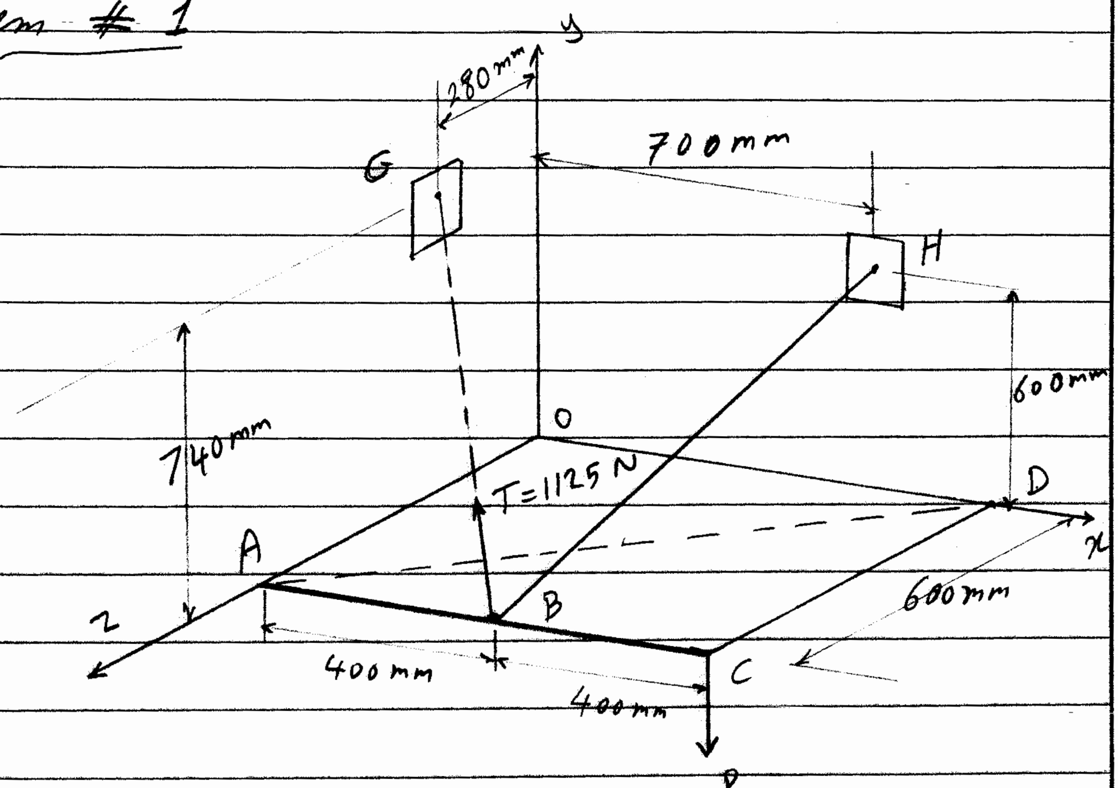
CE 201-9 (061)

Solution of HW # 5

Problem # 1



Given: The system shown in the figure

Required: a) The moment of T about AD.

b) The perpendicular distance between BG & AD.

Solution: $M_{AD} = \vec{u}_{AD} \cdot (\vec{r}_{AB} \times \vec{T})$

$A(0, 0, 600)$ & $D(800, 0, 0)$

$B(400, 0, 600)$ & $G(0, 740, 280)$

$\vec{AD} = +800\vec{i} - 600\vec{k}$ & $AD = 1000$ mm

$\vec{u}_{AD} = \dfrac{\vec{AD}}{AD} = 0.8\vec{i} - 0.6\vec{k}$

$\vec{r}_{AB} = 400\vec{i}$ (mm) or $\vec{r}_{AB} = 0.4\vec{i}$ (m)

