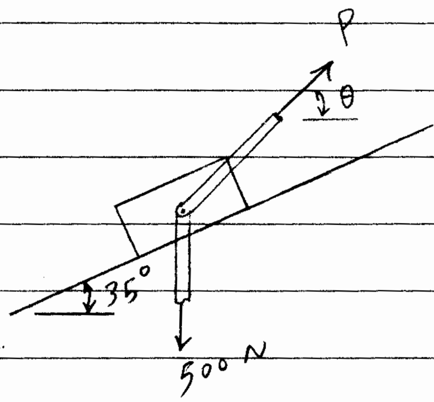
Solution of HW # 12

Problem # 1



Given: The system shown with

$\mu_s = 0.3$ & $\mu_k = 0.25$

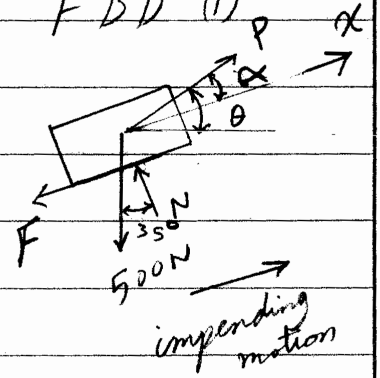
Required: The magnitude & direction of the smallest force P required to

a) start the block up the rail,

b) keep it from moving down.

Solution: to find the smallest value of P we need to find the value of θ.

a) From FBD(1)



$\Sigma F_y = 0 \Rightarrow$

$-500\cos 35 + N + P\sin\alpha = 0$

$\Rightarrow N = 500\cos 35 - P\sin\alpha$

$\Sigma F_x = 0 \Rightarrow$

$P\cos\alpha - 500\sin 35 - F = 0$

but $F = \mu_s N = 0.3(500\cos 35 - P\sin\alpha)$

$\Rightarrow P\cos\alpha - 500\sin 35 - 0.3\times 500\cos 35 + 0.3P\sin\alpha = 0$

$\Rightarrow P(\cos\alpha + 0.3\sin\alpha) = 500(\sin 35 + 0.3\cos 35)$

$$\therefore P = \frac{500(\sin 35 + 0.3\cos 35)}{(\cos\alpha + 0.3\sin\alpha)} \quad \text{---} \quad (1)$$

$\therefore$ To find the value of α for P to be minimum

$\frac{dP}{d\alpha} = 0 \Rightarrow -\sin\alpha + 0.3\cos\alpha = 0$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = 0.3 \Rightarrow \tan\alpha = 0.3$

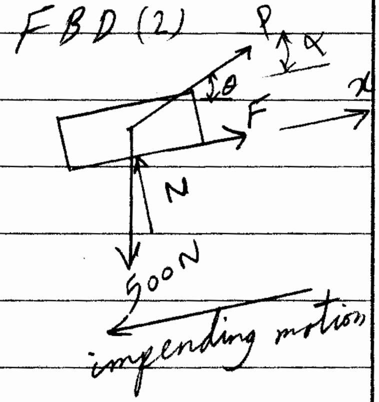
$\Rightarrow \alpha = 16.7^\circ \Rightarrow \boxed{\theta = 51.7^\circ}$

from (1) $\Rightarrow \boxed{P = 392.4}$ N

Continue problem # 1

b) The smallest value of P to keep the box from moving down.

From FBD (2)



$\Sigma F_y = 0$

$-500\cos 35 + N + P\sin\alpha = 0$

$N = 500\cos 35 - P\sin\alpha$

$\Sigma F_x = 0$

$P\cos\alpha - 500\sin 35 + F = 0$

$F = \mu_s N = 0.3(500\cos 35 - P\sin\alpha)$

$\Rightarrow P\cos\alpha - 500\sin 35 + 0.3\times 500\cos 35 - 0.3P\sin\alpha = 0$

$$\Rightarrow P = \frac{500\sin 35 - 0.3\times 500\cos 35}{0.3\sin\alpha - \cos\alpha}$$

