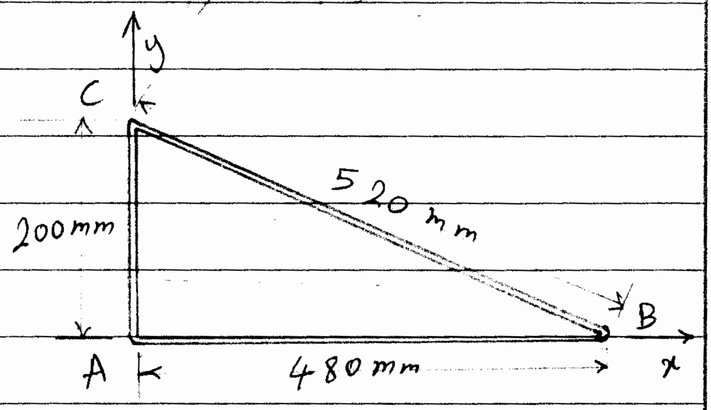
Solution of HW # 13

Problem # 1



Given: The wire ABC shown in the figure.
Required: The centroid of the wire.
Solution: Using composite method

$$\bar{x} = \frac{\Sigma \bar{x}_i L_i}{\Sigma L_i} = \frac{0 * 200 + \frac{480}{2}(480) + \frac{480}{2}(520)}{200 + 480 + 520}$$

$$\boxed{\bar{x} = 200 \text{ mm}}$$

$$\bar{y} = \frac{\Sigma \bar{y}_i L_i}{\Sigma L_i} = \frac{100(200) + 0 \times (480) + \frac{200}{2}(520)}{200 + 480 + 520}$$

$$\Rightarrow \boxed{\bar{y} = 60 \text{ mm}}$$

Note that the centroid is not on the body/area/volume. It is outside → ?

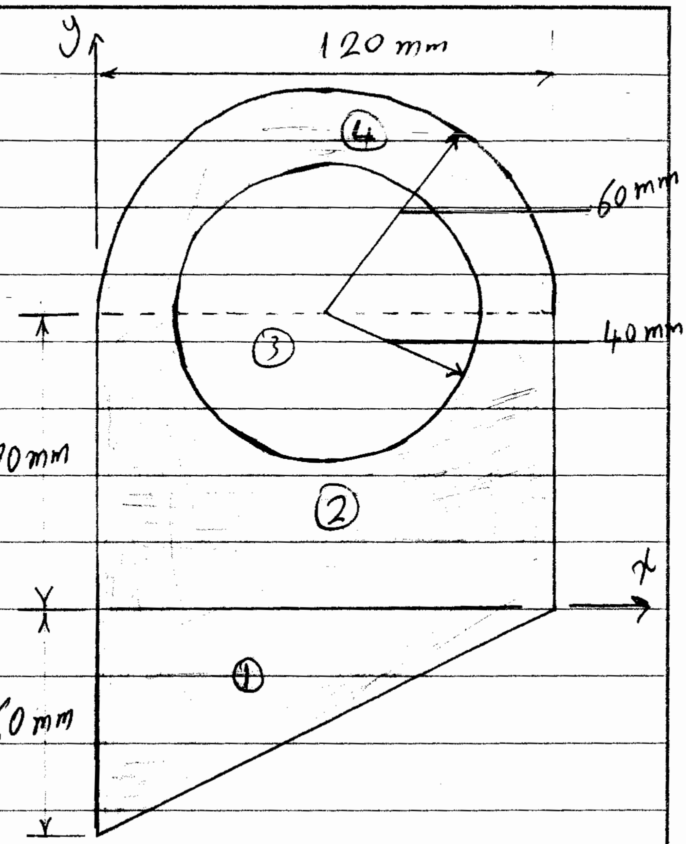
Note: You can use a table for for the solution

Problem # 2

Given; The shaded area shown.

Required: The centroid.

Solution: The area is divided into 4 segments as shown. Note that area 3 is negative.



