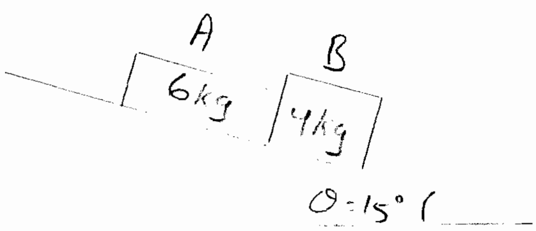
CE-201-485 (071)

H.W # 11

Key Solution

Problem 1:

Given:- $\mu_s = 0.2$, $\mu_k = 0.15$ } Package A



$\mu_s = 0.3$, $\mu_k = 0.25$ } package B

Fig P_1

Required:-
a) check any motion
b) the friction on each package

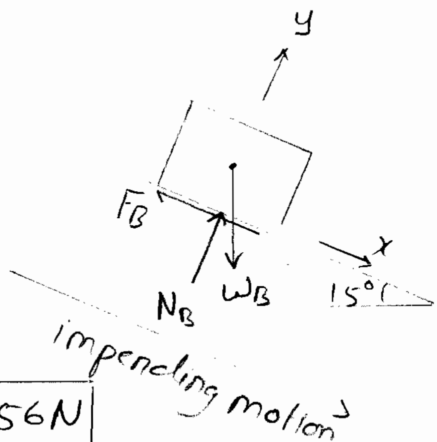
Solution:-

a) First, check the sliding of package B only

(Why ?!)

In FBD ①

$\searrow^{+} \Sigma F_x = 0 \Rightarrow$ (Why x in this direction?!)



FBD ①

$W_B \sin 15° - F_B = 0$

$4(9.81) \sin 15° = F_B \Rightarrow \boxed{F_B = 10.156 N}$

$\nearrow^{+} \Sigma F_y = 0$

$N_B - W_B \cos 15^\circ \Longrightarrow \boxed{N_B = 37.903 N}$

<u>Check</u> $F_B \le \mu_B N_B$

$\mu_B N_B = 0.3(37.903) = 11.371 \ N$

$\Longrightarrow F_B = 11.371 N < 10.540 N$

$\Longrightarrow$ $\boxed{\text{B will not slide alone.}}$

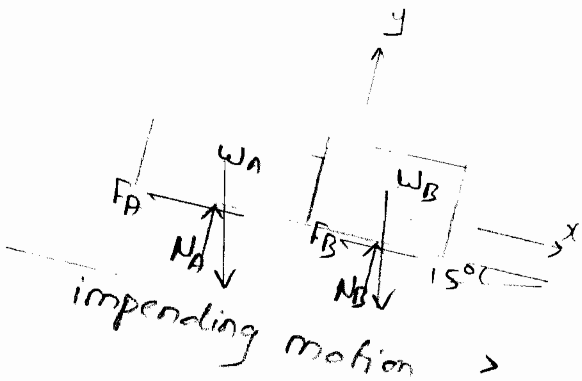
<u>Important Note:</u>

$F_B \neq \mu_B N_B$ (Why?!!!)

Second, Check the Sliding of package A and B together.

(Why did not we check the sliding of package A only ?!!!)

In FBD ②



FBD ②

As before $N_B = 37.903 N$

For N_A,

$+\uparrow^{y} \Sigma F_y = 0$

$N_A = W_A \cos 15 \Longrightarrow \boxed{N_A = 56.854 N}$

For F_A and F_B,

$\overset{+}{\searrow} \sum F_x = 0$

$\Rightarrow W_A \sin 15 + W_B \sin 15 - F_A - F_B = 0$

$\Rightarrow (F_A + F_B) = 25.390\,N.$

$(F_A + F_B)_{max} = \mu_{sA} N_A + \mu_{sB} N_B$

$= 0.2(56.854) + 0.3(37.903)$

$= 22.742\,N$

$\Rightarrow 25.390 > (F_A + F_B)_{max} = 22.742\,N.$

$\Rightarrow$ $\boxed{\text{Packages A and B will both move}}$

Note: that there is at least one other method for the solution. (What is it ?!!)