$\vec{BG} = -400\vec{i} + 740\vec{j} - 320\vec{k}$ & $BG = 900$

$\vec{u}_{BG} = \dfrac{\vec{BG}}{BG} = -0.44444\vec{i} + 0.82222\vec{j} - 0.35556\vec{k}$

Continue problem # 1

$$\vec{T}_{BG} = T_{BG}\,\vec{U}_{BG}$$
$$= -500\,\vec{i} + 925\,\vec{j} - 400\,\vec{k}$$

$\therefore$

$$M_{AD} = \vec{U}_{AD} \cdot \left(\vec{r}_{AB} \times \vec{T}_{BG}\right)$$

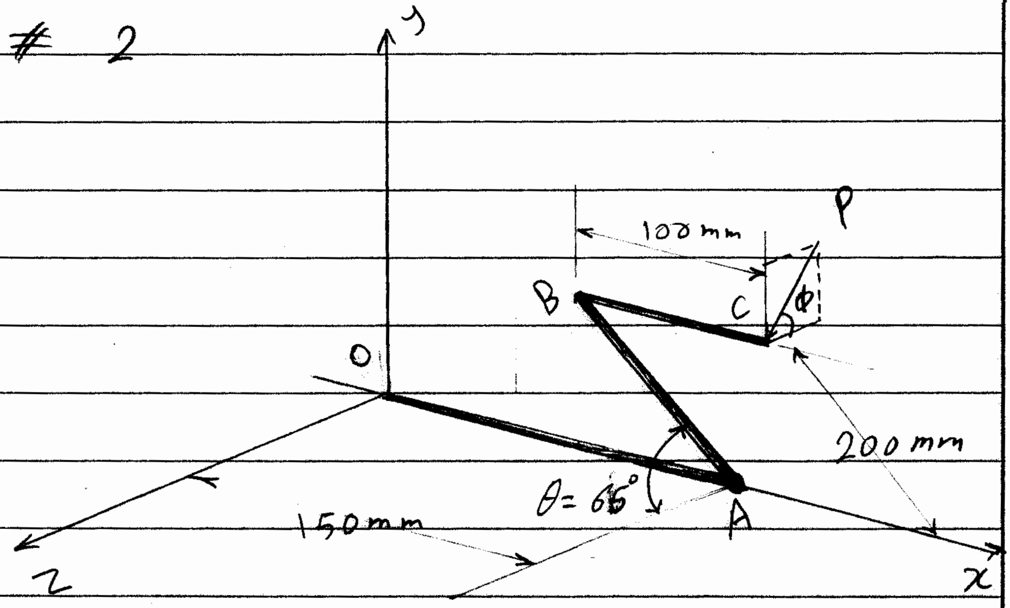
$$= \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.4 & 0 & 0 \\ -500 & 925 & -400 \end{vmatrix}$$

$$M_{AD} = -222 \ \text{N}\cdot\text{m} \quad \text{(Scalar)}$$

$$\therefore \vec{M}_{AD} = M_{AD}\,\vec{U}_{AD} = -222\left(0.8\,\vec{i} - 0.6\,\vec{k}\right)$$

$$\boxed{\vec{M}_{AD} = -177.6\,\vec{i} + 133.2\,\vec{k}} \quad \text{(Vector)}$$

Problem # 2



Given: The system shown with $M_{P_y} = -15\ N.m$ and $M_z = -36\ N.m$

Required: $M_x = ?$

Solution: $C\ (250\ ;\ 200 \sin 65\ ;\ 200 \cos 65)$

$$\therefore \vec{r}_{OC} = 250\vec{i} + 200 \sin 65\,\vec{j} + 200 \cos 65\,\vec{k}$$

$$\vec{P} = 0\vec{i} + P_y\vec{j} + P_z\vec{k}$$

$$\therefore \vec{M}_{P_O} = \vec{r}_{OC} \times \vec{P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 250 & 200\sin 65 & 200\cos 65 \\ 0 & -P_y & P_z \end{vmatrix}$$

