CE 201 STATICS (Sections 4&6)

First Semester (081)

H.W# 10 Solutions.

Problem 1.

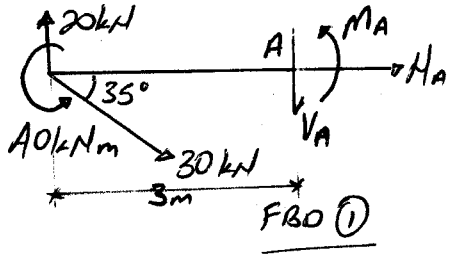
Given: The figure P1 as shown in question sheet

Required: Internal forces at A, B and C.

Solution.

At A, we do not need to find the reactions.

From FBD ①,



$+\uparrow \Sigma F_y = 0 \Rightarrow 20 - 30\sin 35 - V_A = 0$

$\Rightarrow \boxed{V_A = 2.793\ kN}$

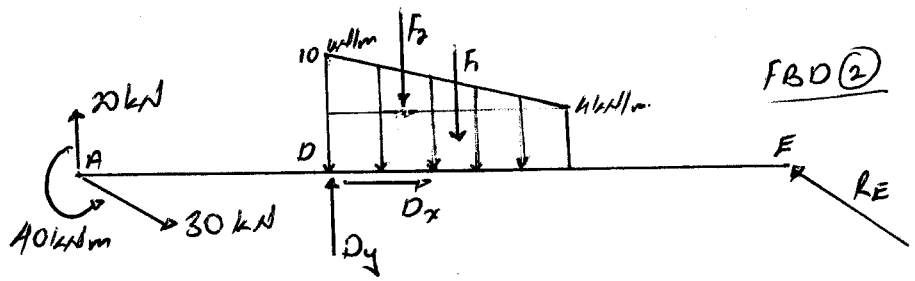
$\pm \rightarrow \Sigma F_x = 0 \Rightarrow 30\cos 35 + H_A = 0$

$\Rightarrow \boxed{H_A = -24.57\ kN}$

$+\circlearrowleft M_A = 0 \Rightarrow -20(3) + 30\sin 35(3) + 40 + M_A = 0$

$\Rightarrow \boxed{M_A = -31.62\ kN.m}$

To find the internal forces at B and C, we need to find the reactions at D and E.



From FBD ②

$F_1 = 4 \times 6 = 24\ kN$, acts 3m from D

$F_2 = 6 \times \frac{6}{2} = 18\ kN$, acts 2m from D

$+\circlearrowleft \sum M_E = 0 \Rightarrow 40 - 20(15) + 30\sin 35(15) + D_y(11) + 18(9) + 24(8) = 0$

$\Rightarrow D_y = 32.01\ kN.$

$+\uparrow \sum F_y = 0 \Rightarrow 20 - 30\sin 35 + 32.01 - 18 - 24 + R_E \cos 75^\circ = 0$

$\Rightarrow R_E = 27.8082\ kN$

$\pm \rightarrow \sum F_x = 0 \Rightarrow 30\cos 35 + D_x - R_E \sin 75 = 0$

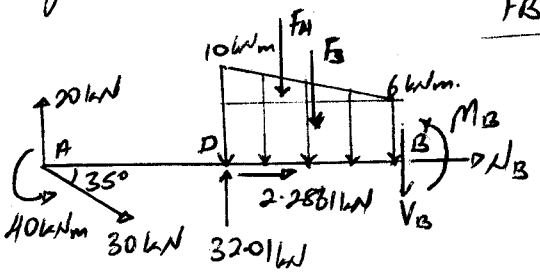
$\Rightarrow D_x = 2.2861\ kN$

Taking the left section from B.

FBD ③

From FBD ③,

$F_3 = 6 * 4 = 24\ kN$
acts 2m from D



$F_4 = 4 \times \frac{4}{2} = 8\ kN$ (acts $\frac{4}{3}$ m from D)

$+\uparrow \sum F_y = 0 \Rightarrow 20 - 30\sin 35 + 32.01 - 24 - 8 - V_B = 0$

$\Rightarrow \boxed{V_B = 2.803\ kN}$

$\pm \rightarrow \sum F_x = 0 \Rightarrow 30\cos 35 + 2.2861 + N_B = 0$

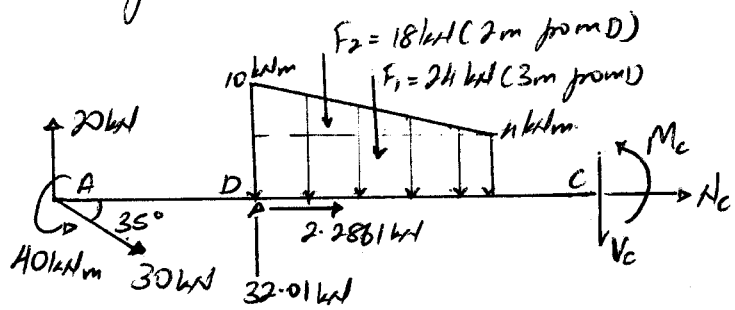
$\Rightarrow \boxed{N_B = -26.8607\ kN}$

