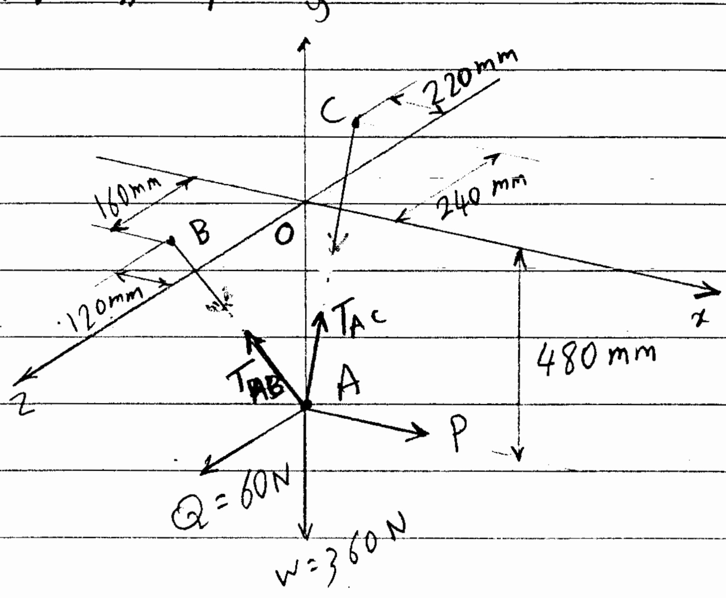
Solution of HW # 4

Problem # 1



Given: The force and cables shown in the figure.

Required: a) The magnitude of P

b) The tension in cables AB & AC

Solution: A(0, −480, 0)

B(−120, 0, 160)

C(−220, 0, −240)

$\vec{AB} = -120\vec{i} + 480\vec{j} + 160\vec{k} \Rightarrow AB = 520\text{ mm}$

$\Rightarrow \vec{T}_{AB} = \frac{T_{AB}}{520}(-120\vec{i} + 480\vec{j} + 160k)$

$\vec{AC} = -220\vec{i} + 480\vec{j} - 240\vec{k} \Rightarrow AC = 580\text{ mm}$

$\therefore \vec{T}_{AC} = \frac{T_{AC}}{580}(-220\vec{i} + 480\vec{j} - 240\vec{k})$

From the FBD for A we get

$\Sigma F_x = 0 \Rightarrow P - \frac{120}{520}T_{AB} - \frac{220}{580}T_{AC} = 0 \quad \text{--- (1)}$

$\Sigma F_y = 0 \Rightarrow -360 + \frac{480}{520}T_{AB} + \frac{480}{580}T_{AC} = 0 \quad \text{--- (2)}$

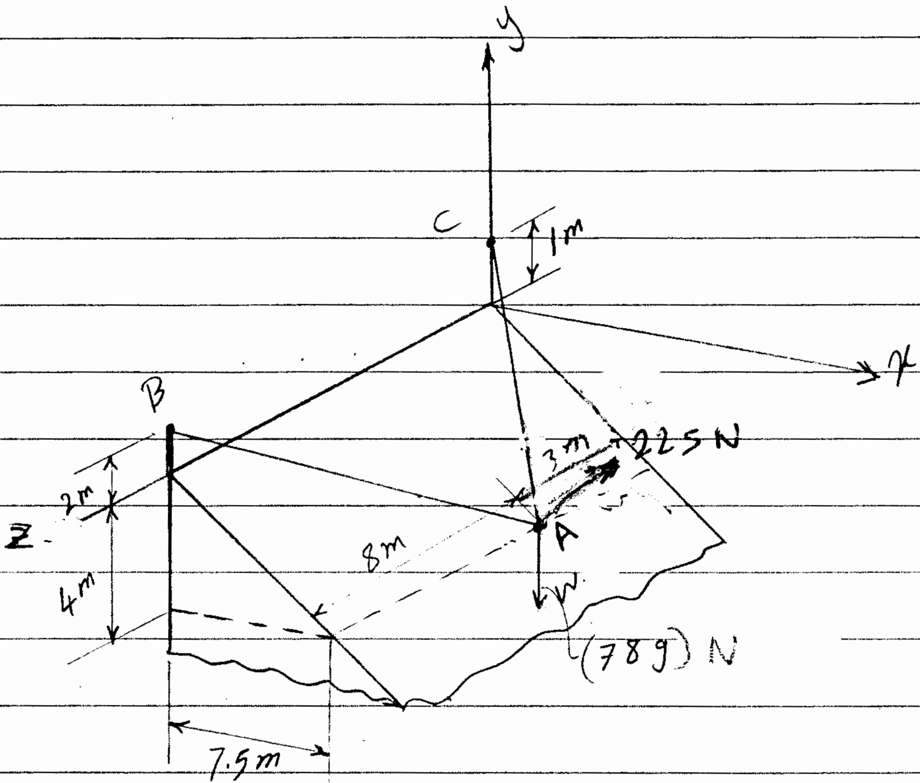
$\Sigma F_z = 0 \Rightarrow 60 + \frac{160}{520}T_{AB} - \frac{240}{580}T_{AC} = 0 \quad \text{--- (3)}$