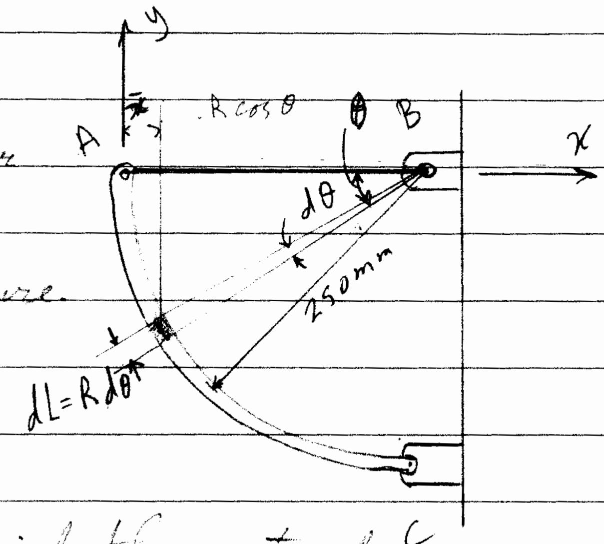
Area #	$A\ (mm^2)$	$\bar{x}\ (mm)$	$\bar{y}\ (mm)$	$\bar{x}A\ (mm^3)$	$\bar{y}A\ (mm^3)$
1	3600	$\frac{120}{3}=40$	$\frac{-60}{3}=-20$	144000	-72000
2	9600	60	40	576000	384000
3	$-\pi\, 40^2$	60	80	-301592.9	-402123.9
4	$\frac{\pi}{2}60^2$	60	$80+\frac{4}{3\pi}60$	339292.0	596389.3
Σ	13828.3			757699.1	506265.4

$$\therefore \bar{x} = \frac{\sum_{i=1}^{4} \bar{x}_i A_i}{\sum_{i=1}^{4} A_i} = \frac{757699.1}{13828.3} \Rightarrow \boxed{\bar{x} = 54.79\ mm}$$

$$\bar{y} = \frac{\sum_{i=1}^{4} \bar{y}_i A_i}{\sum_{i=1}^{4} A_i} = \frac{506265.4}{13828.3} \Rightarrow \boxed{\bar{y} = 36.61\ mm}$$

Note: the centroid of area 4 can be found by integration or from the book.

Problem # 3



Given: 4-kg uniform circular rod AC supported as shown in the figure.

Required: All reactions.

Solution:

We first need to find the centroid.

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL}$$

$$dL = R\, d\theta \quad \& \quad \tilde{x} = R - R\cos\theta$$

$$\therefore \int_L dL = \int_0^{\frac{\pi}{2}} R\, d\theta = \frac{\pi}{2} R$$

$$\int_L \tilde{x}\, dL = \int_0^{\frac{\pi}{2}} (R - R\cos\theta) R\, d\theta = R^2 \int_0^{\frac{\pi}{2}} (1 - \cos\theta)\, d\theta$$

$$\int_L \tilde{x}\, dL = R^2 \left[(\theta - \sin\theta) \right]_0^{\frac{\pi}{2}} = R^2 \left(\frac{\pi}{2} - 1 \right)$$

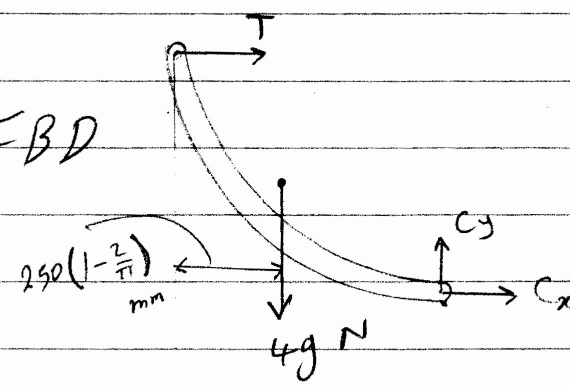
$$\therefore \bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{R^2 \left(\frac{\pi}{2} - 1 \right)}{\frac{\pi}{2} R} = R \left(1 - \frac{2}{\pi} \right)$$

$$\bar{x} = 250 \left(1 - \frac{2}{\pi} \right) \text{ mm}$$

Note: You may use the table at the end of the book (directly) to get $\bar{x}$.