$+\circlearrowleft M_B = 0 \Rightarrow 40 + 30\sin 35(8) - 20(8) - 32.01(4) + 24(2) + 8(8/3) + M_B = 0$

$\Rightarrow \boxed{M_B = 41.11\ kNm}$

Taking the left section from C

FBD (4)



From FBD (4),

$+\uparrow \Sigma F_y = 0 \Rightarrow 20 - 30\sin 35 + 32.01 - 18 - 24 - V_c = 0$

$\Rightarrow \boxed{V_c = -7.19\ kN}$

$\pm \Sigma F_x = 0 \Rightarrow 30\cos 35 + 2.2861 + H_c = 0$

$\Rightarrow \boxed{H_c = -26.8607\ kN}$

$+) \Sigma M_c = 0 \Rightarrow 40 + 30\sin 35(12) - 20(12) - 32.01(8) + 18(6) + 24(5) + M_c = 0$

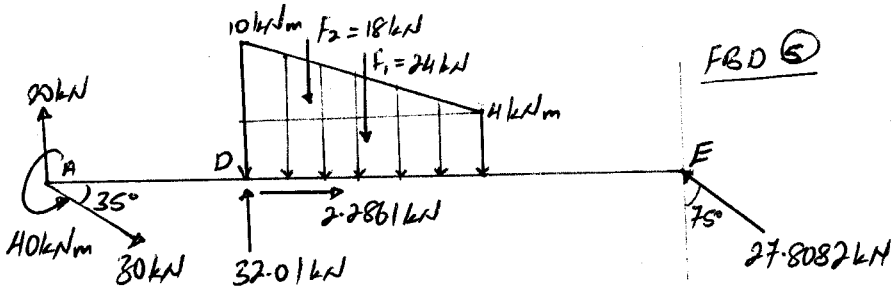
$\Rightarrow \boxed{M_c = 21.52\ kNm}$

Note that for consistency, we take the left section at the points of interest.

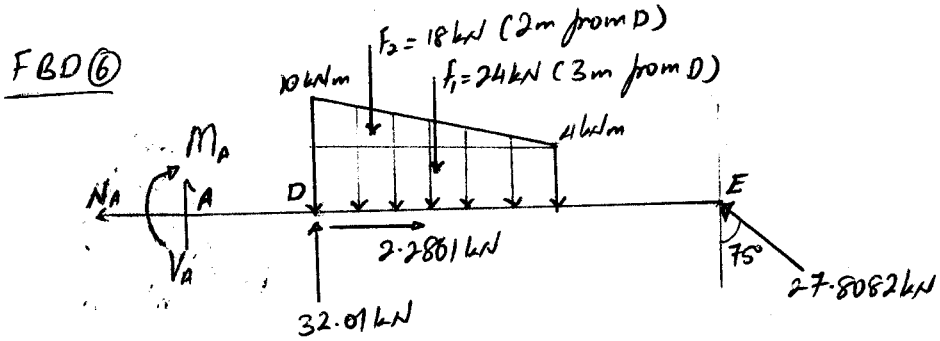
Problem 1 is also worked out when taking the right section at the points of interest.

Note that in this case, we have to first find the reactions.

FBD⑤ of beam with reactions as computed before



Taking the right section from A.



In FBD ⑥

$+\uparrow \sum F_y = 0 \Rightarrow V_A + 32.01 - 18 - 24 + 27.8082 \cos 75 = 0$

$\Rightarrow \boxed{V_A = -2.793 \text{ kN}}$

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -N_A + 2.2861 - 27.8082 \sin 75 = 0$

$\Rightarrow \boxed{N_A = -24.57 \text{ kN}}$

$\circlearrowleft \sum M_A = 0 \Rightarrow -M_A + 32.01(1) - 18(3) - 24(4) + 27.8082 \cos 75 (12) = 0$

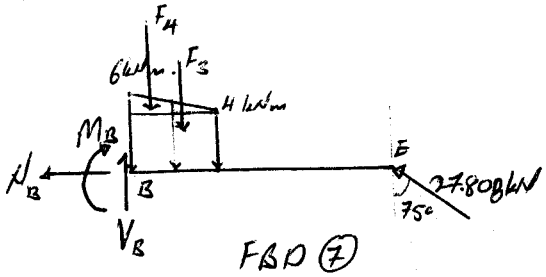
$\Rightarrow \boxed{M_A = -31.62 \text{ kNm}.}$

Taking the right section from B

In FBD ⑦,

$F_3 = 2 \times 4 = 8\,kN$, (acts 1m from B)

$F_4 = 2 \times \frac{2}{2} = 2\,kN$, (acts $\frac{2}{3}$ m from B)



FBD ⑦

$+\uparrow \Sigma F_y = 0 \Rightarrow V_B - 2 - 8 + 27.808(\cos 75) = 0$

$\Rightarrow \boxed{V_B = 2.802\,kN}$

$\pm \rightarrow \Sigma F_x = 0 \Rightarrow N_B - 27.808 \sin 75 = 0$

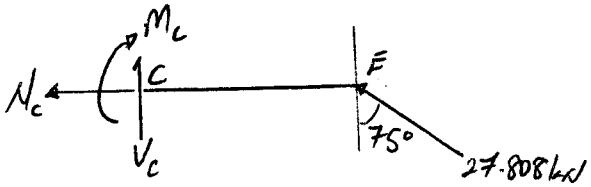
$\Rightarrow \boxed{N_B = -26.8607\,kN}$

