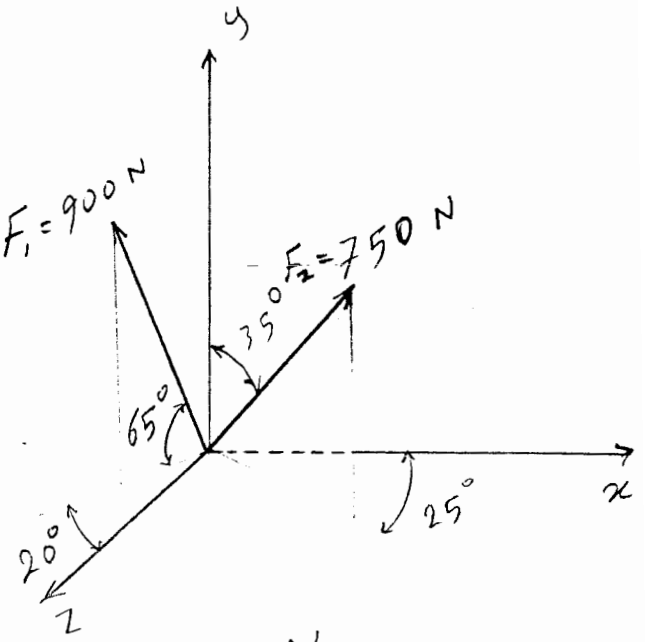
CE 201-9 (061)

Solution of HW # 2

Problem #1

Given: The forces shown in the figure

Required: The value and direction of the resultant force.



Solution: $F_{1x} = -(900 \cos 65) \sin 20 = -130.1$ N

$F_{1y} = 900 \sin 65 = 815.8$ N

$F_{1z} = 900 \cos 65 \cos 20 = 357.4$ N

$F_{2x} = (750 \sin 35) \cos 25 = 389.9$ N

$F_{2y} = 750 \cos 35 = 614.36$ N

$F_{2z} = (750 \sin 35) \sin 25 = 181.8$ N

$\therefore \vec{R} = \vec{F_1} + \vec{F_2} = (-130.1 + 389.9)\vec{i} + (815.8 + 614.36)\vec{j} + (357.4 + 181.8)\vec{k}$

$\therefore \vec{R} = 259.8\vec{i} + 1430.16\vec{j} + 539.2\vec{k}$

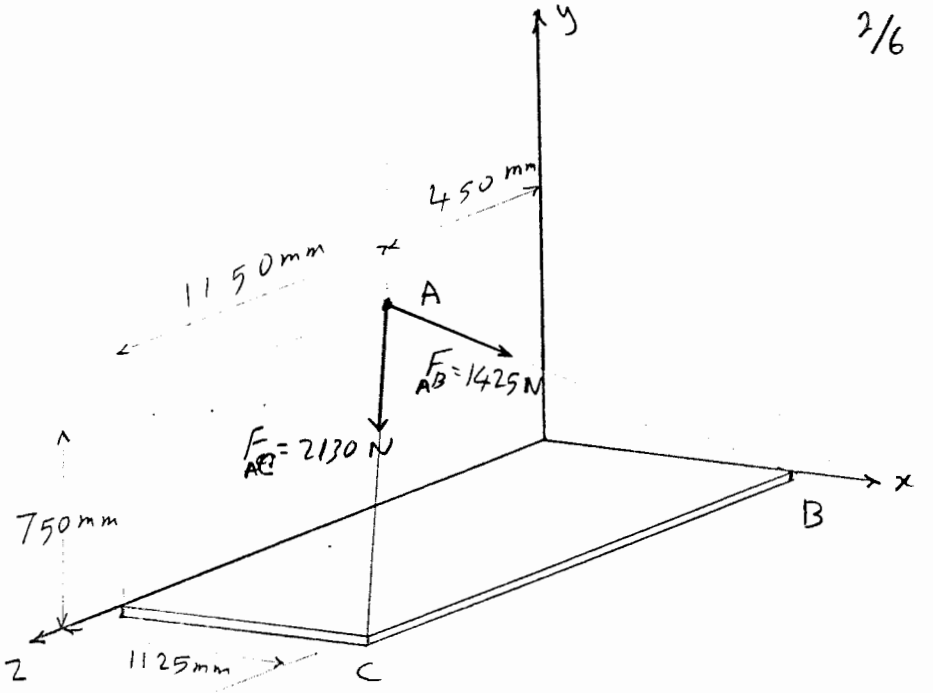
$\therefore R = \sqrt{259.8^2 + 1430.16^2 + 539.2^2} = 1550.35$ N

$\cos\theta_x = \frac{R_x}{R} = \frac{259.8}{1550.35} \Rightarrow \theta_x = 80.35^\circ$

$\cos\theta_y = \frac{R_y}{R} = \frac{1430.16}{1550.35} \Rightarrow \theta_y = 22.7^\circ$

$\cos\theta_z = \frac{R_z}{R} = \frac{539.2}{1550.35} \Rightarrow \theta_z = 69.65^\circ$

Problem #2



Given: The forces shown in the figure.

