CE 201 Sections (4&5) [071]

H.W # 7

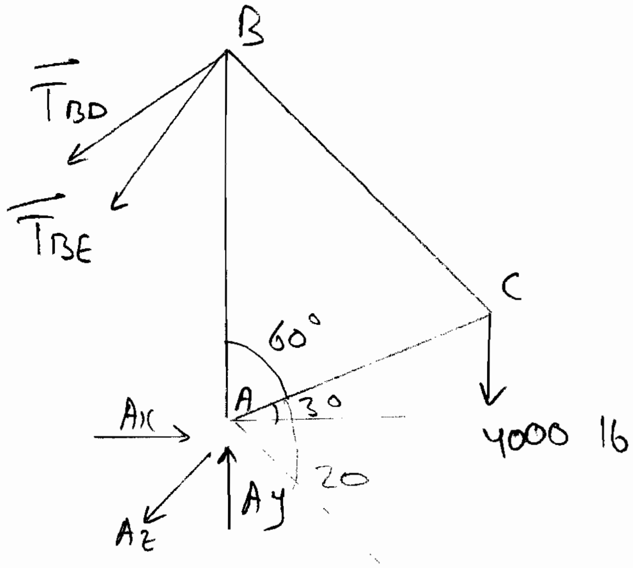
Problem 1

Given:

Fig P_1

load = 4000 lb

$\phi = 20°$



FBD

Required:

Tension in each cable & Reactions at A

Solution

A (0, 0, 0)

B (0, 12, 0)

C (cos30 cos20, sin 30, cos30 sin 20)

D (−8, 0, 6)

E (−8, 0, −6)

$\vec{BD} = -8\vec{i} - 12\vec{j} + 6\vec{k} \Longrightarrow BD = \sqrt{244}$ ft

$\vec{BE} = -8\vec{i} - 12\vec{j} - 6\vec{k} \Longrightarrow BE = \sqrt{244}$ ft

$\vec{AC} = 12(\cos30 . \cos20\vec{i} + \sin30\vec{j} + \cos30 . \sin20\vec{k}]$

$\vec{AC} = 9.7656\vec{i} + 6\vec{j} + 3.5544\vec{k}$

$$\therefore \quad \vec{T_{BD}} = \frac{T_{BD}}{BD}\left(-8\vec{i} - 12\vec{j} + 6\vec{k}\right)$$

$$\vec{T_{BD}} = \frac{T_{BD}}{\sqrt{244}}\left(-8\vec{i} - 12\vec{j} + 6\vec{k}\right)$$

$$\vec{T_{BE}} = \frac{T_{BE}}{BE}\left(-8\vec{i} - 12\vec{j} - 6\vec{k}\right)$$

$$\vec{T_{BE}} = \frac{T_{BE}}{\sqrt{244}}\left(-8\vec{i} - 12\vec{j} - 6\vec{k}\right)$$

$$\text{Load}(\vec{F}) = 4000\vec{j}$$

$\sum \vec{M}_A = 0$

You can combine

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ \frac{-8}{\sqrt{244}}T_{BD} & \frac{-12}{\sqrt{244}}T_{BD} & \frac{6}{\sqrt{244}}T_{BD} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ \frac{-8}{\sqrt{244}}T_{BE} & \frac{-12}{\sqrt{244}}T_{BE} & \frac{-6}{\sqrt{244}}T_{BE} \end{vmatrix}$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9.7656 & 6 & 3.5544 \\ 0 & -4000 & 0 \end{vmatrix}$$

$$= 4.6093\,T_{BD}\vec{i} + 6.1458\,T_{BD}\vec{k} - 4.6093\,T_{BE}\vec{i} + 6.1458\,T_{BE}\vec{k}$$

$$+ 14217.6\,\vec{i} - 39062.4\,\vec{k}$$

$$= (4.6093\,T_{BD} - 4.6093\,T_{BE} + 14217.6)\,\vec{i}$$

$$+$$

$$(6.1458\,T_{BD} + 6.1458\,T_{BE} - 39062.4)\,\vec{k}$$

solving them together

$$28.3278\,T_{BD} - 28.3278\,T_{BE} + 87378.5 = 0$$

$$28.3278\,T_{BD} + 28.3278\,T_{BE} - 180050.3 = 0$$

$$56.6556\,T_{BD} = 92671.8$$

$$\boxed{\begin{array}{l} T_{BD} = 1635.7 \text{ lb} \\ T_{BE} = 4720.2 \text{ lb} \end{array}}$$