$+\circlearrowleft M_B = 0 \Rightarrow -M_B - 2(2/3) - 8(1) + 27.808 \cos 75(7) = 0$

$\Rightarrow \boxed{M_B = 41.048\,kNm}$

Taking the right section from C

In FBD ⑧



FBD ⑧

$+\uparrow \Sigma F_y = 0 \Rightarrow V_C + 27.808 \cos 75°$

$\Rightarrow \boxed{V_C = 7.197\,kN}$

$\pm \rightarrow \Sigma F_x = 0 \Rightarrow -N_C - 27.808 \sin 75° = 0$

$\boxed{N_C = -26.861\,kN}$

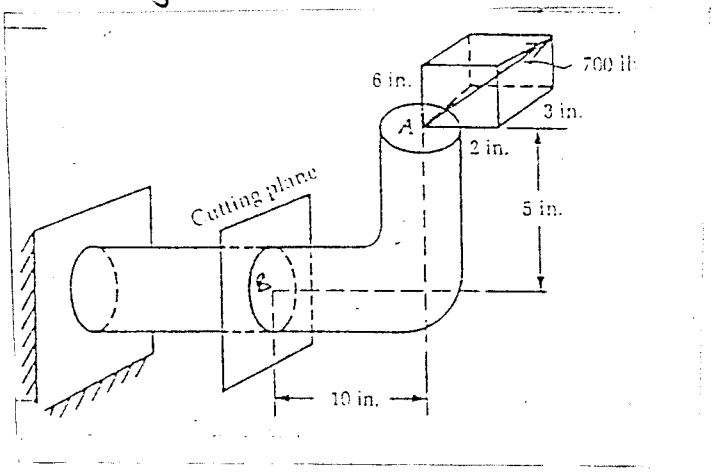
$+\circlearrowleft \Sigma M_C = 0 \Rightarrow -M_C + 27.808 \cos 75(5) = 0$

$\Rightarrow \boxed{M_C = 21.59\,kNm}$

Problem 2

Given: Figure P.2 as shown in the question sheet.

Required: The internal forces acting on the cutting plane.



Solution

A(10, 5, 0), B(0, 0, 0)

From Fig ①,

$$\vec{F}_A = 700\left(\frac{2\vec{i} + 6\vec{j} - 3\vec{k}}{7}\right)$$

$$\vec{F}_A = 200\vec{i} + 600\vec{j} - 300\vec{k}$$

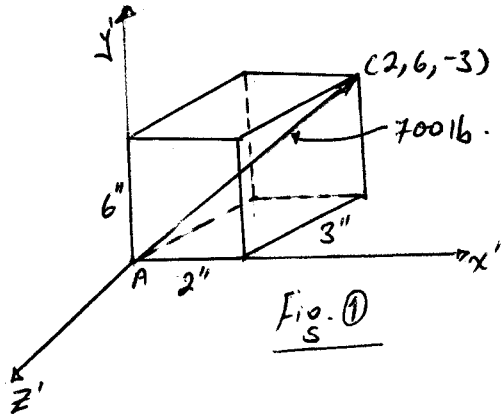


Fig. ①

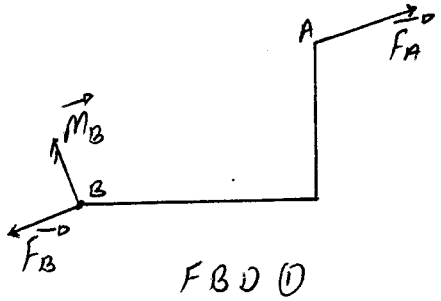
From FBD ①,

$$\Sigma F = 0 \Rightarrow \vec{F}_A + \vec{F}_B = 0$$

$$\Rightarrow \boxed{\vec{F}_B = -200\vec{i} - 600\vec{j} + 300\vec{k}} \text{ (lb)}$$

$$\Sigma \vec{M}_B = 0; \Rightarrow (\vec{r}_{BA} \times \vec{F}_A) + \vec{M}_B = 0$$

$$\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 5 & 0 \\ 200 & 600 & -300 \end{vmatrix} + \vec{M}_B = 0$$



FBD ①

$$\Rightarrow -1500\vec{i} + 3000\vec{j} + 5000\vec{k} + \vec{M}_B = 0$$

$$\Rightarrow \boxed{\vec{M}_B = 1500\vec{i} - 3000\vec{j} - 5000\vec{k} \text{ in. lb}}$$

Note: You may use scalers to solve the problem, however it is not recommended.

Problem 3

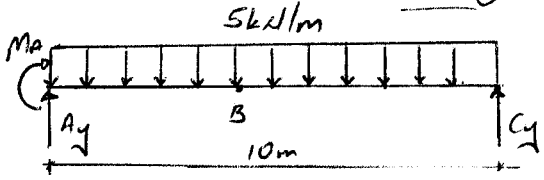
Given: Figure P3 as shown in the question sheet.

