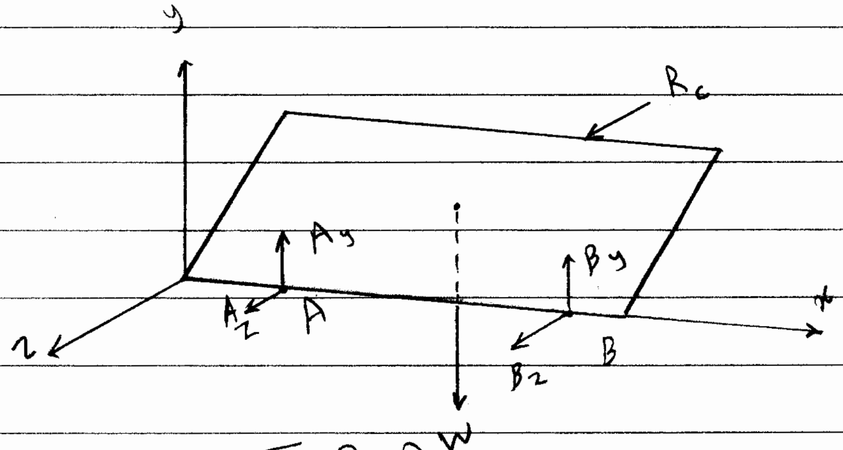
Solution of HW # 8

Problem # 1

Given: The figure shown in the HW sheet (Fig P1), neglecting friction at all surfaces of contact.

Required: The reactions at A, B & C.

Solution:



FBD

$$\sum \vec{M}_A = \vec{0}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 4\sin 60 & -4\cos 60 \\ 0 & 0 & R_c \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2\sin 60 & -2\cos 60 \\ 0 & -50 & 0 \end{vmatrix}$$

$$(4\sin 60 \times R_c - 50 \times 2\cos 60)i + (-5B_z - 4R_c)j + (5B_y - 2\times 50)k = 0$$

$\sum M_x = 0 \Rightarrow 4\sin 60 \times R_c - 50\times 2\cos 60 = 0 \Rightarrow$ $\boxed{R_c = 14.43\ lb}$

$\sum M_y = 0 \Rightarrow -5B_z - 4\times 14.43 = 0 \Rightarrow$ $\boxed{B_z = -11.54\ lb}$

$\sum M_z = 0 \Rightarrow 5B_y - 2\times 50 = 0 \Rightarrow$ $\boxed{B_y = 20\ lb}$

$\sum F_y = 0 \Rightarrow A_y + 20 - 50 = 0 \Rightarrow$ $\boxed{A_y = 30\ lb}$

$\sum F_z = 0 \Rightarrow -11.54 + A_z + 14.43 = 0$

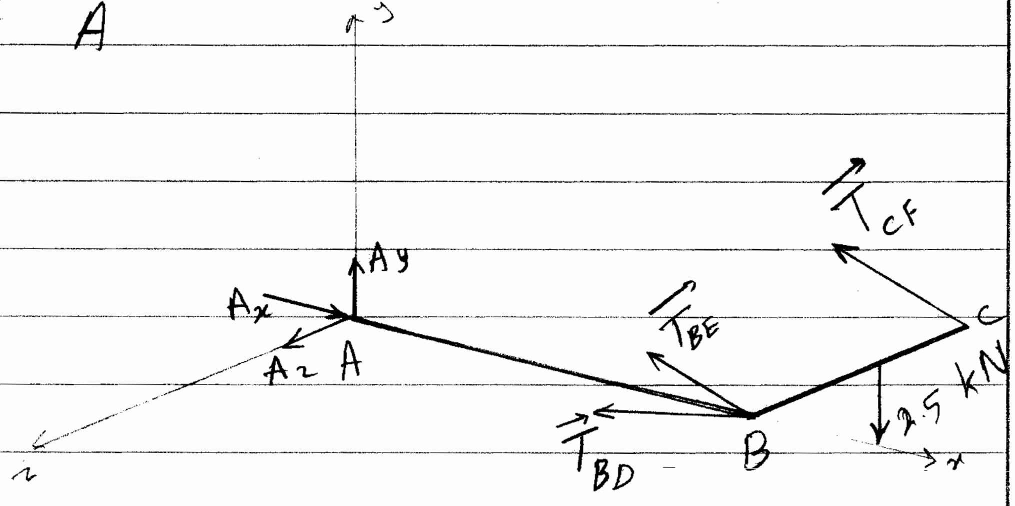
$\Rightarrow$ $\boxed{A_z = -2.89\ lb}$ #