b) Since both packages move, then

$F_A = F_{A_K} \cancel{max} \; (\mu_K N)_A$

$= (0.15)(56.854)$

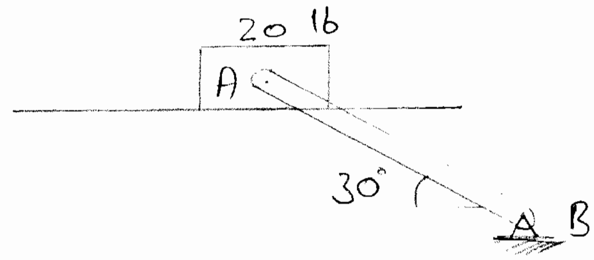
$\Rightarrow \boxed{F_A = 8.528\,N}$

$F_B = F_{B_K} = (\mu_K N)_B$

$= (0.25)(37.903) \Rightarrow \boxed{F_B = 9.476\,N}$

Problem 2:-

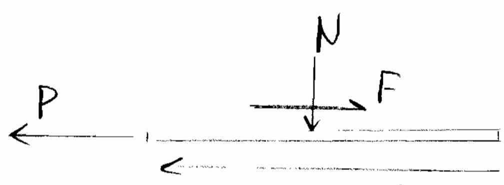
Given: 20 lb block
$\mu_s = 0.25$
Fig P_2



Required:- magnitude of the force P required to move the belt to the left.

Solution:-

From FBD 1 [Conveyor belt], we draw FBD 2 [block]

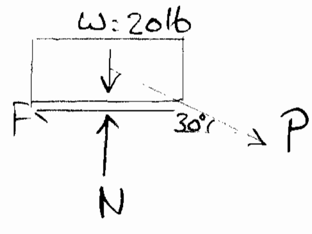


FBD 1

In FBD 2

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$P\cos 30 - F = 0 \implies P = F/\cos 30°$$



FBD 2

$\overset{+}{\uparrow} \Sigma F_y = 0$

$$N - 20 - P\sin 30 = 0$$

$$\implies N - 20 - F\frac{\sin 30}{\cos 30} = 0$$

$$\implies N - 20 - F\tan 30 = 0$$

Since we need to move the belt to the left, it means that we will first have impending motion where

$$F = F_{max} = \mu_s N$$

Thus,

$$N - 20 - 0.25 N \tan 30 = 0$$

$$\Longrightarrow N = 23.374 \text{ lb}$$

$$\Longrightarrow F = 0.25(23.374) = 5.8434 N$$

In FBD (1),

$\xrightarrow{+} \Sigma F_x = 0$

$$F - P = 0$$

$$\boxed{P = 5.843 N}$$

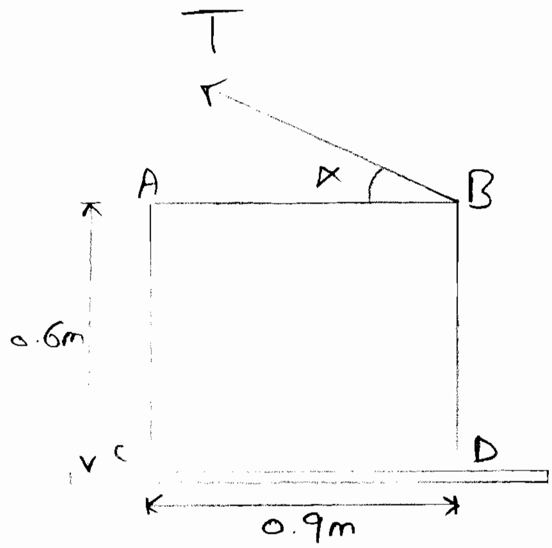
Problem 3:-

Given:- mass $= 30\,kg$

$\mu_s = 0.35$

$\alpha = 30^\circ$

Fig P_3



Required:-

a) The tension T required to move the crate.

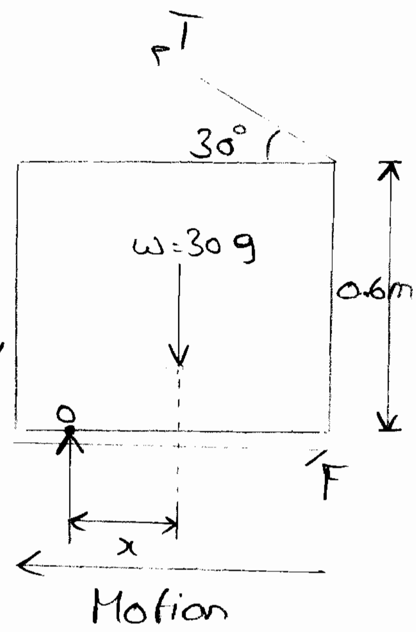
b) Whether the crate will slide or tip.

