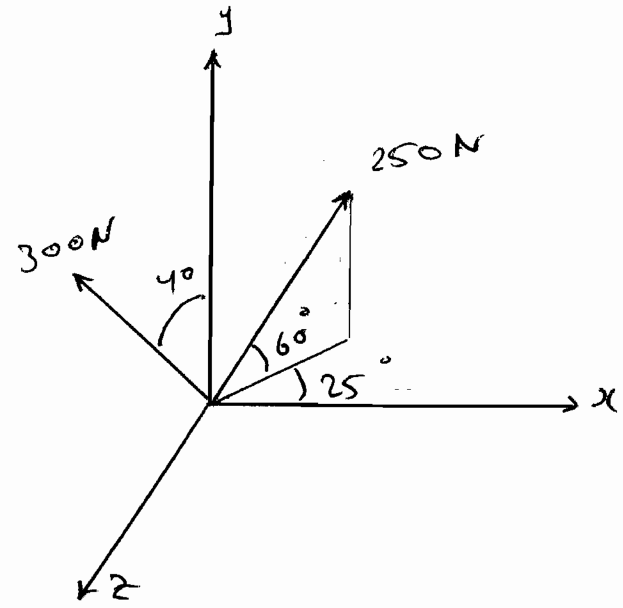
CE 201-485 (071)
H.W. # 2 Key Solution

Problem 1

Given: Fig P_1

Required: Value and resultant of the two forces

Solution:



$$\cos\theta_x = \frac{F_{1x}}{F_1}$$

$$\cos\theta_y = \frac{F_{1y}}{F_1}$$

$$\cos\theta_z = \frac{F_{1z}}{F_1}$$

$$F_{1x} = (300\sin 40)\sin 20 = 65.95\ N$$

$$F_{1y} = 300\cos 40 = 229.8\ N$$

$$F_{1z} = (300\sin 40)\cos 20 = 181.2\ N$$

$$\therefore \boxed{\vec{F_1} = \{65.95\vec{i} + 229.8\vec{j} + 181.2\vec{k}\}}$$

$$F_{2x} = (250\cos 60)\cos 25 = 113.3\ N$$

$$F_{2y} = 250\sin 60 = 216.5\ N$$

$$F_{2z} = (250\cos 60)\sin 25 = -52.83\ N$$

$$\therefore \boxed{\vec{F_2} = \{113.3\vec{i} + 216.5\vec{j} - 52.83\vec{k}\}}$$

$$\vec{R} = \vec{F_1} + \vec{F_2}$$

$$\vec{R} = \left[(65.95 + 113.3)\vec{i} + (229.8 + 216.5)\vec{j} + (181.2 - 52.83)\vec{k} \right]$$

$$\vec{R} = (179.25)\vec{i} + (446.3)\vec{j} + (128.37)\vec{k}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$R = \sqrt{(179.25)^2 + (446.3)^2 + (128.37)^2}$$

$$\boxed{R = 497.8\ N}$$

$$\vec{U_R} = \frac{R_x}{R}\vec{i} + \frac{R_y}{R}\vec{j} + \frac{R_z}{R}\vec{k}$$

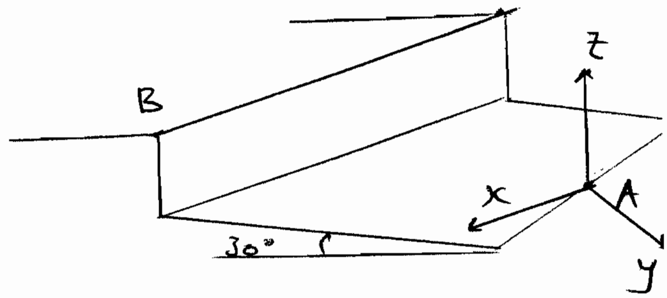
$$= \frac{179.25}{497.8}\vec{i} + \frac{446.3}{497.8}\vec{j} + \frac{128.37}{497.8}\vec{k}$$

$$\theta_x = \cos^{-1}(0.360) \Longrightarrow \boxed{\theta_x = 68.89^\circ}$$

$$\theta_y = \cos^{-1}(0.8965) \Longrightarrow \boxed{\theta_y = 26.29^\circ}$$

$$\theta_z = \cos^{-1}(0.2579) \longrightarrow \boxed{\theta_z = 75.06^\circ}$$

Problem 2 :-



Given:-

Forces at fig P2, $T_{AB} = 10.0 \, KN$, $T_{AC} = 7.5 \, KN$

Required:

Magnitude and direction of resultant force on A.