Required: Equations, Shear force and Bending Moment diagrams.

Solution.

Note that in FBD ①; there are 3 unknown reactions and only 2 equations: ($\Sigma F_y = 0$; $\Sigma M = 0$;), therefore we cannot solve for the unknowns. ($\Sigma F_x = 0$ is useless).

FBD ①



So we have to separately analyze the BC section first.

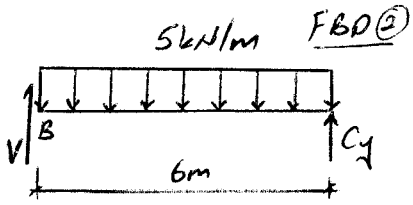
→ Take the Right Section (Why??)

From FBD ②;

$\Sigma M_B = 0 \Rightarrow C_y \times 6 - (5 \times 6)\frac{6}{2} = 0$

$\Rightarrow C_y = 15\,kN \uparrow$

FBD ②



From FBD ①;

$\Sigma M_A = 0 \Rightarrow C_y \times 10 - (5 \times 10)\left(\frac{10}{2}\right) - M_A = 0$

$\Rightarrow M_A = -100$ or $M = 100\,kN\cdot m$ ↺

Note that no M at B. Why?!

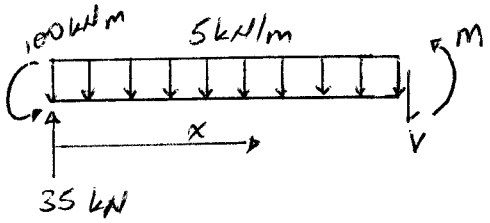
$+\uparrow \Sigma F_y = 0 \Rightarrow 15 - (5 \times 10) + A_y = 0$

$\Rightarrow A_y = 35\,kN \uparrow$

From FBD ③,

AC ($0 \le x \le 10$)

