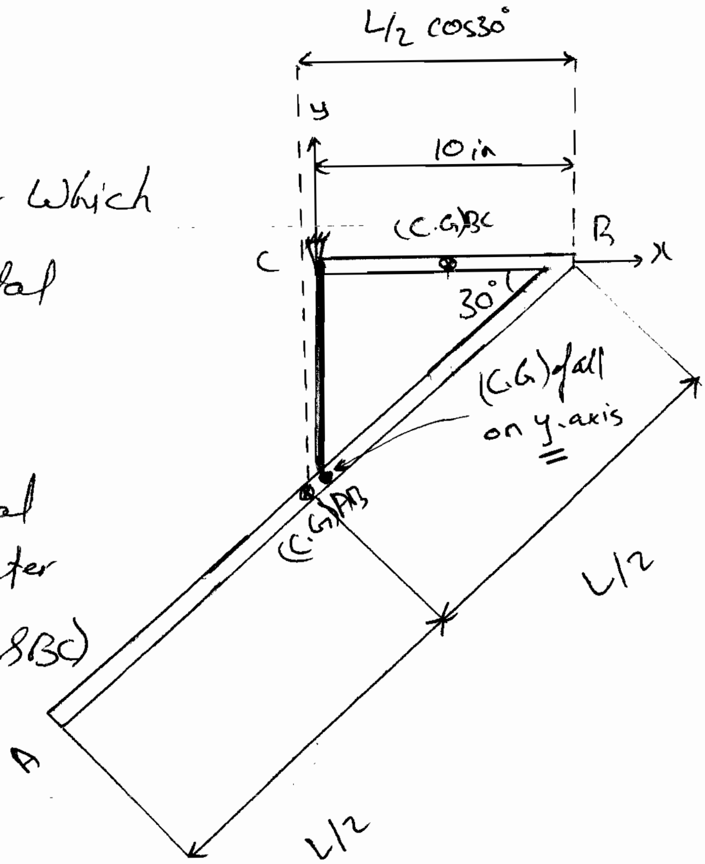
CE 201 - Statics
sections 4 & 5 (071)
H.W 12
Key Solution

Problem 1

Given: Fig P_1

Required: Required length L for which BC becomes horizontal



Solution:

To have BC in the horizontal direction as shown, the center of gravity (C.G) of all (AB & BC) must be on the y-axis (i.e, $\bar{x} = 0$), so that ΣM will be zero. (why?!)

Segment	Length i	$\bar{x}_i$	$\bar{x}_i L_i$
AB	L	$10 - (L/2)\cos 30$	$10L - (L^2/2)\cos 30$
BC	10	5	50
Σ	$L+10$		$(10L - L^2/2 \cos 30) + 50$

$$\bar{X} = \frac{\Sigma \bar{x}_i L_i}{\Sigma L_i} = 0, \implies \Sigma \bar{x}_i L_i = 0$$

$$\Rightarrow (10L - L^2/2 \cos 30^\circ) + 50 = 0$$

Rearranging and simplifying,

$$(\cos 30^\circ) L^2 - 20L - 100 = 0$$

$$\Rightarrow \quad L = \frac{20 \pm \sqrt{(-20)^2 - (4)(\cos 30)(-100)}}{2 \cos 30}$$

$$L = 27.32 \quad \text{or} \quad \underbrace{-4.227}_{\text{Impossible}}$$

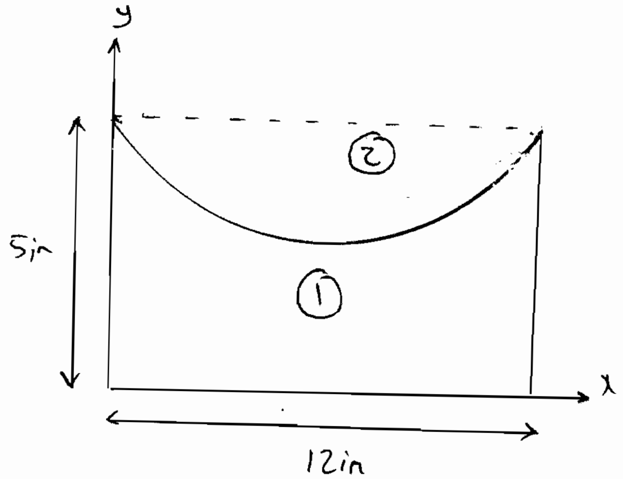
$$\therefore \boxed{L = 27.32 \text{ in}}$$

Problem 2:

Given: Fig P2

Required: Locate the centroid.