Problem # 2

Given: The system shown in Fig. P2 in the HW sheet

Required: The tension in each cable and the reaction at A

Solution:



FBD

From the FBD

$$\sum \vec{M}_A = \vec{0}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 0 & 0 \\ -\frac{15}{16.25}T_{BD} & 0 & \frac{6.25}{16.25}T_{BD} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 0 & 0 \\ -\frac{15}{16.25}T_{BE} & \frac{6.25}{16.25}T_{BE} & 0 \end{vmatrix} +$$

{ Note: You can combine T_{BD} & T_{BE}. Why? }

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 0 & -0.75 \\ -\frac{15}{16.25}T_{CF} & \frac{6.25}{16.25}T_{CF} & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 0 & -0.375 \\ 0 & -2.5 & 0 \end{vmatrix} = 0$$

$\sum M_x = 0 \Rightarrow 0.75 \times \frac{6.25}{16.25} T_{CF} - 2.5 \times 0.375 = 0 \Rightarrow$ $\boxed{T_{CF} = 3.25\ kN}$

$\sum M_y = 0 \Rightarrow -1.5 \times \frac{6.25}{16.25} T_{BD} + 0.75 \times \frac{15}{16.25} T_{CF} = 0 \Rightarrow$ $\boxed{T_{BD} = 3.9\ kN}$

$\sum M_z = 0 \Rightarrow 1.5 \times \frac{6.25}{16.25} T_{BE} + 1.5 \times \frac{6.25}{16.25} T_{CF} - 1.5 \times 2.5 = 0$

$\Rightarrow$ $\boxed{T_{BE} = 3.25\ kN}$

$\sum F_x = 0 \Rightarrow A_x - \frac{15}{16.25} T_{BE} - \frac{15}{16.25} T_{BD} - \frac{15}{16.25} T_{CF} = 0 \Rightarrow$ $\boxed{A_x = 9.6\ kN}$

$\sum F_y = 0 \Rightarrow A_y + \frac{6.25}{16.25} T_{BE} + \frac{6.25}{16.25} T_{CF} - 2.5 = 0 \Rightarrow$ $\boxed{A_y = 0}$

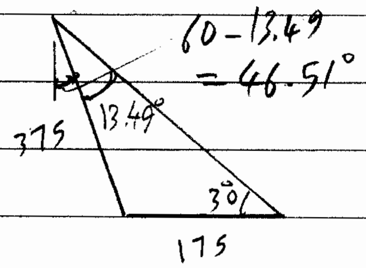
$\sum F_z = 0 \Rightarrow A_z + \frac{6.25}{16.25} T_{BD} = 0 \Rightarrow$ $\boxed{A_z = -1.5\ kN}$ #

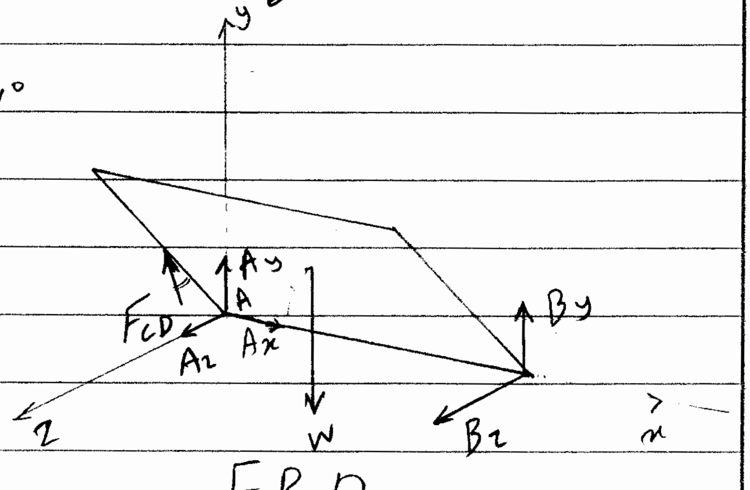
Problem # 3

Given: The system shown in Fig P3 in the HW sheet, the sheet weight is 30 kg and $\alpha = 30°$

Required: a) The magnitude of force exerted by rod CD.
b) The reaction at the hinges.