Solution:

FBD is drawn as shown.

There are more than one way to solve this problem; one method is below; (try others!)

Since we need to move the crate, we must have impending motion first.



FBD ①

Thus, $F = F_{max} = \mu_s N$ ——— ①

OR $X = X_{max} = 0.45\,m$ ——— ②

Let's assume ① above controls (i.e, Sliding will occur before tipping).
[we have to check our assumption latter on as you will see.]

$\xrightarrow{+} \Sigma F_x = 0$ $\quad F - T\cos 30 = 0$ ——— ③

$+\uparrow \Sigma F_y = 0$ $\quad N - 30(9.81) + T\sin 30^\circ = 0$ — ④

From ① into ③

$0.35\,N - T\cos 30^\circ = 0$

$\Rightarrow \quad N = T\dfrac{\cos 30}{0.35}$ ——— ⑤

From ⑤ into ④

$T\left(\dfrac{\cos 30}{0.35}\right) - 30(9.81) + T\sin 30^\circ = 0$

$\Rightarrow \quad T = 98.95\,N$

Now we need to check $x \leq 0.45\,m$ with this value of T.

$\stackrel{+}{\circlearrowright} \Sigma M_o = 0$

$98.95 \cos 30^\circ (0.6) + 98.95 \sin 30 (0.45 + x)$
$- 30 (9.81)(x) = 0$

$\Rightarrow x = 0.3009 \text{ m} < 0.45 \text{ m} \Rightarrow$ OK.

The crate will slide before it tips

$\Rightarrow \boxed{T = 98.95 \text{ N}}$

Note: (If x came out to be > 0.45 m,
What would you do ??!!)

Problem 4:-

Given:- $W = 3000\ lb$

$\mu_s = 0.5$

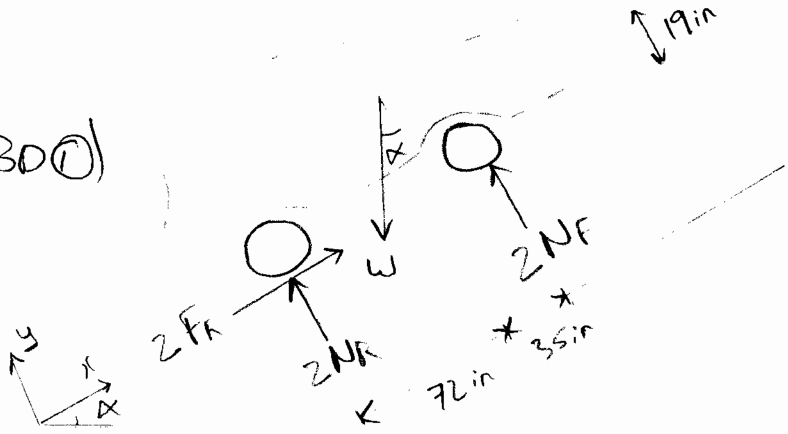
Required: The largest value of angle α if:-

a) the car has rear-wheel drive.
b) the car has front-wheel drive.
c) The car has four-wheel drive.

Solution:-

a) rear-wheel drive (FBD①)

In this case there is friction in the rear wheel only. (Why ?!!)



FBD①

Note "2" in N and R. (Why ?!!)

$$\overset{+}{\nearrow} \Sigma F_x = 0 \implies 2F_R - W \sin \alpha = 0 \quad \text{——} \ (1)$$

Since we need the steepest grade, it means impending motion is assumed

$$\implies F = \mu_s N$$

$$2F_R = \mu_s (2N_R) \implies F_R = \mu_s N_R \quad \text{——} \ (2)$$

$\Rightarrow$ From (2) in (1)

$$2(\mu_s N_R) = W \sin\alpha$$

$$\Rightarrow N_R = (W \sin\alpha)/2\mu_s \quad \text{——} \; (3)$$

Taking $\circlearrowleft^{+} \Sigma M_F = 0$ (Why Front ?!!)

$$(W \sin\alpha)(19) + (W\cos\alpha)(35) - 2N_R(72+35) = 0$$

$$\Rightarrow N_R = \frac{W}{2(107)}(19\sin\alpha + 35\cos\alpha) \quad \text{——} \; (4)$$