Solution:



1 = total rectangle
= (12 X 5)

From Symmetry $\bar{x} = 6in$

$\therefore \boxed{\bar{x} = 6in}$

Segment	$A_i\ (in^2)$	$\bar{y}_i\ (in)$	$A_i\bar{y}_i\ (in^3)$
1	60	2.5	150
2	-24	$5 - \frac{2}{5}(3) = 3.8$	-91.2
Σ	36		58.8

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{58.8}{36} = 1.633in$$

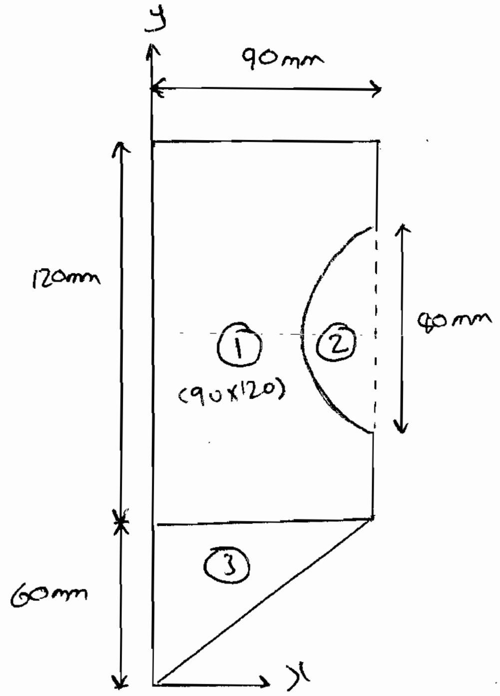
$$\therefore \boxed{\bar{y} = 1.633in}$$

Problem 3:

Given: Fig P_3

Required: Locate center of gravity

Solution:



Segment	A_i (mm^2)	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	$\bar{x}_i A_i$ (mm^3)	$\bar{y}_i A_i$ (mm^3)
1	10800	45	120	486000	1296000
2	-2513.274	73.023	120	-183526.81	-301592.88
3	2700	30	40	81000	108000
Σ	10986.726			383473.190	1102407.120

$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = \frac{383473.190}{10986.726} = 34.903\,mm$$

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{1102407.120}{10986.726} = 100.339\,mm$$

$$\boxed{\begin{aligned} \bar{x} &= 34.90\,mm \\ \bar{y} &= 100.34\,mm \end{aligned}}$$

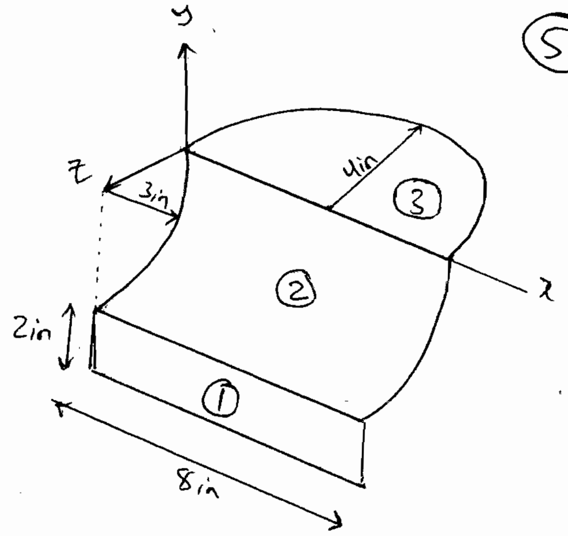


Fig P4

Problem 4

Given: Fig P4

Required: Locate the center of Gravity

Solution:-

From Symmetry $\boxed{\bar{x} = 4\,in}$

Segment	A (in²)	$\bar{y}_i$ (in)	$\bar{z}_i$ (in)	$A_i \bar{y}_i$	$A_i \bar{z}_i$
1	16	-4	3	-64	48
2	$\frac{\pi(3)}{2} \times 8$	$-2(3)/\pi$	$3 - 2(3)/\pi$	-72	41.09734
3	$\pi(4)^2/2$	0	$-4(4)/(3\pi)$	0	-42.66667
Σ	78.83185			-136	46.43067