$\frac{dP}{d\alpha} = 0 \Rightarrow 0.3\cos\alpha + \sin\alpha = 0 \Rightarrow$

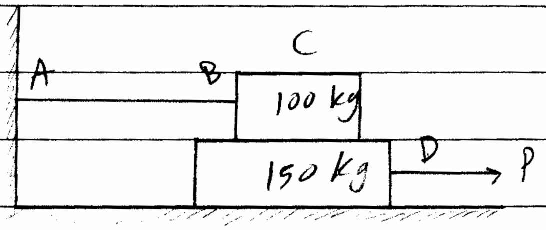
$\tan\alpha = -0.3 \Rightarrow \alpha = -16.7^\circ$

$\Rightarrow \boxed{\theta = 18.3^\circ}$

$\therefore \boxed{P = 157}$ N

Note that $\gamma = \tan^{-1}\mu$,
can you solve the problem in a more direct/short way??

Problem # 2



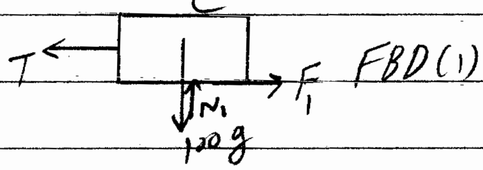
Given: The system shown. $\mu_s = 0.3$ & $\mu_k = 0.25$

Required: The smallest force P required to start block D moving if

a) block C is restrained by cable AB

b) cable AB is removed

Solution a) From FBD (1)

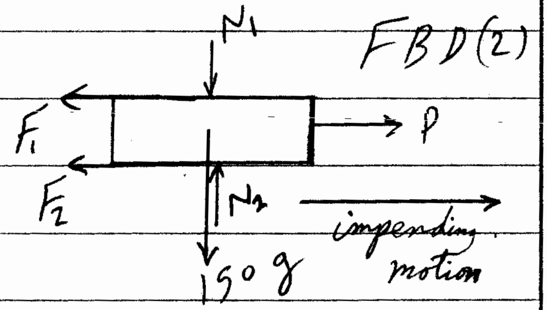


$\Sigma F_y = 0 \Rightarrow$

$N_1 - 100g = 0 \Rightarrow N_1 = 100g$ N

$F_1 = \mu_s N_1 = 0.3 \times 100 g = 30 g$ N

From FBD(2)



$\Sigma F_y = 0 \Rightarrow$

$N_2 - 150g - N_1 = 0$

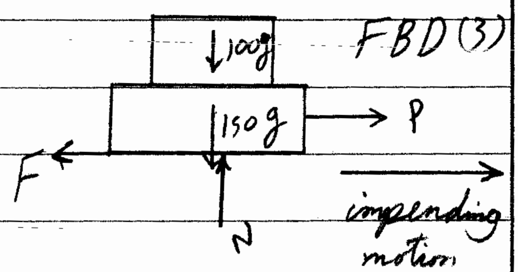
$N_2 = 150g + 100g = 250g$ N

$\therefore F_2 = \mu_s N_2 = 0.3 \times 250 g$ N

$\Sigma F_x = 0 \Rightarrow P - F_1 - F_2 = 0 \Rightarrow P = 30g + 0.3 \times 250 g$ N

$\therefore$ **P = 1030.05 N**

b) From FBD (3)



$\Sigma F_y = 0 \Rightarrow$

$-100g - 150g + N = 0$

$N = 250 g$ N $\Rightarrow$

$\therefore F = \mu_s N = 0.3 \times 250 g = 75 g$ N

$\Sigma F_x = 0 \Rightarrow P - F = 0 \Rightarrow P = F$

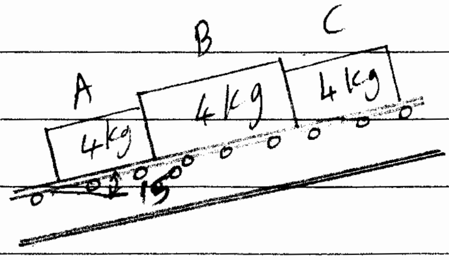
$\therefore P = 75 \times 9.81 = 735.75$ N

P = 735.75 N

Note: Why we took C & D together ?!