From eq (3) into (4)

$$\frac{W}{2\mu_s}\sin\alpha = \frac{W}{2(107)}(19\sin\alpha + 35\cos\alpha) \quad \text{——} \; (5)$$

Multiply both sides of (5) by $\frac{2\mu_s(107)}{W\cos\alpha}$

$$107\tan\alpha = \mu_s(19\tan\alpha + 35)$$

Rearranging:

$$(107 - 19\mu_s)\tan\alpha = 35\mu_s$$

$$\Rightarrow \boxed{\tan\alpha = \frac{35}{\frac{107}{\mu_s} - 19}}$$

By replacing μ_s by 0.5

$$\boxed{\alpha = 10.18^\circ}$$

b) Front-wheel drive [FBD 2]

(Similar to FBD (1) except the friction is at the front wheel)

$\nearrow^{+} \Sigma F_x = 0$

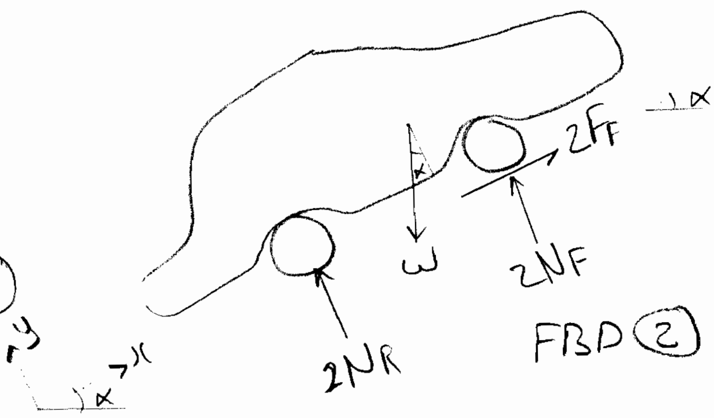
$$2F_F - W \sin\alpha = 0 \quad \text{——} (1)$$

Since we need the steepest gradeit means impending motion is assumed

$\Longrightarrow \quad F = \mu_s N$

$\Longrightarrow \quad 2F_F = \mu(2N_F)$

$$F_F = \mu_s N_F \quad \text{——} (2)$$



From (2) into (1)

$$2(\mu_s N_F) = W \sin\alpha \quad \text{——} (3)$$

$$N_F = (W \sin\alpha)/2\mu_s$$

taking $\circlearrowright^{+} \Sigma M_R = 0$

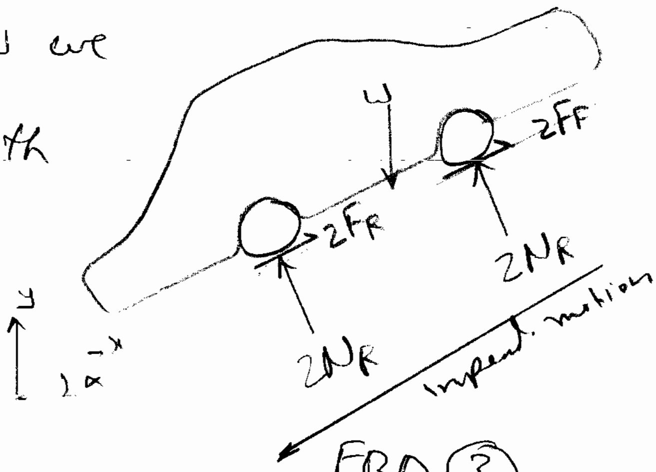
$$N_F = \frac{W}{2(107)}(-19 \sin\alpha + 72 \cos\alpha) \quad \text{——} (4)$$

$$\Longrightarrow \boxed{\tan\alpha = \frac{72}{107/\mu_s + 19}}$$

Since $\mu_s = 0.5$ $\therefore$ $\boxed{\alpha = 17.17^\circ}$

c) Four Wheel drive: [FBD 3]

As before, but now we have friction at both front and rear wheels as shown in FBD 3



FBD 3

$\nearrow^{+} \sum F_x = 0$

$$2F_F + 2F_R - W \sin\alpha = 0$$

$$\Rightarrow 2\mu_s N_F + 2\mu_s N_R - W\sin\alpha = 0 \quad \text{— A}$$

$\circlearrowright^{+} \sum M_F = 0$ [as in part (a)!]