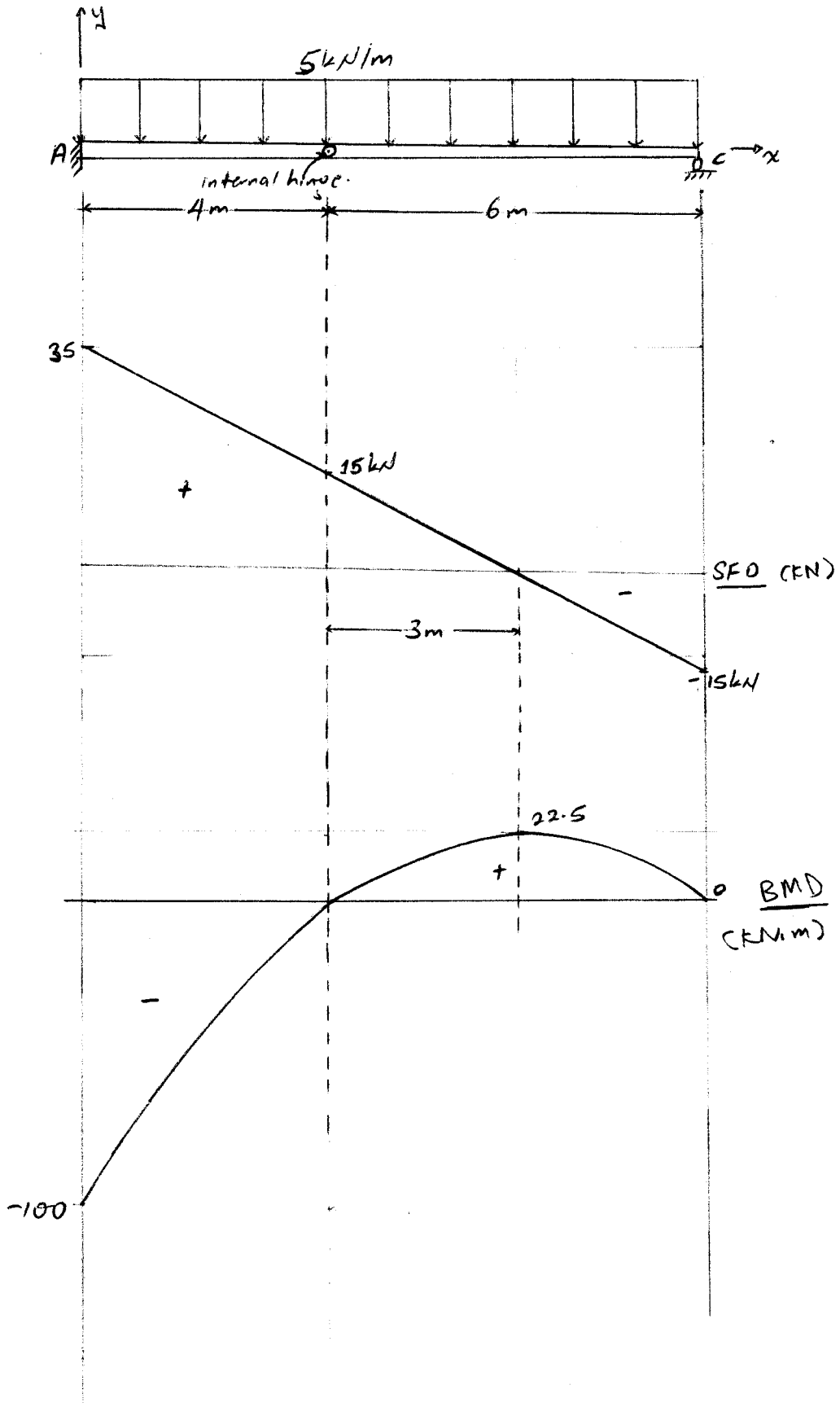
FBD ③



$+\uparrow \Sigma F_y = 0$; $35 - 5x - V = 0 \Rightarrow \boxed{V = 35 - 5x}$

$+\circlearrowleft \Sigma M = 0$; $100 - 35x + \frac{5x^2}{2} - M = 0$

$\boxed{M = 35x - 2.5x^2 - 100}$

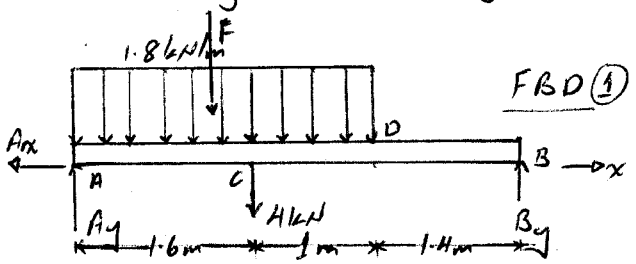

y
5kN/m
A
C
x
internal hinge
4m
6m
35
15kN
+
SFD (kN)
-
3m
-15kN
22.5
+
0
BMD
(kN.m)
-
-100

Problem 4

Given: A beam as shown in Figure P4 of the question sheet.

Required: Equations, Shear Force and Bending Moment Diagrams

Solution.



From FBD (1);

$F = 1.8 \times 2.6 = 4.68\ kN$ (acts 1.3m from A)

First we obtain the reactions;

$\rightarrow$ By inspection, $A_x = 0$.

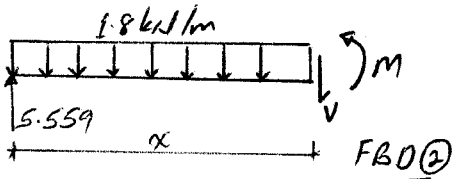
$-\circlearrowright^{+} \sum M_A = 0 \Rightarrow -4.68(1.3) - 4(1.6) + B_y(4) = 0$

$\Rightarrow B_y = 3.121\ kN$

$+\uparrow \sum F_y = 0 \Rightarrow A_y - 4.68 - 4 + 3.121 = 0$

$\Rightarrow A_y = 5.559\ kN.$

AC ($0 \le x \le 1.6$) – FBD (2)

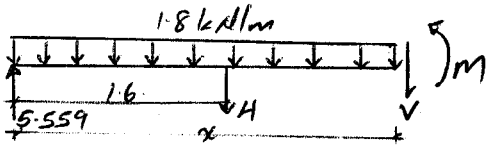


$+\uparrow \sum F_y = 0 \Rightarrow 5.559 - (1.8 \times x) - V = 0$

$\Rightarrow \boxed{V = 5.559 - 1.8x}$

$+\circlearrowright \sum M = 0 \Rightarrow M - 5.559(x) + (1.8 \times x)\frac{x}{2} = 0 \Rightarrow \boxed{M = 5.559x - 0.9x^2}$

CD ($1.6 \le x \le 2.6$) – FBD (3)



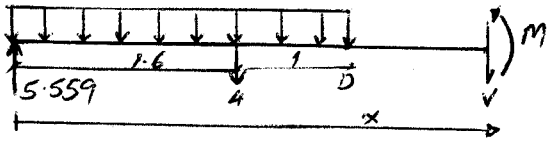
FBD (3)

$\sum F_y = 0 \Rightarrow 5.559 - 4 - 1.8x - V = 0$

$\Rightarrow \boxed{V = 1.559 - 1.8x}$

$\sum M = 0 \Rightarrow M - 5.559(4) + 1.8\left(\frac{x^2}{2}\right) + 4(x - 1.6) \Rightarrow \boxed{M = 1.559x - 0.9x^2 + 6.4}$

DB ($2.6 \le x \le 4$) – FBD (4)



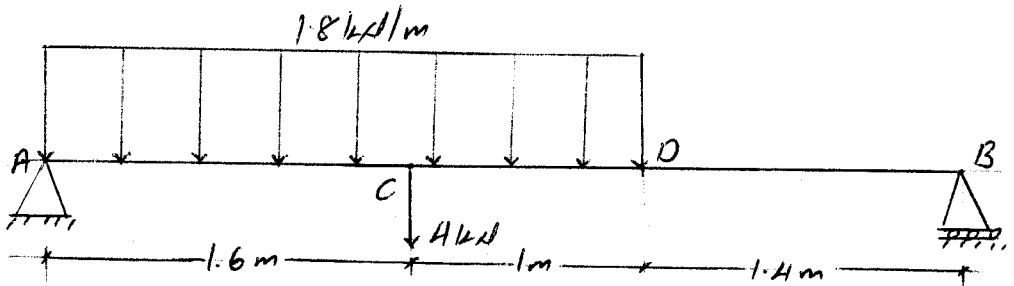
FBD (4)