Solving these equations we get

$\boxed{P = 135\text{ N}}$

$\boxed{T_{AC} = 261\text{ N}}$ $\boxed{T_{AB} = 156\text{ N}}$ #

Problem # 2



Given: The system shown in the figure.

Required: Determine the tension in ropes AB & AC.

Solution: $A(7.5, -4, 3)$

$B(0, 2, 11)$

$C(0, 1, 0)$

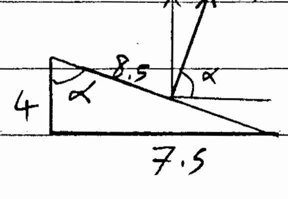
$$\overrightarrow{AB} = -7.5\vec{i} + 6\vec{j} + 8\vec{k} \Rightarrow AB = 12.5\ m$$

$$\therefore \overrightarrow{T_{AB}} = \frac{T_{AB}}{12.5}(-7.5\vec{i} + 6\vec{j} + 8\vec{k})$$

$$\overrightarrow{AC} = -7.5\vec{i} + 5\vec{j} - 3\vec{k} \Rightarrow AC = 9.5\ m$$

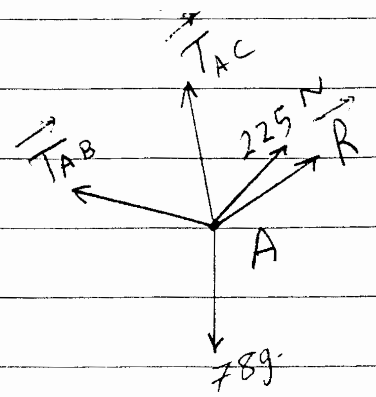
$$\therefore \overrightarrow{T_{AC}} = \frac{T_{AC}}{9.5}(-7.5\vec{i} + 5\vec{j} - 3\vec{k})$$

$$\vec{R} = R\left(\frac{4}{8.5}\vec{i} + \frac{7.5}{8.5}\vec{j} + 0\vec{k}\right)$$



$\therefore$ From FBD for A

Continue problem # 2



$$\sum F_x = 0 \Rightarrow -\frac{7.5}{12.5} T_{AB} - \frac{7.5}{9.5} T_{AC} + \frac{4}{8.5} R = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0 \Rightarrow \frac{6}{12.5} T_{AB} + \frac{5}{9.5} T_{AC} + \frac{7.5}{8.5} R - 765.18 = 0 \quad \text{--- (2)}$$

$$\sum F_z = 0 \Rightarrow \frac{8}{12.5} T_{AB} - \frac{3}{9.5} T_{AC} - 225 = 0 \quad \text{--- (3)}$$

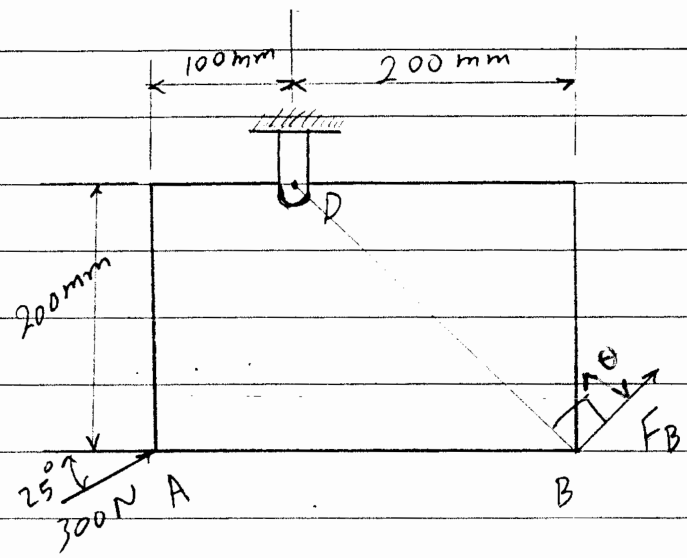
Solving these equation yield

$T_{AB} = 386.91$ N

$T_{AC} = 71.796$ N

$R = 613.85$ N (not required)