$$\Rightarrow N_R = \frac{W}{2(107)}(19\sin\alpha + 35\cos\alpha) \quad \text{— B}$$

$\circlearrowright^{+} \sum M_R = 0$ [as in part (b)!]

$$N_F = \frac{W}{2(107)}(-19\sin\alpha + 72\cos\alpha) \quad \text{C}$$

Thus, from eq B and C, into A

$$\frac{2\mu_s W}{2(107)}(-19\sin\alpha + 72\cos\alpha) +$$

$$+ \frac{2\mu_s W}{2(107)}(19\sin\alpha + 35\cos\alpha) - W\sin\alpha = 0 \quad \text{— (D)}$$

Simplifying eq (D)

$$\frac{\mu_s}{107}(72+35)\cos\alpha - \sin\alpha = 0$$

$$\frac{\mu_s}{107}(107)\cos\alpha - \sin\alpha = 0$$

$$\sin\alpha = \mu_s \cos\alpha$$

Dividing by $\cos\alpha$, $\boxed{\tan\alpha = \mu_s}$

Raplacing μ_s by 0.5,

$\tan\alpha = 0.5 \implies \boxed{\alpha = 26.57^\circ}$

Conclusions :-

The following conclusions can be drawn :-

1) The steepest slope $(\alpha) \neq f(W)$!

2) α is propertional to μ; that is the bigger the μ the bigger the slope α.

3- In this particular case, the best car is the four wheel drive, followed by the front-wheel drive.

4- Rear-wheel-drive could be better than front-wheel drive if the center of gravity (centroid) is closer to it (72" vs. 35").

5- If you have a rear-wheel drive car, and you have a problem climbing the slope (when the road is slippery), then try to shift the centroid to the rear by adding more loads on the back (e.g trunck back seats)

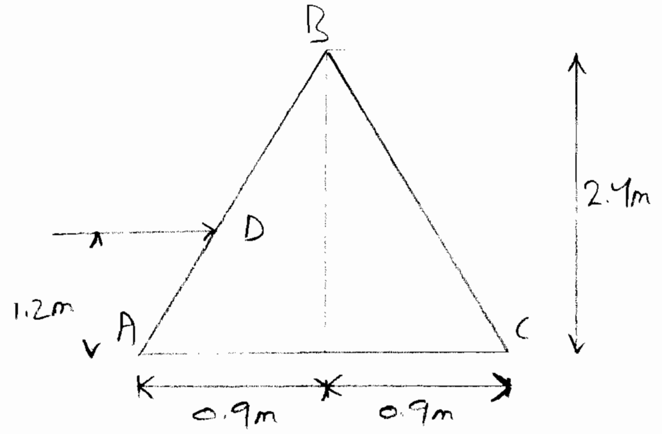
6- In four-wheel drive, the steepest slope is a function of μ_s only; it is not a function of weight, centroid, or geometry. It becomes like any single object on an incline in which $\tan \alpha = \mu_s$ (Always).

7- To be able to climb steeper slope, try to increase μ by making the tires and/or the road rougher « less smooth » (How?!)

8- Note that driving on normal highways, you need (in general) to minimize the friction (for smooth driving, saving gas, going faster, ---etc); thus, rear-wheel drive cars are better (if the centroid is closer to the front wheels). But driving on steep slippey slopes, four-wheel & front-wheel drive cars are better.

Problem 5:-

Given:- mass = 20 kg
μ_s = 0.4
Fig P_5



Required:-
a) The largest magnitude of the Force P. for which equilibrium will be maintained.

b) The surface at which motion will impend.

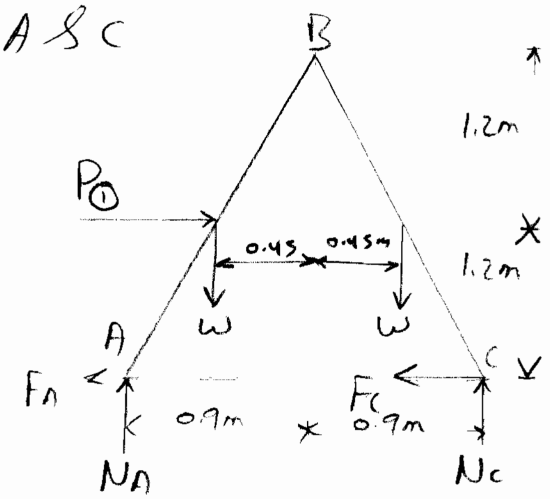
Solution:-

There are several possible motions !!

