vec{AB} = (0-6)\vec{i} + (6-0)\vec{j} + (8-10)\vec{k} = -6\vec{i} + 6\vec{j} - 2\vec{k}$, $AB = \sqrt{76}$ m

$\vec{AC} = (8-6)\vec{i} + (6-0)\vec{j} + (0-10)\vec{k} = 2\vec{i} + 6\vec{j} - 10k$, $AC = \sqrt{140}$ m

$$\vec{F}_{AB} = F_{AB}\vec{U}_{AB} = F_{AB}\frac{\vec{AB}}{AB} = \frac{200}{\sqrt{76}}(-6i + 6j - 2k)$$

$$\vec{F}_{AB} = (-137.65\vec{i} + 137.65\vec{j} - 45.8831k)\,N$$

$$\vec{F}_{AC} = \frac{100}{\sqrt{140}}(2i + 6j - 10k),$$

$$\vec{F}_{AC} = (16.9031i + 50.7093j - 84.5154k)\,N$$

$$\vec{R} = \vec{F}_{AB} + \vec{F}_{AC} = -120.7469i + 188.3593j - 130.3985k\ N$$

$$|\vec{R}| = \left[(-120.7469)^2 + (188.3593)^2 + (-130.3985)^2\right]^{1/2}$$

$$\boxed{R = 259.0N}$$

Direction

$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

$$\theta = \cos^{-1}\left[\frac{-120.7469}{259}\right] \qquad \theta_j = \cos^{-1}\left[\frac{188.3593}{259}\right] \qquad \theta_z = \cos^{-1}\left[\frac{130.3985}{259}\right]$$

$$\boxed{\theta = 117.8^\circ} \qquad \boxed{\theta_y = 43.34^\circ} \qquad \boxed{\theta_z = 120.23^\circ}$$

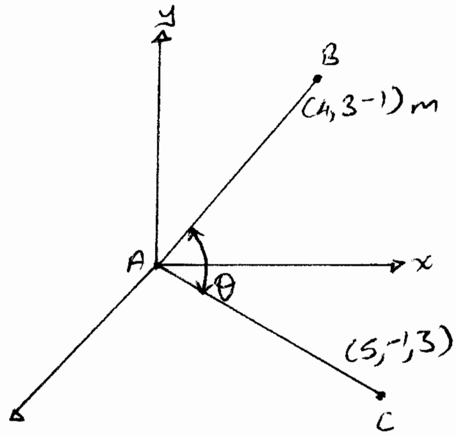
CE 201 STATICS (Section 4 & 6)

First Semester - 081, H.W. #1.

Problem 3

Given: The diagrams shown

Required: θ between lines AB and AC

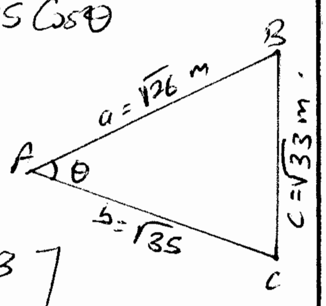


Solution

$$\vec{AB} = (4-0)\vec{i} + (3-0)\vec{j} + (-1-0)\vec{k}$$
$$= 4\vec{i} + 3\vec{j} - \vec{k}$$
$$AB = [(4)^2 + (3)^2 + (1)^2]^{1/2} = \sqrt{26}\ m$$
$$\vec{AC} = (5-0)\vec{i} + (-1-0)\vec{j} + (3-0)\vec{k}$$
$$= 5\vec{i} - \vec{j} + 3\vec{k}$$
$$AC = [(5)^2 + (-1)^2 + (3)^2]^{1/2} = \sqrt{35}\ m$$
$$\vec{BC} = (5-4)\vec{i} + (-1-3)\vec{j} + (3 - -1)\vec{k}$$
$$= \vec{i} - 4\vec{j} + 4\vec{k}$$
$$BC = \sqrt{(1^2) + (-4)^2 + (4)^2} = \sqrt{33}\ m$$

a) By Using Law of Cosine.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$
$$33 = 26 + 35 - 2\sqrt{26}\sqrt{35}\cos\theta$$
$$\cos\theta = \frac{26 + 35 - 33}{2\sqrt{26}\sqrt{33}}$$