Problem # 3



Given: The system in the figure.

Required: a) The moment of 300N force about D.

b) The smallest force applied at B which creates the same moment about D.

Solution a)

$+\circlearrowleft$ $M_A = 300\cos 25 * 0.2 - 300\sin 25 * 0.1$

$M_A = 41.700 N.m$ $\circlearrowleft$

b) To find the smallest force at B, the force should be ⊥ on BD

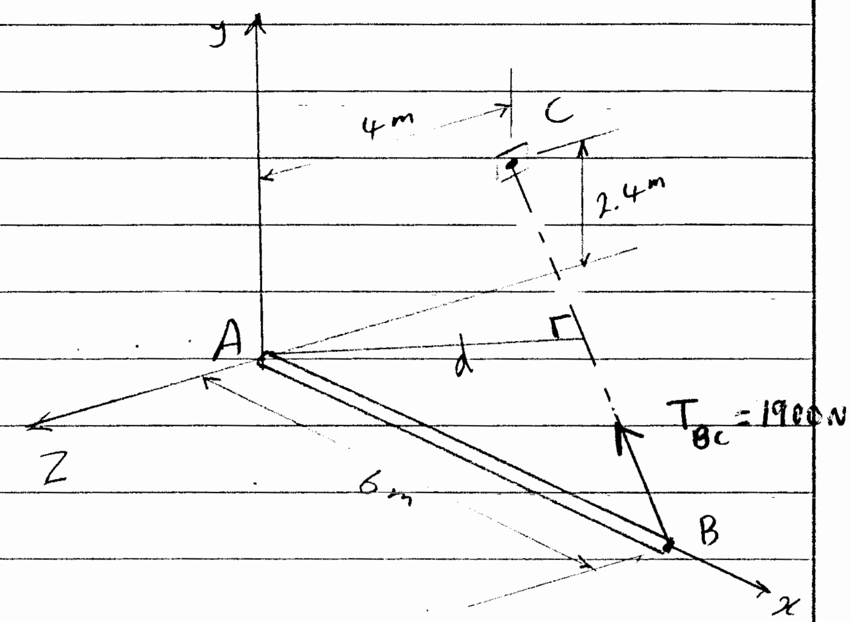
$\Rightarrow d = BD\ \sqrt{0.2^2+0.2^2} = \sqrt{0.08}$ m

$M = Fd \Rightarrow 41.7 = \sqrt{0.08}\ F_B$

$\Rightarrow F_B = 147.43$ N #

$\theta = 45^\circ$ from Fig above

Problem # 4



Given: The system shown in the figure, with $T_{BC} = 1900N$

Required: a) The moment of T_{BC} about A

b) The perpendicular distance from point A to cable CB.