$$= (200 \sin 65\, P_z - 200 \cos 65\, P_y)\,\vec{i} - (250\, P_z)\,\vec{j} + (250\, P_y)\,\vec{k}$$

$$\therefore M_y = -15 = -0.25\, P_z \Rightarrow P_z = 60\ N$$

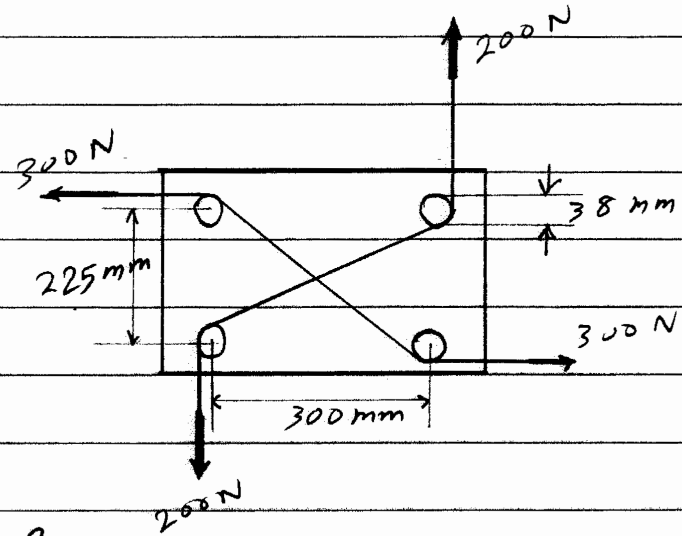
$$\&\ M_z = -36 = 0.25\, P_y \Rightarrow P_y = -144\ N$$

$$\therefore M_x = 0.2 \sin 65 * 60 - 0.2 \cos 65 * (-144)$$

$$= 23.05\ N.m \quad \#$$

Note (how to find P & Φ if needed?)

Problem # 3



Given The system shown

Required
a) The resultant couple acting on the board.
b) if only one string is used, around which pegs should it pass and in what direction should it be pulled to create the same couple with the minimum tension in the string?
c) The value of that minimum tension.

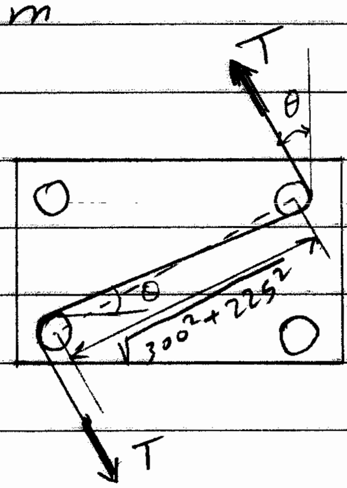
Solution a)

$$M = 200\left[0.3 + 2\,\frac{0.038}{2}\right] + 300\left[0.225 + 2\,\frac{0.038}{2}\right]$$
$$= 146.5\ N.m$$

b)

$$\theta = \tan^{-1}\left(\frac{225}{300}\right)$$

$$\boxed{\theta = 36.87^\circ}$$



note: The other string AD can be used.