Solution:-

A (0, 0, 0)

B (12, − 18 cos 30 , 6 + 18 sin 30)

C (−15, − 18 cos 30 , 9.60 + 18 sin 30)

∴ coordinates:-

A (0, 0, 0)

B (12, −15.6, 15)

C (−15, −15.6, 18.6)

$\vec{AB} = 12\vec{i} - 15.6\vec{j} + 15.0\vec{k}$ m $\Longrightarrow$ AB = 24.74 m

$\vec{AC} = -15\vec{i} - 15.6\vec{j} + 18.6\vec{k}$ m $\Longrightarrow$ AC = 28.5 m

$$\vec{F}_{AB} = F_{AB}\,\vec{U}_{AB} = F_{AB}\,\frac{\vec{AB}}{AB}$$

$$F_{AB} = \frac{10}{24.75}\left(12\vec{i} - 15.6\vec{j} + 15.0\vec{k}\right)$$

$$\boxed{\vec{F}_{AB} = 4.85\vec{i} - 6.30\vec{j} + 6.06\vec{k} \quad (KN)}$$

$$\vec{F}_{AC} = F_{AC}\,\vec{U}_{AC} = F_{AC}\,\frac{\vec{AC}}{AC}$$

$$F_{AC} = \frac{7.5}{28.50}\left(-15\vec{i} - 15.6\vec{j} + 18.6\vec{k}\right)$$

$$\boxed{\vec{F}_{AC} = -3.95\vec{i} - 4.10\vec{j} + 4.89\vec{k} \quad (KN)}$$

$$\therefore \vec{R} = \Sigma\vec{F} = \vec{F}_{AB} + \vec{F}_{AC}$$

$$= (4.85 - 3.95)\vec{i} + (-6.30 - 4.10)\vec{j} + (6.06 + 4.89)\vec{k}$$

$$\vec{R} = 0.90\vec{i} - 10.4\vec{j} + 10.95\vec{k}$$

$$R = \sqrt{(0.90)^2 + (-10.4)^2 + (10.95)^2} \Rightarrow \boxed{R = 15.13\ KN}$$

$$\cos \theta_x = \frac{R_x}{R} = \frac{0.9}{15.13} \implies \boxed{\theta_x = 86.0^\circ}$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{-10.4}{15.13} \implies \boxed{\theta_y = 133^\circ}$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{10.95}{15.13} \implies \boxed{\theta_z = 43.6^\circ}$$

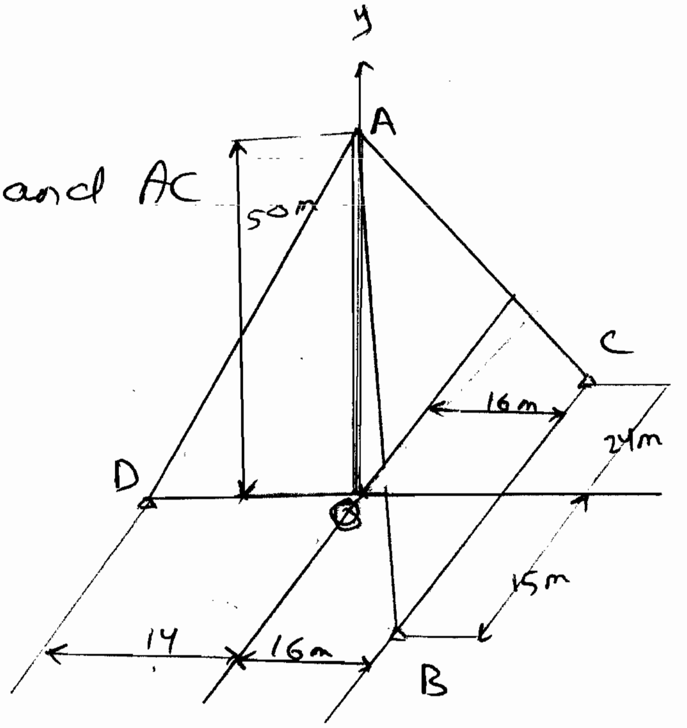
Problem 3

Given :-

fig P_3

Required :-

Angle formed between AB and AC



Solution :-

$A(0, 50, 0)$

$B(16, 0, 15)$

$C(16, 0, -24)$

$$\vec{r}_{AB} = (16-0)\vec{i} + (0-50)\vec{j} + (15-0)\vec{k}$$

$$\vec{r}_{AB} = 16\vec{i} - 50\vec{j} + 15\vec{k}$$

$$r_{AB} = \sqrt{16^2 + (-50)^2 + 15^2} \Longrightarrow \boxed{r_{AB} = 54.59m}$$

$$\vec{r}_{AC} = (16-0)\vec{i} + (0-50)\vec{j} + (-24-0)\vec{k}$$

$$\vec{r}_{AC} = 16\vec{i} - 50\vec{j} - 24\vec{k}$$

$$r_{AC} = \sqrt{16^2 + (-50)^2 + (-24)^2} \Longrightarrow \boxed{r_{AC} = 57.72m}$$

$$\cos\theta = \frac{\vec{r}_{AB} \cdot \vec{r}_{AC}}{r_{AB}\, r_{AC}}$$

$$\cos\theta = \frac{(16)(16) + (-50)(-50) + (15)(-24)}{(54.59)(57.72)}$$

$$\boxed{\theta = 40.5^\circ}$$

Problem (4)