(a) Sliding of A and C, FBD is drawn as shown

Since we are assuming motion of A & C

$F = \mu N$ @ A and C



$\xrightarrow{+} \Sigma F_x = 0$

$P - F_A - F_C = 0$

$P - 0.4N_A - 0.4N_C = 0$ —— (1)

$+\uparrow \Sigma F_y = 0$

$N_A + N_C - 2w = 0$ —— (2)

$\overset{+}{\curvearrowleft} \Sigma M_c = 0$

$0.45W + 1.35W - 1.8N_A - 1.2P = 0$

$1.8W - 1.8N_A - 1.2P = 0$ —— (3)

From (3), $P = 1.5W - 1.5N_A$ —— (4)

From (4) into (1), $1.5W - 1.5N_A - 0.4N_A - 0.4N_c = 0$

$\Longrightarrow$ $1.5W - 1.9N_A - 0.4N_c = 0$ —— (5)

From (2), $N_A = 2W - N_c$ —— (6)

From (6) into (5), $1.5W - 1.9(2)W - 1.9(-N_c) - 0.4N_c = 0$

$\Longrightarrow$ $-2.3W + 1.5N_c = 0$

$\Longrightarrow$ $N_c = 1.53333\ W$ —— (7)

From (7) into (2), $N_A = 2W - 1.53333\,W$

$N_A = 0.466667\,W$ —— (8)

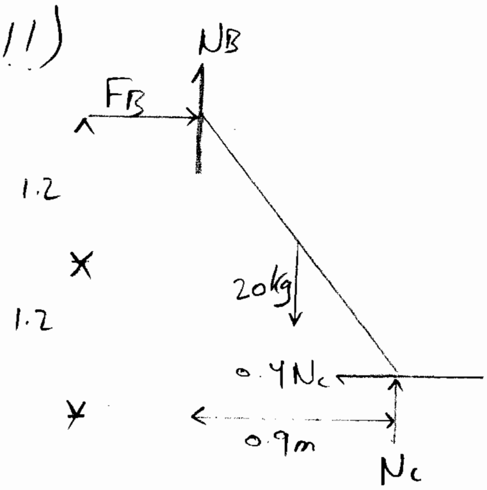
From (8) into (4), $P = 1.5W - 1.5(0.466667W)$

$= 0.65W = 0.65(20)(9.81)$

$$\boxed{P_{①} = 127.53\ N}$$

(b) Sliding of A and C ((Another Method))

FBD of BC is known. (Why ?!!)



FBD (BC)

Since motion is at C and no motion at B

$F_C = \mu N_C = 0.4 N_C$

$F_B < \mu N_B$
$= ?$

$\circlearrowright^{+} \sum M_B = 0$

$0.9\, N_C - (2.4)(0.4) N_C - 20(0.4) = 0$

$\Rightarrow N_C = \dfrac{9.96}{0.9} = 11.07\, g$

Check $F_B \overset{?}{\leq} \mu N_B$

$\overset{+}{\rightarrow} \sum F_x = 0 \qquad N_B = 0.4 N_C = 4.427 g$

$\overset{+}{\uparrow} \sum F_y = 0 \qquad F_B = -N_C + 20 = 8.93\, g$

$F_{B\,max} = \mu N_B = 1.77 g < 8.93 g \Rightarrow$ Not ok

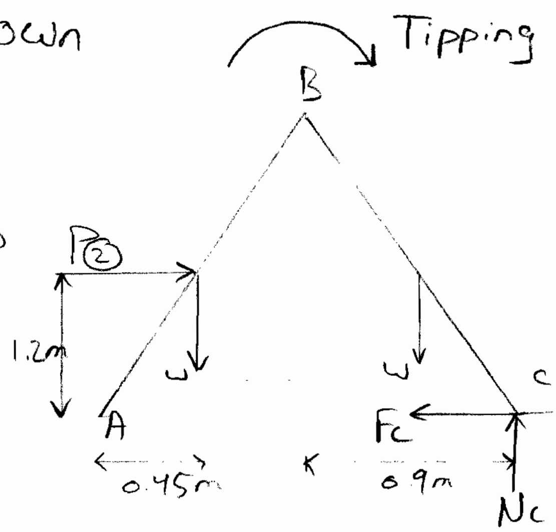
Which means it is not possible to slide at A before sliding B

Thus, sliding of A and C will not occur first.