Solution





$\Sigma F_x = 0 \Rightarrow$

$A_x = 0$

Using the sine rule to find the direction of F_{CD} $\quad \frac{375}{\sin 30} = \frac{175}{\sin\theta} \Rightarrow \theta = 13.49°$

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma \vec{M}_A = \vec{0} \Rightarrow$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.65 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.325 & 0.2 & 0.3464 \\ 0 & -294.3 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0.175 \\ 0 & F_{CD}\cos 46.51 & F_{CD}\sin 46.51 \end{vmatrix}$$

$\Sigma M_x = 0 \Rightarrow 0.3464 \times 294.3 - 0.175 \times F_{CD}\cos 46.51 = 0 \Rightarrow F_{CD} = 846.4\ N$

$\Sigma M_y = 0 \Rightarrow -0.65 B_z = 0 \Rightarrow B_z = 0$

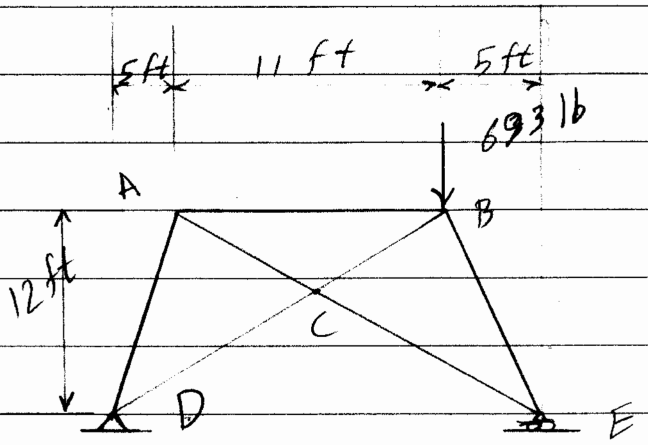
$\Sigma M_z = 0 \Rightarrow 0.65 B_y - 0.325 \times 294.3 = 0 \Rightarrow B_y = 147.2\ N$

$\Sigma F_y = 0 \Rightarrow 147.2 - 294.3 + 846.4\cos 46.51 + A_y = 0$

$A_y = -435.4\ N$

$\Sigma F_z = 0 \Rightarrow 846.4 \times \sin 46.51 + A_z = 0 \Rightarrow A_z = -614\ N$

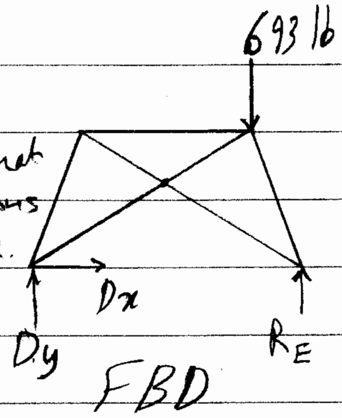
Problem # 4



Given: The system shown in the figure

Required: The force in each member.

Solution: From FBD



* Note that reactions are needed. Why?!

$\Sigma F_x = 0 \Rightarrow D_x = 0$

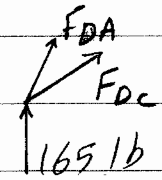
$\Sigma M_D = 0 \Rightarrow$ (+↺)

$R_E \times 21 - 693 \times 16 = 0$

$\Rightarrow R_E = 528 \text{ lb}$

$\Sigma F_y = 0 \Rightarrow D_y - 693 + R_E = 0 \Rightarrow D_y = 165 \text{ lb}$

Joint D FBD



$\Sigma F_y = 0 \Rightarrow$

$165 + F_{DA}\frac{12}{13} + F_{DC}\frac{12}{20} = 0$ ---- (1)

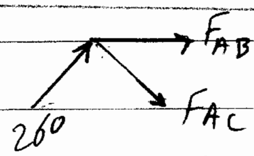
$\Sigma F_x = 0 \Rightarrow \frac{5}{13}F_{DA} + \frac{16}{20}F_{DC} = 0$ ---- (2)

$\Rightarrow F_{DA} = -260 \text{ lb}$

$\Rightarrow F_{DC} = +125 \text{ lb}$

$\Rightarrow$ **$F_{DA} = 260 \text{ lb (C)}$, $F_{DC} = 125 \text{ lb (T)}$**

Joint A



$\Sigma F_x = 0 \Rightarrow$

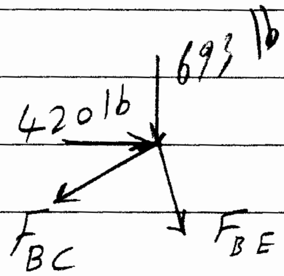
$\frac{5}{13}260 + \frac{16}{20}F_{AC} + F_{AB} = 0$ ---- (3)