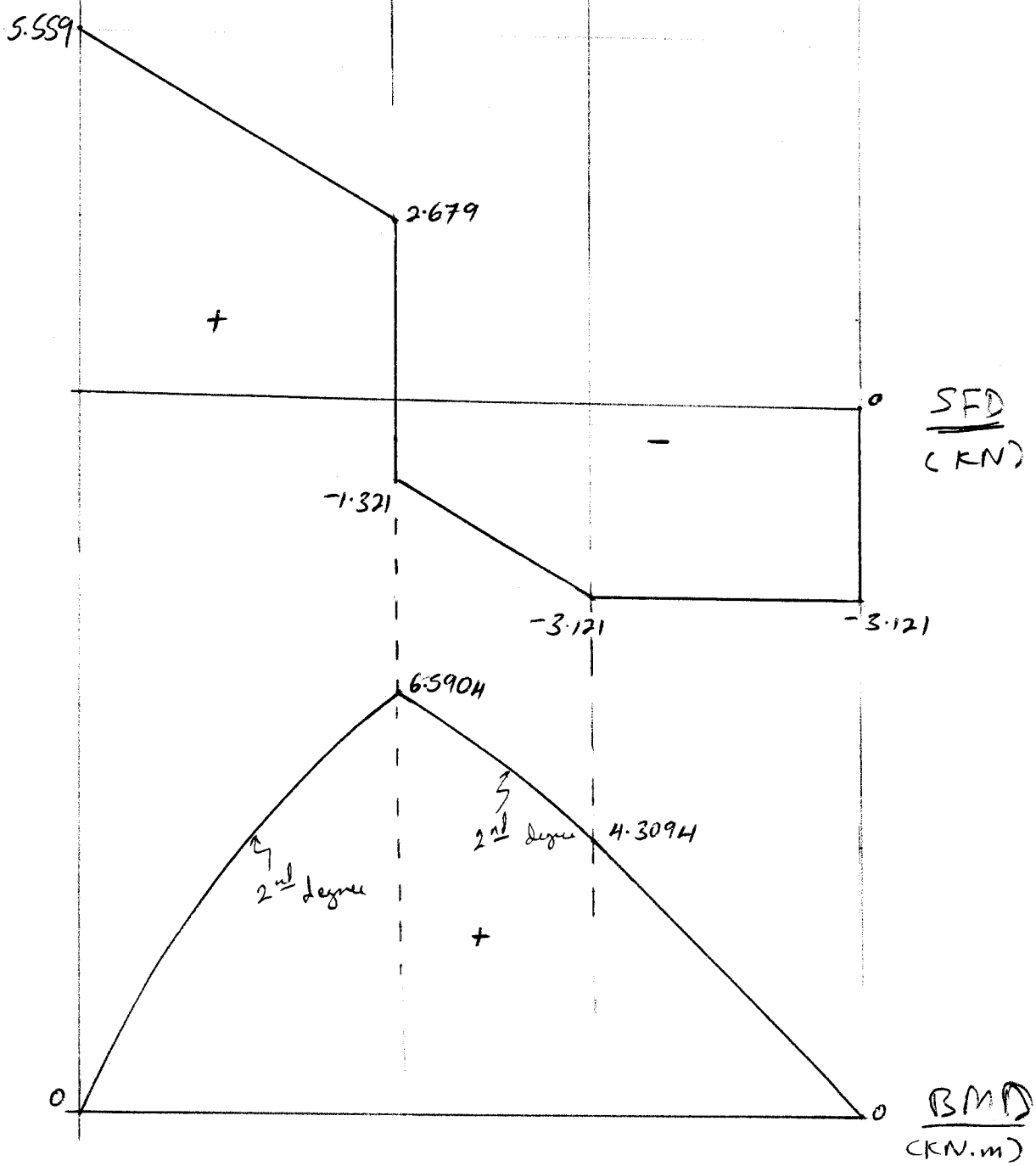
$+\uparrow \sum F_y = 0 \Rightarrow 5.559 - 4 - (1.8 \times 2.6) - V = 0$

$\Rightarrow \boxed{V = -3.121\ kN}$

$+\circlearrowright M = 0 \Rightarrow M - 5.559x + (1.8 \times 2.6)(x - 1.3) + 4(x - 1.6) = 0$

$\Rightarrow \boxed{M = 12.484 - 3.121x}$


1.8kN/m
A
C
D
B
4kN
1.6 m
1 m
1.4 m


5.559
2.679
+
0
SFD
(KN)
-1.321
-3.121
-3.121
6.590H
2nd degree
4.309H
2nd degree
+
0
0
BMD
(KN.m)

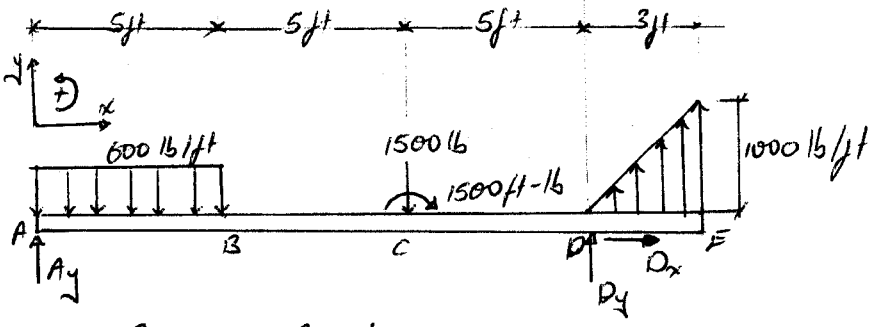
Problem 5

Given: A beam as shown in figure 1.5 of question sheet.