$\sum F_x = 0 \xrightarrow{+}$

$$A_x - \frac{8}{\sqrt{244}} T_{BD} - \frac{8}{\sqrt{244}} T_{BE} = 0$$

$$A_x = \boxed{3255.2 \text{ lb}}$$

$\sum F_y = 0 \uparrow +$

$$A_y - \frac{12}{\sqrt{244}} T_{BD} - \frac{12}{\sqrt{244}} T_{BE} - 4000 = 0$$

$$\boxed{A_y = 8882.7 \text{ lb}}$$

$\sum F_z = 0$

$$A_z + \frac{6}{\sqrt{244}} T_{BD} - \frac{6}{\sqrt{244}} T_{BE} = 0, \quad \boxed{A_z = 1184.8 \text{ lb}}$$

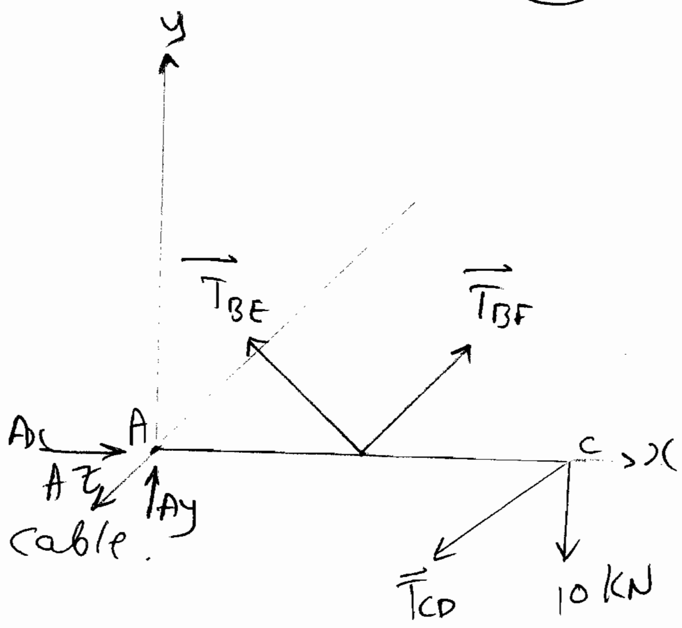
Problem 2

Given:

Fig P2

Required:

Determine tension in each cable.



FBD.

Solution:

A (0, 0, 0)

B (1, 0, 0)

c (2.5, 0, 0)

D (0, 0, 1.5)

E (0, 1, 0)

F (0, 0, -2)

$\overrightarrow{AC} = 2.5\vec{i}$

$\overrightarrow{BE} = -\vec{i} + \vec{j} \Longrightarrow BE = 2$

$\overrightarrow{BF} = -\vec{i} - 2\vec{k} \Longrightarrow BF = \sqrt{5}$

$\overrightarrow{CD} = -2.5\vec{i} + 1.5\vec{k} \Longrightarrow CD = \sqrt{8.5}$

$$\vec{F} = -10\vec{j}$$

$$\vec{T}_{BE} = \frac{T_{BE}}{\sqrt{2}}\left(-\vec{i}+\vec{j}\right)$$

$$\vec{T}_{BF} = \frac{T_{BF}}{\sqrt{5}}\left(-\vec{i}-2\vec{k}\right)$$

$$\vec{T}_{CD} = \frac{T_{CD}}{\sqrt{8.5}}\left(-2.5\vec{i}+1.5\vec{k}\right)$$

By Taking Moment around A:

$$\sum \vec{M}_A = 0$$

can combine

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -\frac{T_{BE}}{\sqrt{2}} & \frac{T_{BE}}{\sqrt{2}} & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -\frac{T_{BF}}{\sqrt{5}} & 0 & \frac{-2T_{BF}}{\sqrt{5}} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2.5 & 0 & 0 \\ 0 & -10 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2.5 & 0 & 0 \\ \frac{-2.5T_{CD}}{\sqrt{8.5}} & 0 & \frac{1.5T_{CD}}{\sqrt{8.5}} \end{vmatrix}$$

can combin

$$\left(\frac{T_{BE}}{\sqrt{2}} \; - \; 25 \right) \vec{k} + \left(\frac{2 T_{BF}}{\sqrt{5}} - \frac{2.5 \times 1.5}{\sqrt{8.5}} T_{CD} \right) \vec{j} = 0$$

$$\therefore \quad T_{BE} = 25 \times \sqrt{2} = 35.355$$

$$T_{BE} = 35.355 \text{ KN}$$

$$T_{BF} = T_{BE}$$

$$T_{BF} = 35.355 \text{ KN}$$

$$T_{CD} = 24.585 \text{ KN}.$$

$$\therefore \quad \boxed{\begin{aligned} T_{BE} = T_{BF} &= 35.4 \text{ KN} \\ T_{CD} &= 24.6 \text{ KN}. \end{aligned}}$$

Problem 3

Given:

$\omega = 20\,kg$, No axial thrust at A,

Fig P_3, Assume hinges at A & B are properly aligned.

Required:

$\therefore M_A = M_B = 0$

Magnitude of P &
reactions A & B when $\theta = 90°$

Solution:

A (0, 1.75, 0)

B (0, 0.25, 0)

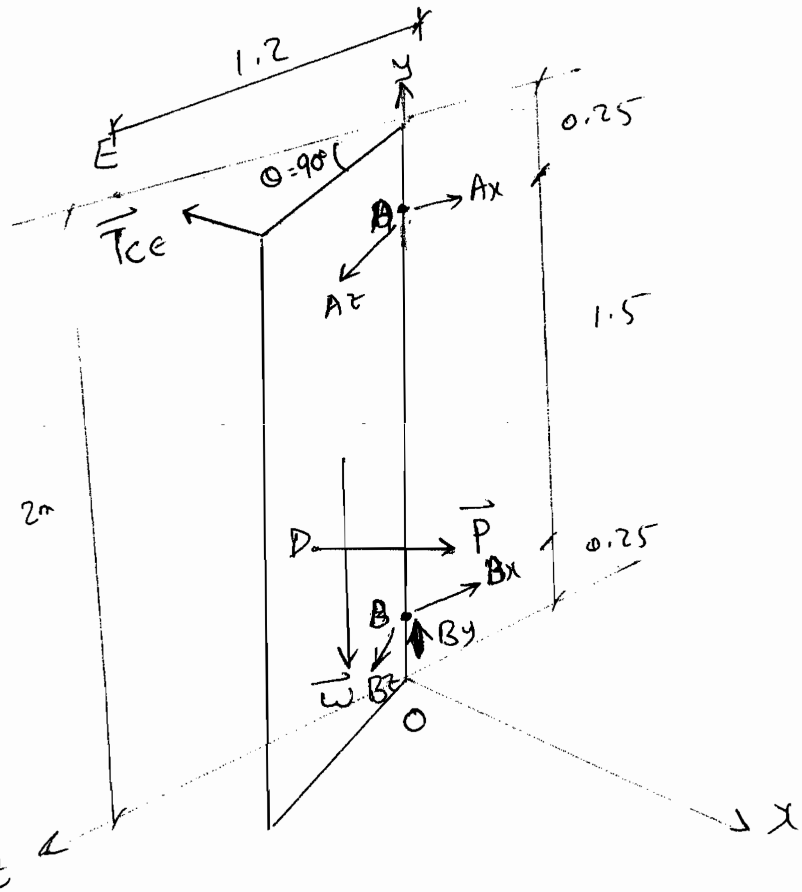
C (1.2, 2, 0)