2) Tipping of <u>both</u> boards. (Why?!)

Due to the location and direction of P, the tipping will occur to the right at the instant the reactions at A (N and F) are becoming zero. (Why?!)

$\Longrightarrow$ FBD is drawn as shown



$\circlearrowright^{+} \Sigma M_C = 0$ (Why?!)

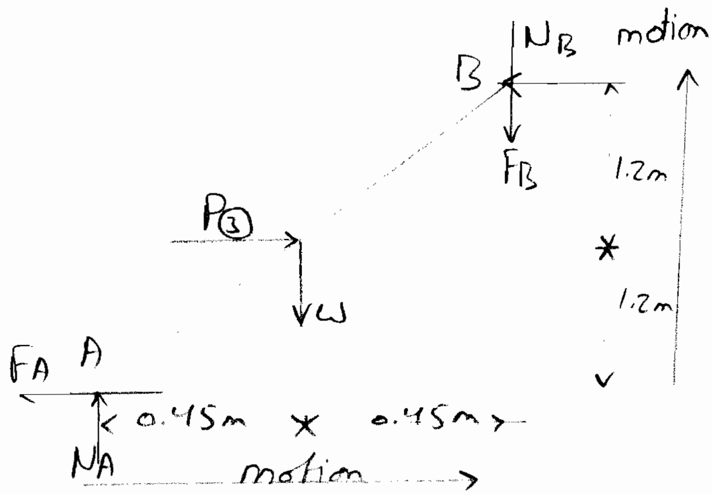
$\Longrightarrow -1.2 P_{②} + 1.35W + 0.45W = 0$

$\Longrightarrow P_{②} = 1.5W$

$\Longrightarrow \boxed{P_{②} = 294.3\ N}$

3) Motion at A and B :

FBD is drawn.



$F = \mu N$ « impending motion is assumed »

$F_A = 0.4 N_A$

$F_B = 0.4 N_B$

$\xrightarrow{+} \Sigma F_x = 0 \Longrightarrow P_{③} - N_B - 0.4 N_A = 0$ —— (1)

$\overset{+}{\uparrow}\Sigma F_y = 0 \implies N_A - 0.4\,N_B - W = 0$ — (2)

$\overset{+}{\circlearrowright}\Sigma M_B = 0 \implies 1.2\,P_{③} + 0.45W - 0.9N_A - 2.4(0.4N_A) = 0$

$\implies 1.2\,P_{③} + 0.45W - 1.86\,N_A = 0$ — (3)

From (1), $N_B = P_{③} - 0.4\,N_A$ — (4)

From (2), $N_A = W - 0.4\,N_B$ — (5)

From (4) into (5), $N_A = W - 0.4(P_{③} - 0.4N_A)$

$\implies N_A = (W - 0.4\,P_{③})/0.84$ — (6)

From (6) into (3),

$$1.2\,P_{③} + 0.45W - \frac{1.86}{0.84}(W - 0.4\,P_{③}) = 0$$

$$\implies P_{③} = 0.84589\ W$$

$$\boxed{P_{③} = 166.0\ \text{N}}$$

4) Motion at B and C:-

(Is it possible?! Is it possible to control??!)