$$\theta = \cos^{-1}\left[\frac{26 + 35 - 33}{2\sqrt{26}\sqrt{35}}\right]$$

$$\boxed{\theta = 62.35^\circ}$$

b) By Using the dot product

$$\cos\theta = \frac{\vec{AB}\cdot\vec{AC}}{(AB)(AC)}$$
$$= \frac{(4\vec{i} + 3\vec{j} - \vec{k})\cdot(5\vec{i} - \vec{j} + 3\vec{k})}{(\sqrt{26})(\sqrt{35})}$$
$$\cos\theta = \frac{(4)(5) + (3)(-1) + (-1)(3)}{\sqrt{26}\sqrt{35}}$$

$$\boxed{\theta = 62.35^\circ}$$

CE 201 STATICS (Sections 4&6)

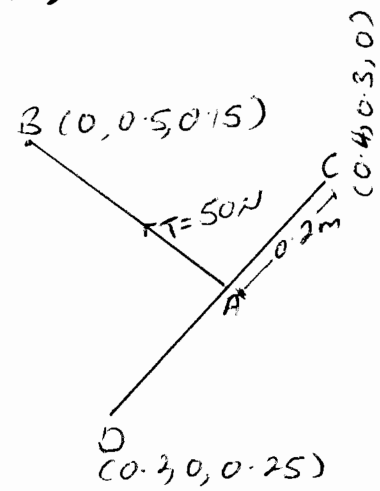
FIRST SEMESTER - 081, H.W. #2

Problem 4

Given: As in the diagram shown

Required: a) Angle between AB and AD

b) Component of T normal to CD.



Solution:

B (0, 0.5, 0.15)

C (0.4, 0.3, 0)

D (0.2, 0, 0.25)

a)

$\vec{CD} = -0.2\vec{i} - 0.3\vec{j} + 0.25\vec{k}$

$CD = \sqrt{0.1925} \approx 0.43875m$

$AD \approx 0.43875 - 0.2 \approx 0.23875m$

$$A_x = \left[\frac{0.2}{0.43875}(0.2) + \frac{0.23875}{0.43875}(0.4)\right] = 0.30883$$

$$A_y = \left[\frac{0.2}{0.43875}(0) + \frac{0.23875}{0.43875}(0.3)\right] = 0.16325$$

$$A_z = \left[\frac{0.2}{0.43875}(0.25) + \frac{0.23875}{0.43875}(0)\right] = 0.11396$$

$\vec{AB} = -0.30883\vec{i} + 0.33675\vec{j} + 0.03604\vec{k}$

$AB = 0.45834 m$

$\vec{AD} = -0.10883\vec{i} - 0.16325\vec{j} + 0.13604\vec{k}$

$AD = 0.23875m$

$$\vec{U}_{AD} = \frac{\vec{AD}}{AD} = -0.45583\vec{i} - 0.68377\vec{j} + 0.36780\vec{k}$$

H.W #2

Problem n

$$\cos\theta = \frac{\vec{AB}\cdot\vec{AD}}{(AB)(AD)}$$

$$= -0.15043$$

$$\boxed{\theta = 98.652^\circ}$$

b) (i) Component of T normal to CD utilizing result in (a) above

$$T_\perp = T\sin\theta$$

$$= 50\sin(98.652)$$

$$\boxed{T_\perp = 49.43N}$$

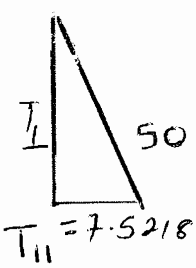
(ii) Component of T normal to CD using general method (assuming angle in (a) is not known.

$$T_{11} = \vec{T}\,\vec{U}_{AD}$$

$$T = \frac{T}{AB} \quad = -33.690\vec{i} + 36.736\vec{j} + 3.9316\vec{k}$$

$T_{11} = -7.5218N$ (− means in AC not AD

$$T_\perp = \sqrt{50^2 - (-7.5218)^2}$$



$$\boxed{T_\perp = 49.43N}$$

as in b(i) above.