D (1.05, 0.9, 0)

E (0, 2, 1.2)

G (0.6, 1, 0)

O (0, 0, 0)



By taking Moment about OA

$\Sigma M_{OA} = 0$

$$\vec{U}_{OA} \cdot (\vec{OD} \times \vec{P}) + \vec{U}_{OA} \cdot (\vec{OG} \times \vec{\omega}) + \vec{U}_{OA} \cdot (\vec{OC} \times \vec{T}_{CE}) = 0$$

$$\vec{U}_{OA} = 1.75\vec{j}$$

$$\vec{OD} = 1.05\vec{i} + 0.9\vec{j}$$

$$\vec{P} = -\vec{P}K$$

$$\vec{OG} = 0.6\vec{i} + \vec{1j}$$

$$\vec{\omega} = -20 \times 9.81\vec{j} = -196.2\vec{j}$$

$$\vec{OC} = 1.2\vec{i} + 2\vec{j}$$

$$\vec{T}_{CE} = \frac{T_{CE}(-1.2\vec{i} + 1.2\vec{k})}{\sqrt{2.88}}$$

$$M_{OA} = \begin{vmatrix} 0 & 1.75 & 0 \\ 1.05 & 0.9 & 0 \\ 0 & 0 & -P \end{vmatrix} + \begin{vmatrix} 0 & 1.75 & 0 \\ 0.6 & 1 & 0 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1.75 & 0 \\ 1.2 & 2 & 0 \\ \frac{-T_{CE}(1.2)}{\sqrt{2.88}} & 0 & \frac{T_{CE}(1.2)}{\sqrt{2.88}} \end{vmatrix}$$

Since $T_{CE} = 15 \times 9.81 = 147.15$ N

$\therefore \sum M_{OA} = 0$

$$= -1.75(-P \times 1.05) - 1.75 \frac{(1.2 \times 1.2 \times 147.15)}{\sqrt{2.88}} = 0$$

$$\boxed{P = 118.915 \text{ N}}$$

$\overrightarrow{\sum M_B} = 0$

can combine

$$= \underbrace{(\overrightarrow{BA} \times \overrightarrow{Ax}) + (\overrightarrow{BA} \times \overrightarrow{Az})} + (\overrightarrow{BC} \times \overrightarrow{T_{CE}}) + (\overrightarrow{BG} \times 196.2) + (\overrightarrow{BD} \times 118.915) = 0$$

$\overrightarrow{BA} = 1.5\vec{j}$

$\overrightarrow{BC} = 1.2\vec{i} + 1.75\vec{j}$

$\overrightarrow{BG} = 0.6\vec{i} + 0.75\vec{j}$

$\overrightarrow{BD} = 1.05\vec{i} + 0.65\vec{j}$

$\longrightarrow$ PTO

can combine

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.5 & 0 \\ A_x & 0 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.5 & 0 \\ 0 & 0 & A_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.2 & 1.75 & 0 \\ -104.051 & 0 & 104.051 \end{vmatrix}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.6 & 0.75 & 0 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.05 & +0.65 & 0 \\ 0 & 0 & -118.913 \end{vmatrix}$$

$$= -(1.5A_x)\vec{k} + (1.5A_z)\vec{i} + (1.75 \times 104.051)\vec{i} - (1.2 \times 104.051)\vec{j} + (1.75 \times 104.051)\vec{k} + (0.6 \times 196.2)\vec{k} + (0.65 \times 118.915)\vec{i} + (1.05 \times 118.915)\vec{j} = 0$$