5) Motion at A only:-

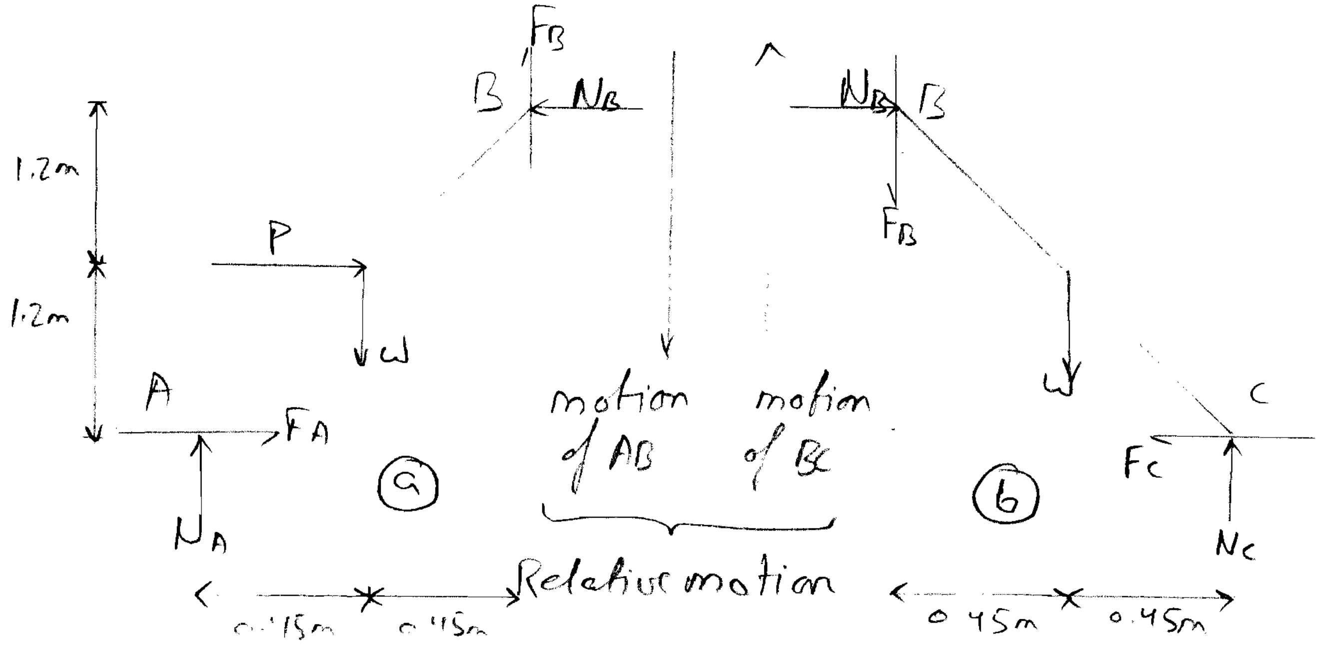
Is it possible? Try it yourself!

6) Motion of C only:-

Is it possible?! Try it yourself!

7) Motion at B only:-

The two FBDs are drawn as shown. Note that we had to sparate AB and BC. (Why?!)



In FBD (b),

$$F_B = F_{max} = \mu N_B \quad (\text{Why?!})$$
$$= (0.4)(N_B)$$
$$F_C < F_{max} \; (= \mu N_C) \quad (\text{Why?!})$$

$\circlearrowright \Sigma M_C = 0$

$$0.45W + (0.9)(0.4N_B) - 2.4N_B = 0$$

$$\Longrightarrow N_B = \frac{0.45W}{2.09} = 43.279\,N$$

Check $F_c \overset{?}{<} \mu N_c$

$\xrightarrow{+} \Sigma F_x = 0$

$$F_c = N_B = 43.279\,N$$

$+\uparrow \Sigma F_y = 0$

$$N_c = W + 0.4(43.279)$$
$$= 213.5\,N$$

$$F_{c\,max} = \mu N_c = (0.4)(213.5)$$
$$= 85.39\,N < F_c = 43.279\,N$$

$\Longrightarrow$ OK (i.e, no sliding at C)

In FBD ②

$$F_B = F_{max} = \mu N_B$$
$$F_A < F_{max} = \mu N_A$$

$+\circlearrowright \Sigma M_A = 0$

$$-0.45W - 1.2P_{⑦} + 2.4(43.279)$$
$$+ 0.9(0.4)(43.279) = 0$$

$$P_{⑦} = 25.93\ N$$

Check $F_A \overset{?}{<} F_{A\,max}$

$+\uparrow \Sigma F_y = 0$

$$N_A = W - F_B = 178.9\,N$$

$\xrightarrow{+} \Sigma F_x = 0$

$$F_A = N_B - P = 17.35\ N$$

$$F_{A\,max} = \mu N_B = 0.4\,(178.9) = 71.56\,N$$

$$F_A < F_{A\,max} \Longrightarrow OK$$

"No motion at A"

$P_{max} = P_{min}\ (P_1, P_2, \ldots\ldots, P_7)$ (Why ?!)

$$\boxed{P_{max} = P_7 = 25.93\,N}$$