Required: The magnitude and directions of the resultant force at A.

Solution: First we find the unit vector in $\vec{AB}$ and $\vec{AC}$

$$\vec{u}_{AB} = \frac{1125\vec{i} - 750\vec{j} - 450\vec{k}}{\sqrt{1125^2 + 750^2 + 450^2}} = 0.789\vec{i} - 0.526\vec{j} - 0.316\vec{k}$$

$$\vec{u}_{AC} = \frac{1125\vec{i} - 750\vec{j} + 1150\vec{k}}{\sqrt{1125^2 + 750^2 + 1150^2}} = 0.634\vec{i} - 0.423\vec{j} + 0.648\vec{k}$$

$$\therefore \vec{F}_{AB} = F_{AB}\,\vec{u}_{AB} = 1425(0.789\vec{i} - 0.526\vec{j} - 0.316\vec{k})$$

$$\therefore \vec{F}_{AB} = 1124.3\vec{i} - 749.6\vec{j} - 450.3\vec{k}$$

$$\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC} = 2130(0.634\vec{i} - 0.423\vec{j} + 0.648\vec{k})$$

$$\therefore \vec{F}_{AC} = 1350.42\vec{i} - 901\vec{j} + 1380.24\vec{k}$$

$$\therefore \vec{R} = \vec{F}_{AB} + \vec{F}_{AC}$$

$$= (1124.3 + 1350.42)\vec{i} - (749.6 + 901)\vec{j} + (1380.2 - 450.3)\vec{k}$$

$$\therefore \vec{R} = 2474.72\vec{i} - 1650.6\vec{j} + 929.9\vec{k}$$

Continue problem # 2

$$\therefore R = \sqrt{2474.72^2 + 1650.6^2 + 929.9^2} = 3116.64\ N$$

$$\therefore \cos\theta_x = \frac{R_x}{R} = \frac{2474.72}{3116.64} \Rightarrow \theta_x = 37.44^\circ$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{-1650.6}{3116.64} \Rightarrow \theta_y = 121.98^\circ$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{929.9}{3116.64} \Rightarrow \theta_z = 72.64^\circ$$

Note: when find $\overrightarrow{AB}$

$$\overrightarrow{AB} = B - A = (1125, 0, 0) - (0, 750, 450)$$

$$\overrightarrow{AB} = 1125\vec{i} - 750\vec{j} - 450\vec{k}$$

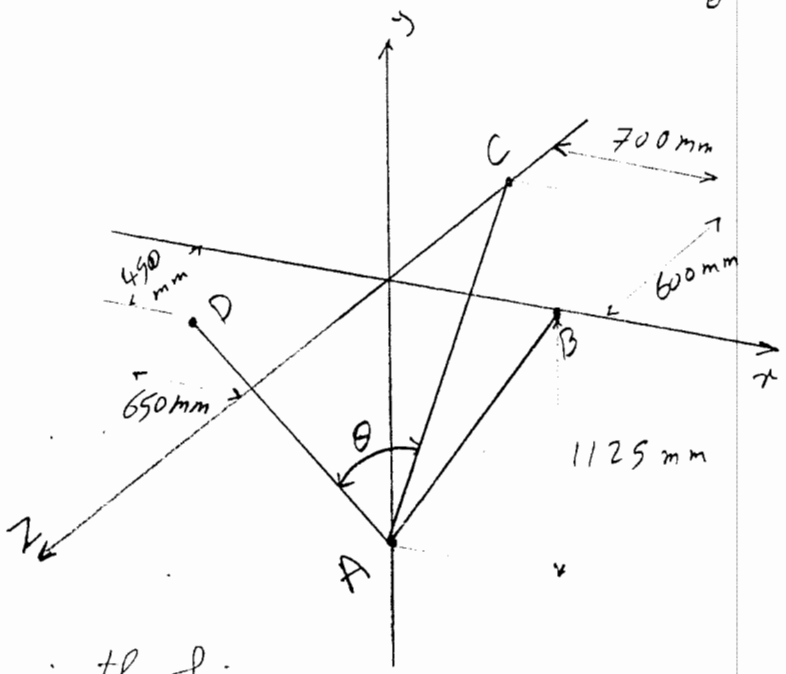
similarly $\overrightarrow{AC}$

check:

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$\Rightarrow$ ok

Problem #3



Given: The cables shown in the figure.