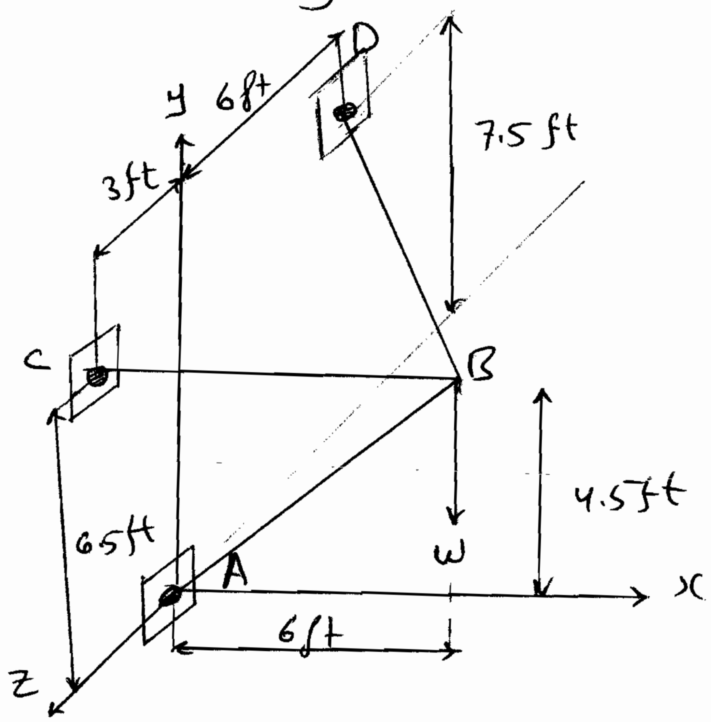
Given:- fig P4 , $T_{BC} = 280$ lb

Required:

a- Angle between BC and BA

b- Projection on BA exerted by cable BC at point B.

Solution:-



a-

A(0, 0, 0)

B(6, 4.5, 0)

C(0, 6.5, 3)

$$\vec{r}_{BC} = (0-6)\vec{i} + (6.5-4.5)\vec{j} + (3-0)\vec{k}$$

$$\vec{r}_{BC} = -6\vec{i} + 2\vec{j} + 3\vec{k}$$

$$r_{BC} = \sqrt{-6^2 + 2^2 + 3^2} \Longrightarrow r_{BC} = 7.0 \text{ ft}$$

$$\vec{r}_{BA} = (0-6)\vec{i} + (0-4.5)\vec{j} + 0\vec{k}$$

$$\vec{r}_{BA} = -6\vec{i} - 4.5\vec{j}$$

$$r_{BA} = (-6^2) + (-4.5)^2 \Longrightarrow r_{BA} = 7.5 \text{ ft}$$

$$\cos\theta = \frac{\vec{r_{BA}} \cdot \vec{r_{BC}}}{r_{BA}\, r_{BC}}$$

$$= \frac{(-6)(-6) + (-4.5)(2) + (0)(3)}{7.5 \times 7} = 0.514$$

$$\boxed{\theta = 59.05^\circ}$$

6-(i) (if Angle θ was not found)

$$\vec{(F_{BC})}_{BA} = F_{BC}\, \vec{U_{BC}}$$

$$\vec{F_{BC}} = \frac{280}{7}(-6\vec{i} + 2\vec{j} + 3\vec{k})$$

$$\vec{F_{BC}} = -240\vec{i} + 80\vec{j} + 120\vec{k}$$

$$(F_{BA})_{BA} = \vec{F_{BC}} \cdot \vec{U_{BA}}$$

$$= (-240\vec{i} + 80\vec{j} + 120\vec{k}) \cdot \left(-\frac{6}{7.5}\vec{i} - \frac{4.5}{7.5}\vec{j}\right)$$

$$= 192 - 48 = 144 \text{ lb}$$

$$\therefore \boxed{F_{BA} = 144 \text{ lb}}$$

(iii) if angle θ was found.

$$(F_{BC})_{BA} = F_{BC} \cos\theta$$

$$= 280 \cos 59.05$$

$$= 144.0 \text{ N}$$

$$\boxed{(F_{BC})_{BA} = 144.\text{ N}}$$