Note: We don't need $\bar{y}$, why ??!

FBD



Continue problem # 3

From the FBD

$\circlearrowleft + \sum M_c = 0 \Rightarrow$

$$4g \times [250 - 250(1 - \tfrac{2}{\pi})] - T * 250 = 0$$

$$\Rightarrow \boxed{T = 24.98 \text{ N}}$$

$\sum F_x = 0 \Rightarrow C_x + T = 0$

$$\Rightarrow \boxed{C_x = -24.98 \text{ N}}$$

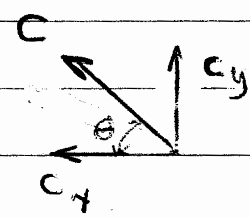
$\sum F_y = 0 \Rightarrow C_y - 4g = 0$

$$\Rightarrow \boxed{C_y = 39.24 \text{ N}} \#$$

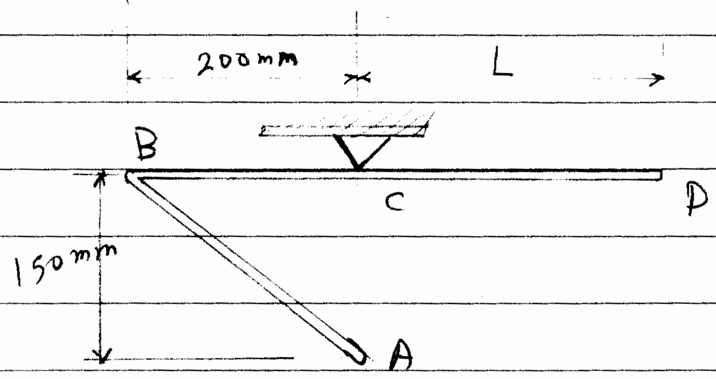
$\Rightarrow C = 46.52$ N

$\theta = 57.52^\circ$

((as shown))



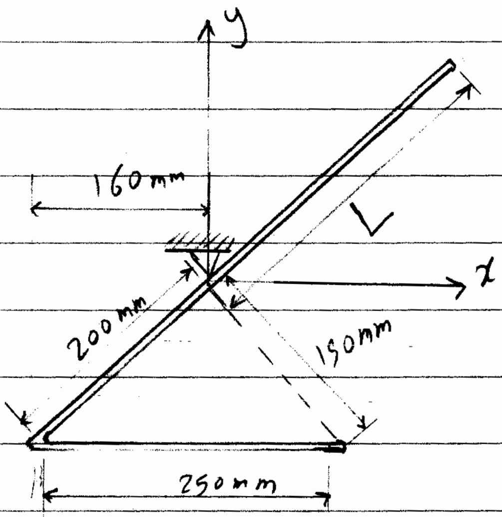
Problem # 4



Given: The wire shown on the figure.

Required: The length L for which portion AB of the wire is horizontal.

Solution: We draw the wire when portion AB is horizontal.



For the wire to be equilibruim in this case $\bar{x}$ should equals to zero. (Why?!!)

$$\therefore \bar{x} = \frac{\sum_{i=1}^{3} \tilde{x}_i L_i}{\sum_{i=1}^{3} L_i} = 0 \Rightarrow \sum_{i=1}^{3} \tilde{x}_i L_i = 0$$