Required: Equations, Shear force and Bending Moment Diagrams.

Solution.

FBD ①



First, we obtain the reactions of the beam.

From FBD ①:

By inspection, $D_x = 0$

$+\circlearrowleft \sum M_D = 0 \Rightarrow -A_y(15) + (600\times5)(12.5) + 1500(5) + 1500(\tfrac{2}{3}\times3) - 1500 = 0$

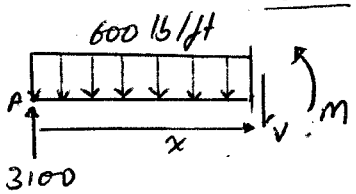
$\Rightarrow A_y = 3100$ lb.

$+\uparrow \sum F_y = 0 \Rightarrow 3100 - (600\times5) - 1500 + 1500 + D_y = 0$

$\Rightarrow D_y = 100 \downarrow$ lb

→ AB $(0 \le x \le 5)$ – FBD ②

FBD ②



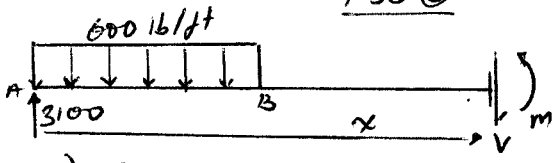
$+\uparrow \sum F_y = 0 \Rightarrow 3100 - 600(x) - V = 0$: $\boxed{V = 3100 - 600x}$

$+\circlearrowleft \sum M = 0 \Rightarrow M - 3100(x) + 600\left(\frac{x^2}{2}\right) = 0$

$\Rightarrow \boxed{M = 3100x - 300x^2}$

→ BC $(5 \le x \le 10)$ – FBD ③

FBD ③



$+\uparrow \sum F_y = 0 \Rightarrow 3100 - 600(5) - V = 0$

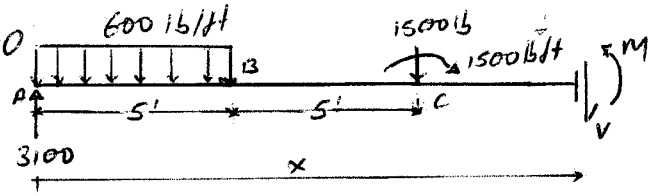
$\Rightarrow \boxed{V = 100}$

$+\circlearrowleft \sum M = 0 \Rightarrow M - 3100(x) + (600\times5)(x - 2.5) = 0$

$\Rightarrow \boxed{M = 100x + 7500}$

→ CD (10 ≤ x ≤ 15) — FBD (4)

FBD (4)



$+\uparrow \Sigma F_y = 0 \Rightarrow 3100 - (600 \times 5) - 1500 - V = 0$

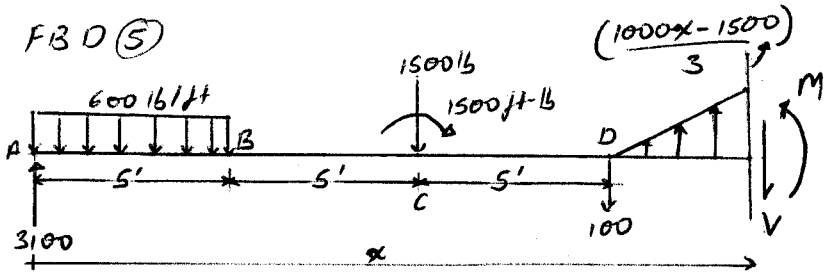
$\Rightarrow \boxed{V = -1400 \text{ or } 1400 \text{ lb} \uparrow}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow$

$\Rightarrow M - 3100(x) + (600 \times 5)(x - 2.5) + 1500(x - 10) - 1500 = 0$

$\Rightarrow \boxed{M = -1400x + 24000}$

→ DE (15 ≤ x ≤ 18) — FBD (5)



FBD (5)

$+\uparrow \Sigma F_y = 0 \Rightarrow 3100 - (600 \times 5) - 1500 - 100 + \left(\frac{1000x - 1500}{3}\right)\left(\frac{x - 15}{2}\right) - V = 0$