Solution a) $B(6, 0, 0)$

$C(0, 2.4, -4)$

$A(0, 0, 0)$

$\vec{BC} = -6\vec{i} + 2.4\vec{j} - 4\vec{k} \Rightarrow BC = 7.6 \text{ m}$

$\vec{AB} = 6\vec{i}$

$$\vec{T_{BC}} = \frac{1900}{7.6}(-6\vec{i} + 2.4\vec{j} - 4\vec{k})$$

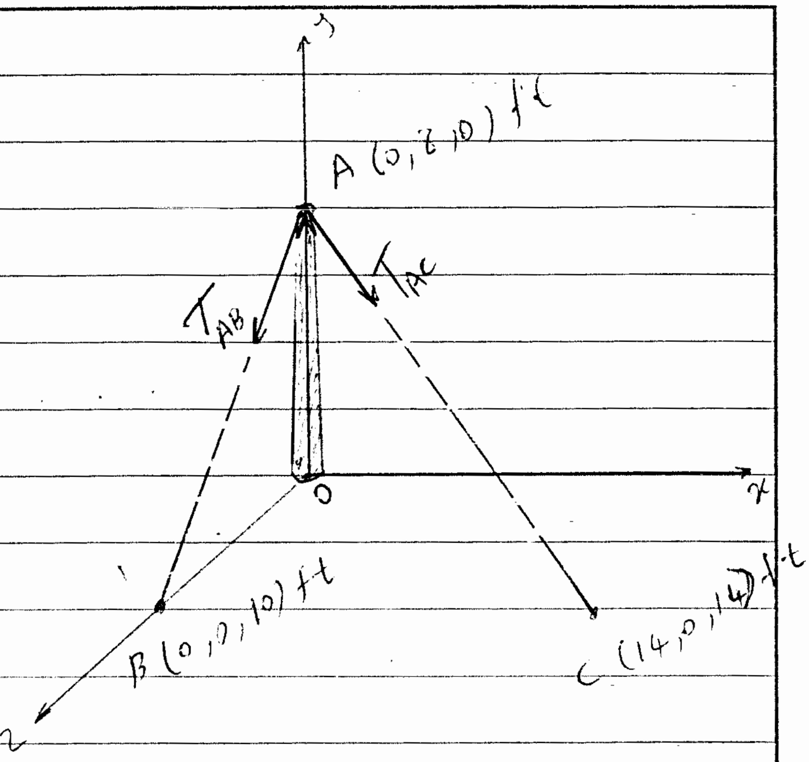
$$\vec{M}_A = \vec{AB} \times \vec{T_{BC}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 0 \\ -\frac{6}{7.6}1900 & \frac{2.4}{7.6}1900 & \frac{-4}{7.6}1900 \end{vmatrix}$$

$\Rightarrow \vec{M}_A = 0\vec{i} + 6000\vec{j} + 3600\vec{k} \Rightarrow$ $\boxed{M_A = 6997.14 \text{ N.m}}$

b) $T_{BC} * d = M_A \Rightarrow 1900 * d = 6997.14$

$\Rightarrow \boxed{d = 3.683 \text{ m}}$ #

Problem # 5



Given: The system shown on the figure with $T_{AB} = T_{AC} = T$ and $M_{O_{max}} = 5000$ ft-lb

Required: The maximum allowable value of T

Solution: $\vec{AC} = 14\vec{i} - 8\vec{j} + 14\vec{k} \Rightarrow AC = 21.354$ ft

$\therefore \vec{T}_{AC} = \frac{T}{21.354}(14\vec{i} - 8\vec{j} + 14\vec{k})$

$\vec{AB} = -8\vec{j} + 10\vec{k} \Rightarrow AB = 12.806$ ft

$\therefore \vec{T}_{AB} = \frac{T}{12.806}(-8\vec{j} + 10\vec{k})$

$\therefore$ The moment of T_{AC} about O is

$\vec{M}_{T_{AC}/O} = \vec{r} \times \vec{T}_{AC}$ & $\vec{r} = 8\vec{j}$

$$\vec{M}_{T_{AC}/O} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 8 & 0 \\ \frac{14}{21.354}T & \frac{-8}{21.354}T & \frac{14}{21.354}T \end{vmatrix} = 5.2449T\vec{i} - 5.2449T\vec{k}$$

Continue problem # 5

$$\vec{M}_{T_{AB}/O} = \vec{r} \times \vec{T}_{AB}$$

$$\therefore \vec{M}_{T_{AB}/O} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 8 & 0 \\ 0 & \frac{-8}{12.806}T & \frac{10}{12.806}T \end{vmatrix}$$

$$= 6.2471T\vec{i}$$

$\therefore$ Total moment about O is

$$\vec{M}_O = \vec{M}_{T_{AC}/O} + \vec{M}_{T_{AB}/O}$$

$$\vec{M}_O = (5.2449 + 6.2471)T\vec{i} - 5.2449T\vec{k}$$

$$\therefore \vec{M}_O = 11.492T\vec{i} - 5.2449T\vec{k}$$

$$\Rightarrow M_O = 12.632\,T \text{ ft-lb}$$

$\therefore$ To find the maximum T

$$\therefore 12.632\,T = 5000$$

$$\Rightarrow T = 395.81 \text{ lb}$$