Required: The angle formed by AC and AD

Solution: $A(0, -1125, 0)$

$D(-650, 0, 450)$

$C(0, 0, -600)$

$$\therefore \vec{AD} = (-650-0)\vec{i} + (0+1125)\vec{j} + (450-0)\vec{k}$$

$$= -650\vec{i} + 1125\vec{j} + 450\vec{k}$$

$$AD = \sqrt{650^2 + 1125^2 + 450^2} = 1375 \text{ mm}$$

$$\vec{AC} = (0-0)\vec{i} + (0+1125)\vec{j} + (-600-0)\vec{k}$$

$$= 0\vec{i} + 1125\vec{j} - 600\vec{k}$$

$$AC = \sqrt{1125^2 + 600^2} = 1275 \text{ mm}$$

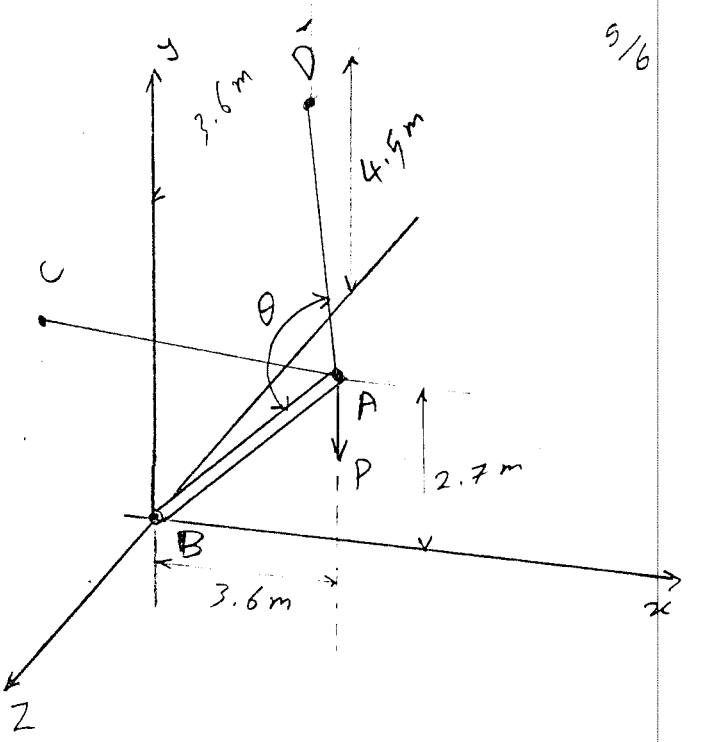
$$\therefore \cos\theta = \frac{\vec{AD} \cdot \vec{AC}}{(AC)(AD)}$$

$$= \frac{(-650\vec{i} + 1125\vec{j} + 450\vec{k}) \cdot (0\vec{i} + 1125\vec{j} - 600\vec{k})}{(1375)(1275)}$$

$$= 0.567914$$

$$\Rightarrow \theta = 55.40^\circ \quad \#$$

Problem # 4



Data: As shown in the figure.

Required: a) The angle between cable AD and boom AB.
b) The projection on AB of the force exerted by cable AD
i) utilizing the result of part (a)
ii) The general method

Solution a)

$A(3.6, 2.7, 0)$ | $B(0,0,0)$ | $D(0, 4.5, -3.6)$

$$\vec{AB} = -3.6\vec{i} - 2.7\vec{j} + 0\vec{k}$$

$$AB = \sqrt{3.6^2 + 2.7^2} = 4.5\ m$$

$$\vec{AD} = -3.6\vec{i} + 1.8\vec{j} - 3.6\vec{k}$$

$$AD = \sqrt{3.6^2 + 1.8^2 + 3.6^2} = 5.4\ m$$

$$\therefore \cos\theta = \frac{\vec{AB}\cdot\vec{AD}}{AB(AD)}$$

$$= \frac{(-3.6\vec{i} - 2.7\vec{j} + 0\vec{k})\cdot(-3.6\vec{i} + 1.8\vec{j} - 3.6\vec{k})}{4.5\ (5.4)}$$

$$= \frac{8.1}{24.3} = 0.333333$$

$$\therefore \theta = 70.53^\circ \quad \#$$

Continue problem #4

b) i) If the force exerted by cable AD is T_D

∴ The projection of T_D on AB is:

$$(T_D)_{AB} = 540 \times \cos\theta = 0.3333 \times 540 = 180 \text{ N}$$

ii) In this case:

$$(T_D)_{AB} = \vec{T}_D \cdot \vec{u}_{AB}$$

$$\vec{u}_{AB} = \frac{\vec{AB}}{AB} = \frac{-3.6\vec{i} - 2.7\vec{j}}{4.5} = -0.8\vec{i} - 0.6\vec{j} + 0\vec{k}$$

$$\vec{T}_D = T_D \vec{u}_{AD}$$

$$\vec{u}_{AD} = \frac{\vec{AD}}{AD} = \frac{-3.6\vec{i} + 1.8\vec{j} - 3.6\vec{k}}{5.4} =$$

$$\vec{u}_{AD} = -0.6666\vec{i} + 0.3333\vec{j} - 0.6666\vec{k}$$

$$\therefore \vec{T}_D = -360\vec{i} + 180\vec{j} - 360\vec{k}$$

$$\Rightarrow (T_D)_{AB} = (-360\vec{i} + 180\vec{j} - 360\vec{k}) \cdot (-0.8\vec{i} - 0.6\vec{j} + 0\vec{k})$$

$$= 180 \text{ N}$$

$$\therefore (T_D)_{AB} = 180 \text{ N}$$

which is the same as what we found in i) ≠