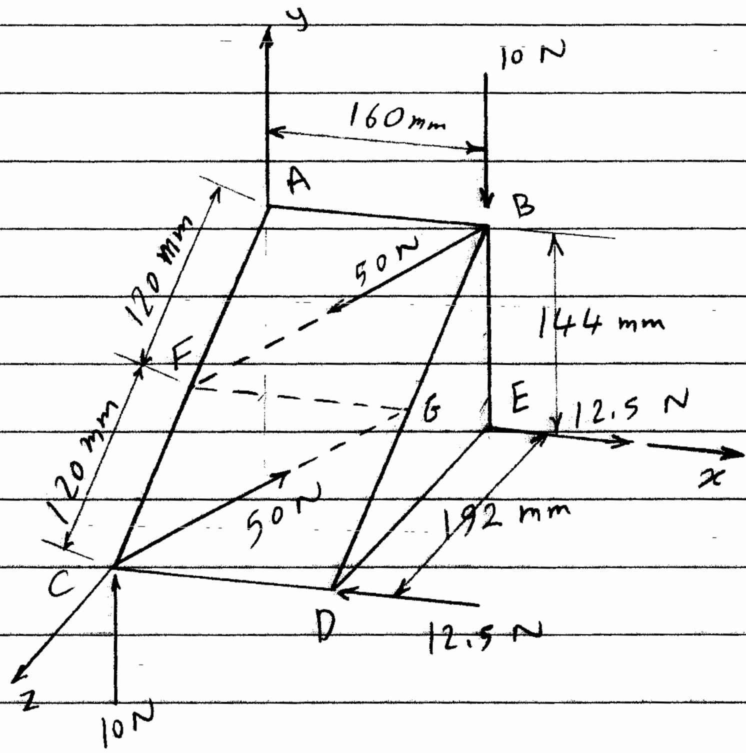
c) $d = \sqrt{300^2 + 225^2} + 2\,\frac{38}{2} = 413\ mm$

$$\therefore T = \frac{M}{d} = \frac{146.5}{0.413} = 354.7\ N$$

$$\therefore \boxed{T = 354.7\ N}$$

\#

Problem # 4



Given: The three couples shown in the figure.
Required: A single equivalent coupled.
Solution:

$F = 12.5\ N$

$\vec{F}_D = -12.5\,\vec{i} \quad (N)$

$\vec{r}_{ED} = 192\,\vec{k} \quad (mm)$

$$M_{E_1} = \vec{r}_{ED} \times F_D = \begin{vmatrix} i & j & k \\ 0 & 0 & 0.192 \\ -12.5 & 0 & 0 \end{vmatrix} = -2.4\,\vec{j} \quad N.m$$

$P = 10\ N$

$P_c = 10\,\vec{j} \quad (N)$

$r_{EC} = -160\,\vec{i} + 192\,\vec{k}$

$$M_{E_2} = \vec{r}_{EC} \times \vec{P}_c = \begin{vmatrix} i & j & k \\ -0.16 & 0 & 0.192 \\ 0 & 10 & 0 \end{vmatrix} = -1.92\,\vec{i} - 1.6\,k$$

Continue problem # 4

$F = 50\,N$

$\vec{CG} = 160\vec{i} + 72\vec{j} - 96\vec{k}$ $\quad$ & $CG = 200\,mm$

$\vec{u}_{CG} = \frac{\vec{CG}}{CG} = \frac{1}{200}(160\vec{i} + 72\vec{j} - 96\vec{k})$

$\therefore \vec{F_c} = \frac{50}{200}(160\vec{i} + 72\vec{j} - 96\vec{k})$

$\vec{F_c} = 40\vec{i} + 18\vec{j} - 24\vec{k}$

$\vec{r}_{FC} = -72\vec{j} + 96\vec{k}$ (mm)

$$\therefore M_F = \vec{r}_{FC} \times \vec{F_c} = \begin{vmatrix} i & j & k \\ 0 & -72 & 96 \\ 40 & 18 & -24 \end{vmatrix}$$

$M_F = 3.84\vec{j} + 2.88\vec{k}$

$\therefore$ The equivalent couple is

$M = M_{E_1} + M_{E_2} + M_F$

$= (-2.4\vec{j}) + (-1.92\vec{i} - 1.6\vec{k}) +$

$+ (3.84\vec{j} + 2.88\vec{k}$

$\vec{M} = -1.92\vec{i} + 1.44\vec{j} + 1.28\vec{k}$ #

its magnitude are

$M = 2.72$ N.m

direction:

$\vec{u} = \frac{\vec{M}}{M} = \frac{1}{2.72}(-1.92\vec{i} + 1.44\vec{j} + 1.28\vec{k})$