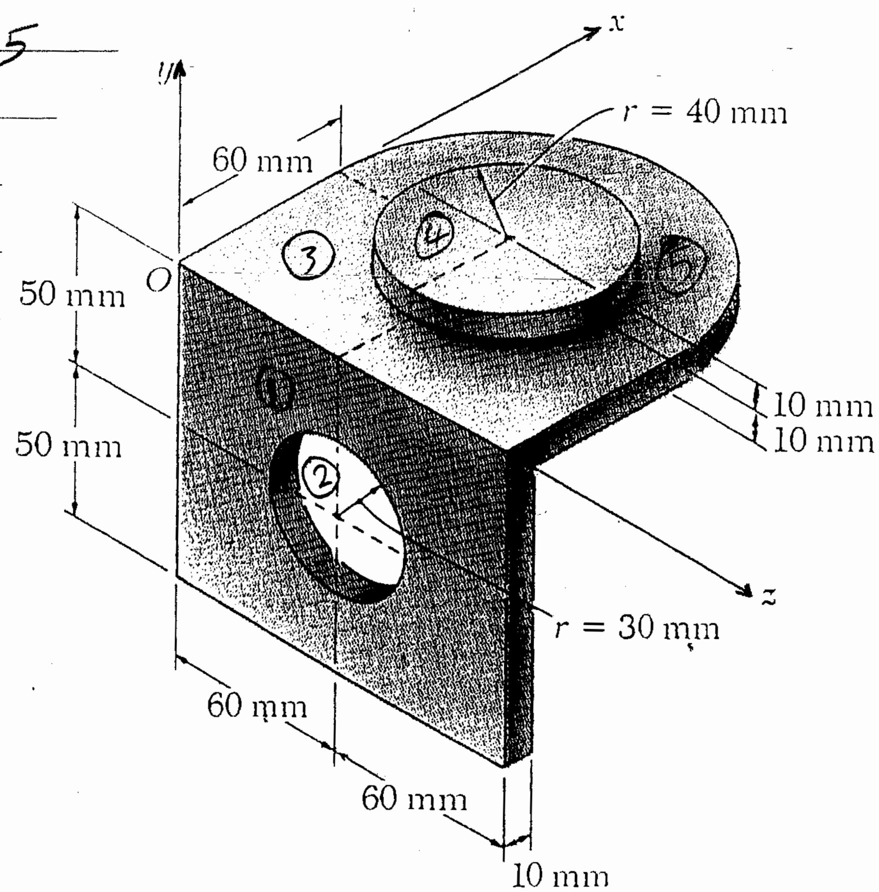
$$\sum_{i=1}^{3} \tilde{x}_i L_i = 250\left[-(160-125)\right] + 200\times\left(-\frac{160}{80}\right) + L\left(\frac{L}{2}\,\frac{200}{250}\right) = 0$$

$$\Rightarrow \boxed{L = 248.75 \text{ mm}}$$

Note: Try to solve the problem for the cases

1. portion BD is horizontal.
2. portion BA is vertical.

Problem # 5



Given The machine element shown in the figure.

Required: The centroid.

Solution Because of the symmetry about a plane parallel to x-y plane at $z = 60$ mm

$\therefore \boxed{\bar{z} = 60 \text{ mm}}$

The volume will be divided into 5 segment.

Note segment 5 is a half cylinder and segment 2 is negative

Segment	volume (mm^3)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}$ V (mm^4)	$\bar{y}$ V (mm^4)
1	$100 \times 120 \times 10 = 120000$	5	-50	600000	-6000000
2	$-\pi \times 30^2 \times 10 = -28274$	5	-50	-141370	1413700
3	$50 \times 120 \times 10 = 60000$	35	-5	2100000	-300000
4	$\pi\, 40^2 \times 10 = 50265$	60	5	3015900	251325
5	$\frac{\pi}{2} 60^2 \times 10 = 56549$	$60 + \frac{4(60)}{3\pi}$	-5	4832948	-282745
Σ	258540			10407478	-4917720

$$\therefore \bar{x} = \frac{\sum_{i=1}^{5} \tilde{x}_i V_i}{\sum_{i=1}^{5} V_i} = \frac{10407478}{258540} \Rightarrow \boxed{\bar{x} = 40.25 \text{ mm}}$$

$$\bar{y} = \frac{\sum_{i=1}^{5} \tilde{y}_i V_i}{\sum_{i=1}^{5} V_i} = \frac{-4917740}{258540} \Rightarrow \boxed{\bar{y} = -19.02 \text{ mm}}$$

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