$\Sigma F_y = 0 \Rightarrow \frac{12}{13}260 - \frac{12}{20}F_{AC} = 0 \Rightarrow$ **$F_{AC} = 400 \text{ lb (T)}$**

from (3) $\Rightarrow F_{AB} = -420 \text{ lb} \Rightarrow$ **$F_{AB} = 420 \text{ lb (C)}$**

Continue problem # 4

Joint B FBD



$\Sigma F_x = 0 \Rightarrow$

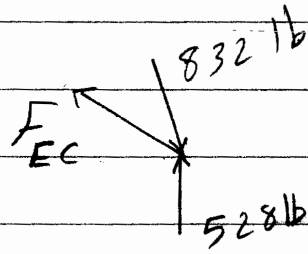
$420 - \frac{16}{20} F_{BC} + \frac{5}{13} F_{BE} = 0$ --- 4

$\Sigma F_y = 0 \Rightarrow -693 - \frac{12}{20} F_{BC} - \frac{12}{13} F_{BE} = 0$ -- (5)

from 4 & 5 $\Rightarrow$

$F_{BC} = 125 \text{ lb } (T)$
$F_{BE} = 832 \text{ lb } (C)$

Joint E FBD

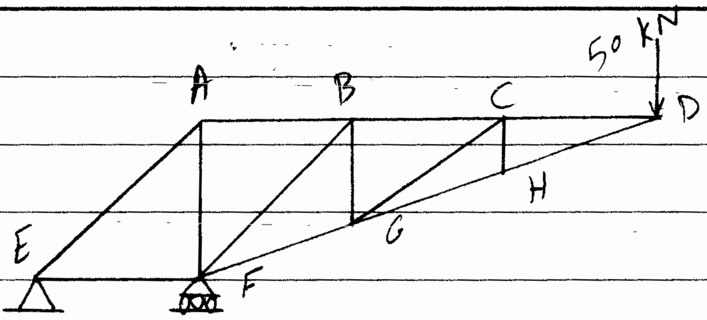


$\Sigma F_x = 0$

$-\frac{16}{20} F_{EC} + \frac{5}{13} 832 = 0$

$\Rightarrow$ $F_{EC} = 400 \text{ lb } T$

Problem # 5



Given: The system shown in the figure

Required: a) all zero-force members by inspection

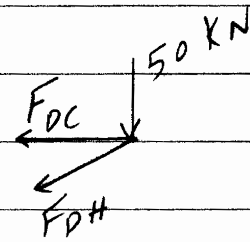
b) the force in all other member

Solution:

a) The zero-force members are (in order)

HC ⇒ CG ⇒ GB ⇒ BF

b) Joint D FBD



$\Sigma F_y = 0$

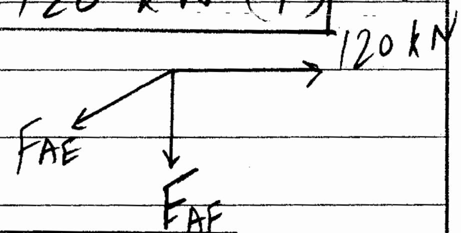
$-50 - \frac{2.5}{6.5} F_{DH} = 0 \Rightarrow F_{DH} = -130 \text{ kN}$

$\Rightarrow \boxed{F_{DH} = F_{GH} = F_{FG} = 130 \text{ kN (C)}}$ Why?!

$\Sigma F_x = 0 \Rightarrow -F_{DC} + \frac{6}{6.5} 130 = 0 \Rightarrow F_{DC} = 120 \text{ kN}$

$\Rightarrow \boxed{F_{DC} = F_{CB} = F_{BA} = 120 \text{ kN (T)}}$ Why?!

Joint A FBD



$\Sigma F_x = 0 \Rightarrow$

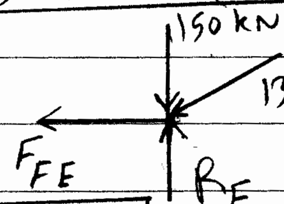
$120 - \frac{2}{\sqrt{10.25}} F_{AE} = 0 \Rightarrow$

$\boxed{F_{AE} = 192.09 \text{ kN (T)}}$

$\Sigma F_y = 0 \Rightarrow -\frac{2.5}{\sqrt{10.25}} 192.09 - F_{AF} = 0 \Rightarrow$

$\boxed{F_{AF} = 150 \text{ kN (C)}}$

Joint F FBD



$\Sigma F_x = 0 \Rightarrow$

$-F_{FE} - \frac{6}{6.5} 130 = 0$

$\Rightarrow \boxed{F_{FE} = 120 \text{ kN C}}$

* Note that the reactions are not needed. Why?!