$$\therefore \quad 1.5A_z + 182.089 - 77.295 = 0$$

$$\boxed{A_z = -69.863 \text{ N}}$$

$$-1.5A_x + 182.089 - 117.720 = 0$$

$$\boxed{A_x = 42.913 \text{ N}}$$

$$\Sigma F_x = 0 \xrightarrow{+} \quad -104.051 + A_x + B_x = 0$$

$$B_x = 104.051 - 42.913$$

$$\boxed{B_x = 61.138 \text{ N}}$$

$\sum F_z = 0 \swarrow +$ $\quad -P + 104.051 + A_z + B_z = 0$

$$B_z = 118.915 - 104.051 + 69.863$$

$$\boxed{B_z = 84.727 \text{ N}}$$

∴ Since No axial thrust at A

$$\boxed{\therefore A_y = 0}$$ «no A_y»

$$\therefore \cancel{A_y}^{0} + B_y = 196.2$$

$$\boxed{B_y = 196.2 \text{ N}}$$

Problem 4

Given:

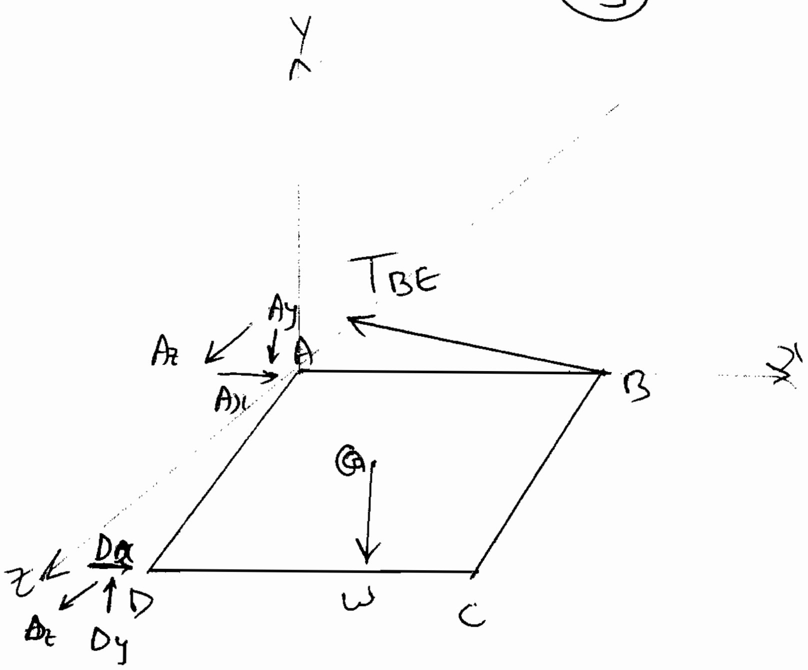
W = 23 kg

Plate = (325 x 450) mm

Fig P4

Required:

Tension in the wire



Solution:

(The hinges along AD are assumed Properly aligned. No moment about the axis)

A (0, 0, 0)

B (450, 0, 0)

C (450, −125, 300)

D (0, −125, 300)

E (0, 225, 150)

G (225, −62.5, 150)

By taking moment around axis AD

$$\sum M_{AD} = 0$$

$$= \vec{U}_{AD} \cdot (\vec{AB} \times \vec{T}_{BE}) + \vec{U}_{AD} \cdot (\vec{AG} \times \vec{W})$$

$$\vec{U}_{AD} = \frac{-125\vec{j} + 300\vec{k}}{325}$$

$$\vec{AB} = 450\vec{i}$$

$$\vec{AG} = 225\vec{i} - 62.5\vec{j} + 150\vec{k}$$

$$\vec{T}_{BE} = T_{BE} \frac{-450\vec{i} + 225\vec{j} + 150\vec{k}}{525}$$

$$\sum M_{AD} = 0$$

$$= \begin{vmatrix} 0 & -125/325 & 300/325 \\ 450 & 0 & 0 \\ \frac{-450T_{BE}}{525} & \frac{225T_{BE}}{525} & \frac{150T_{BE}}{525} \end{vmatrix} + \begin{vmatrix} 0 & \frac{-125}{325} & \frac{300}{325} \\ 225 & -62.5 & 150 \\ 0 & -225.63 & 0 \end{vmatrix}$$