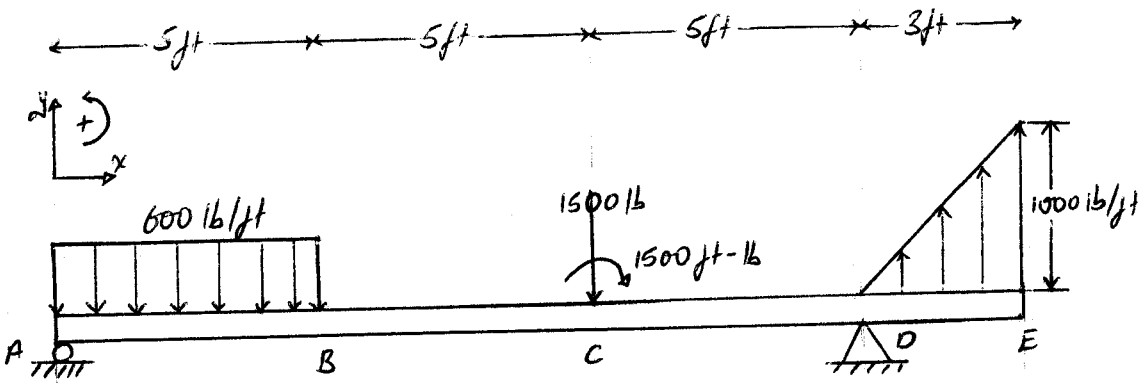
$\Rightarrow \boxed{V = 5000x - \frac{500x^2}{3} - 3600}$

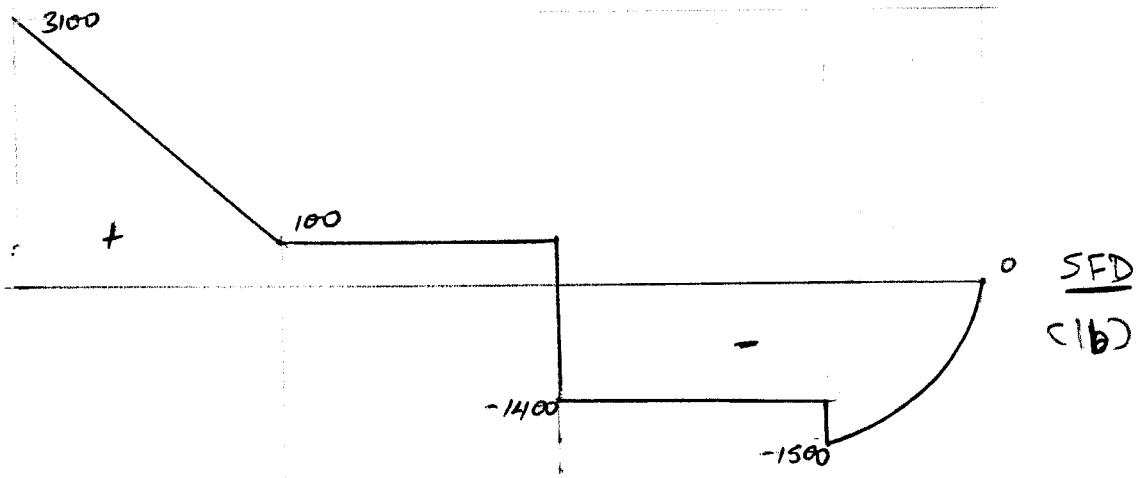
$+\circlearrowleft \Sigma M = 0 \Rightarrow$

$\Rightarrow M - 3100(x) + 3000(x - 2.5) + 1500(x - 10) - 1500 \cdot$

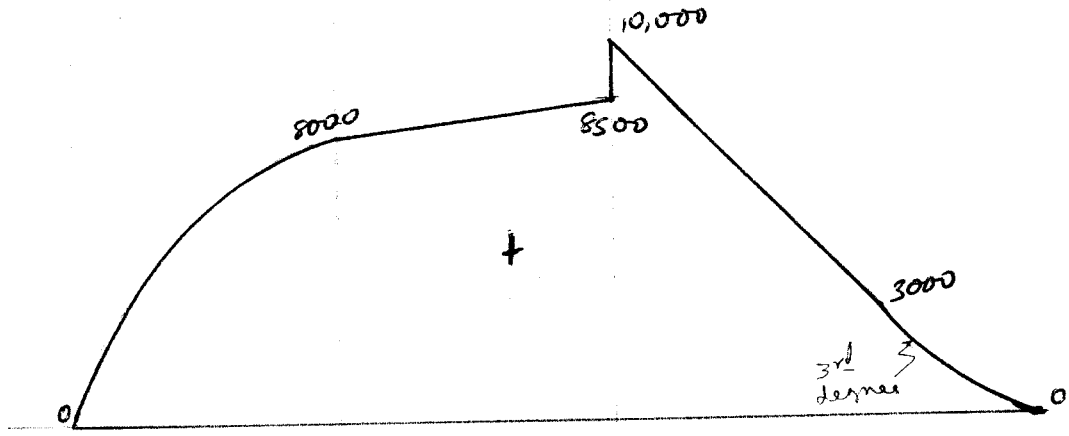
$-\left(\frac{1000x - 15000}{3}\right)\left(\frac{x - 15}{2}\right)\left(\frac{x - 15}{3}\right)$

$\Rightarrow \boxed{M = \frac{500x^3}{9} - 2500x^2 + 38900x - 211500}$





SFD (lb)



BMD (ft-lb)