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{-136}{78.83185} = -1.72519 \text{ in}$$

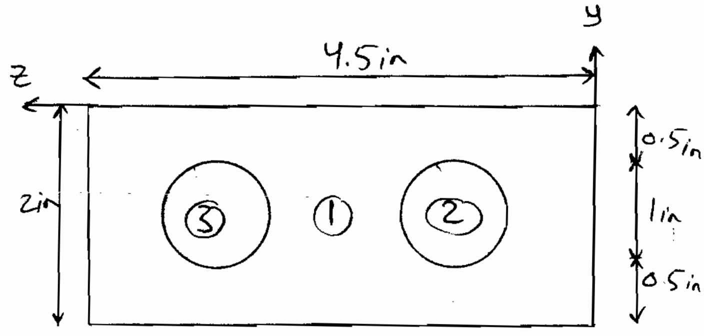
$$\bar{z} = \frac{\Sigma \bar{z}_i A_i}{\Sigma A_i} = \frac{46.43067}{78.83185} = 0.5889 \text{ in}$$

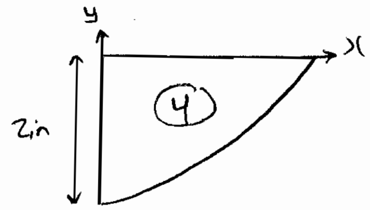
$$\boxed{\begin{aligned} \bar{y} &= -1.725 \text{ in} \\ \bar{z} &= 0.5890 \text{ in} \end{aligned}}$$

Problem 5 :.

Given: Fig P5

Required: Locate centroid.





Solution :

Volume ① = $4.5 \times 0.5 \times 2 = 4.5 \text{ in}^3$

Volume ② = Volume ③ = $\frac{\pi(1)^2}{4} \times 0.5 = 0.3926991 \text{ in}^3$

Volume ④ = $\left(\frac{\pi(4^2)}{4}\right)/4 \times 0.5 = 1.5707963 \text{ in}^3$

Segment	Volume (in³)	$\bar{x}_i$ (in)	$\bar{y}_i$ (in)	$\bar{z}$ (in)	$\bar{x}_i V_i$ (in⁴)	$\bar{y}_i U_i$ (in⁴)	$\bar{z}_i U_i$ (in⁴)
1	4.5	0.25	-1	2.25	1.125	-4.5	10.1250
2	-0.3926991	0.25	-1	1.5	-0.0981748	+0.3926991	-0.5890487
3	-0.3926991	0.25	-1	3.5	-0.0981748	0.3926991	-1.3744469
4	1.5707963	$4(2)/(3\pi)+0.5$	$-4(2)/(3\pi)$	0.25	2.1187315	-1.3333333	0.3926991
Σ	5.2853981				3.0473819	-5.0479351	8.5542035

$$\bar{x} = \frac{\Sigma x_i V_i}{\Sigma U_i} = \frac{3.0473819}{5.2853981} = 0.5765662 \text{ in}$$

$$\bar{y} = \frac{\Sigma y_i U_i}{\Sigma U_i} = \frac{-5.0479351}{5.2853981} = -0.9550718 \text{ in}$$

$$\bar{z} = \frac{\Sigma z_i U_i}{\Sigma V_i} = \frac{8.5542035}{5.2853981} = 1.6184596 \text{ in}$$

$\therefore$ $\boxed{\bar{x} = 0.577 \text{ in}, \ \bar{y} = -0.955 \text{ in}, \ \bar{z} = 1.618 \text{ in}}$