$$= \frac{175}{325}\left(\frac{67500}{525} T_{BE} + \cancel{\frac{1350000}{525}} T_{BE}\right) +$$

$$\frac{300}{325}\left(\frac{450 \times 225}{525} T_{BE}\right) + \frac{300}{325}\left(-50766.75\right) = 0$$

$$\Rightarrow \quad 49.451\, T_{BE} + 178.022\, T_{BE} = 46861.615$$

$$T_{BE} = 206.009$$

$$\therefore \boxed{T_{BE} = 206.01 \text{ N.}}$$

Problem 5:

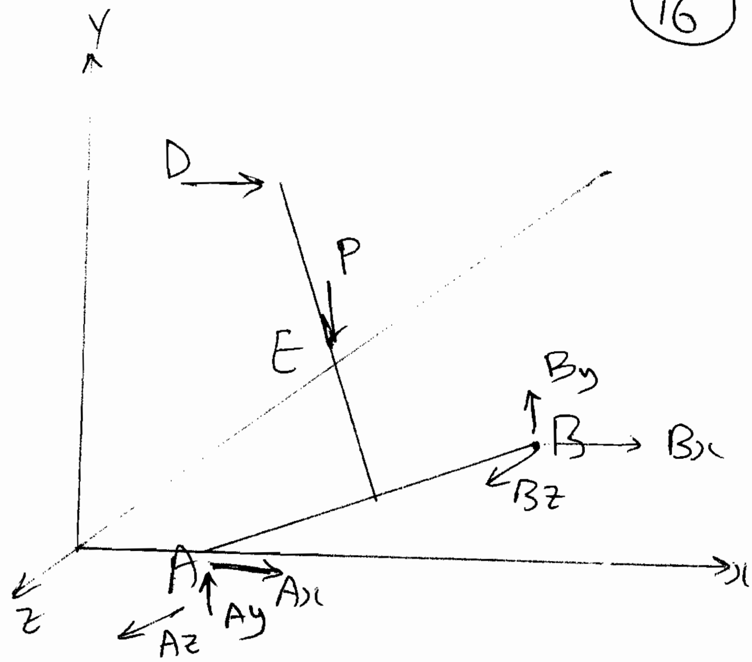
Given:

$P = 400N$

Fig P5

Required:

reaction at D



Solution: (The bearings at A and B are assumed properly aligned)

By taking moment around Axis AB

$\sum \vec{M}_{AB} = 0$

$= \vec{U}_{AB} \cdot (\vec{AE} \times \vec{P}) + \vec{U}_{AB} \cdot (\vec{AD} \times \vec{D}) = 0$

A (125, 0, 0)

B (275, 0, -200)

C (200, 0, -100)

D (0, 250, -250)

E (100, 125, -175)

$$\vec{u}_{AB} = \frac{150\vec{i} - 200\vec{k}}{250}$$

$$\vec{AE} = -25i + 175j - 175k$$

$$P = -400\vec{j}$$

$D = \vec{Di} \Rightarrow$ Reaction at D.

$$\vec{AD} = -125i + 250j - 250k$$

$$\Sigma \vec{M}_{AB} = 0$$

$$= \begin{vmatrix} \frac{150}{250} & 0 & -\frac{200}{250} \\ -25 & 125 & -175 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \frac{150}{250} & 0 & -\frac{200}{250} \\ -125 & 250 & -250 \\ & 0 & 0 \end{vmatrix}$$

$$= -\frac{150}{250}(70000) - \frac{200}{250}(10000) + (-200D) = 0$$

$$\boxed{D = 250 \text{ N.}}$$