Problem # 3

Given: The system shown with

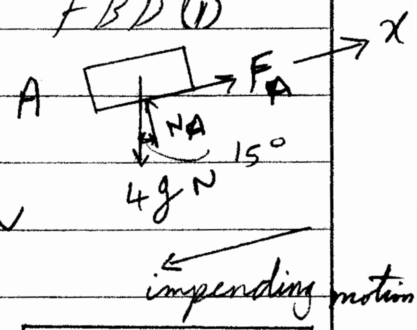


$\mu_s = 0.3$ & $\mu_k = 0.2$ between belt and both packages A & C

$\mu_s = 0.1$ & $\mu_k = 0.08$ between the belt and B

Required: which, if any, of the packages will move and the friction force acting on each package.

Solution: * Start from A why!



$\Sigma F_y = 0$

$-4g\cos 15 + N_A = 0$

$\Rightarrow N_A = 4g\cos 15$ N

$F_{A_{max}} = \mu_s N_A = 0.3 \times 4g\cos 15 = 11.37$ N

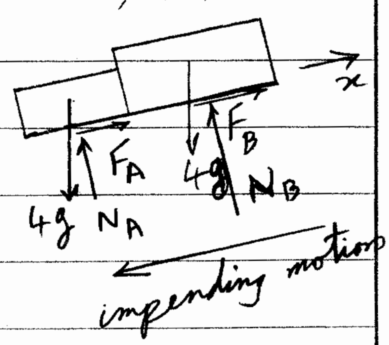
$\Sigma F_x = 0 \Rightarrow F_A - 4g\sin 15 = 0 \Rightarrow \boxed{F_A = 10.16 \text{ N}}$

$\because F_A < F_{A_{max}}$

$\therefore$ package A will not move alone.

* Package B can't move alone, but it can move with package A.

let us take A & B together. FBD (2)



From FBD (2)

$N_A = N_B = N_C = 4g\cos 15$ N

$F_{A_{max}} = F_{C_{max}} = 11.37$ N

$F_{B_{max}} = \mu_s N_B = 0.1 \times 4g\cos 15$ N

$\therefore F_{B_{max}} = 3.79$ N

$\Sigma F_x = 0 \Rightarrow -4g\sin 15 \times 2 + F_B + F_A = 0$

$\therefore F_A + F_B = 20.312$ N

but $F_{A_{max}} + F_{B_{max}} = 11.37 + 3.79 = 15.16$ N which is less than the force effected by weight which is 20.312 **N**.

Continued problem # 3

∴ A & B will move togather and

$F_A = \mu_k N_A = 0.2 \times 4g \cos 15 = 7.58$ N

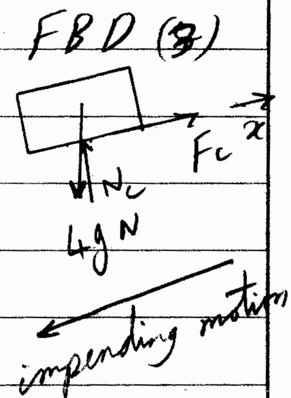
$\boxed{F_A = 7.58}$ N

$F_B = \mu_k N_B = 0.08 \times 4g \cos 15 = 3.03$ N

∴ $\boxed{F_B = 3.03}$ N

let us take package C only why??!!

From FBD (3)



$\Sigma F_y = 0$

$-4g \cos 15 + N_c = 0$

$\Rightarrow N_c = 4g \cos 15$ N

$F_{C_{max}} = \mu_s N_C = 0.3 \times 4g \cos 15$

$F_{C_{max}} = 11.37$ N

$\Sigma F_x = 0$

$\Rightarrow F_C - 4g \sin 15 = 0 \Rightarrow$

$\Rightarrow \boxed{F_C = 10.16}$ N

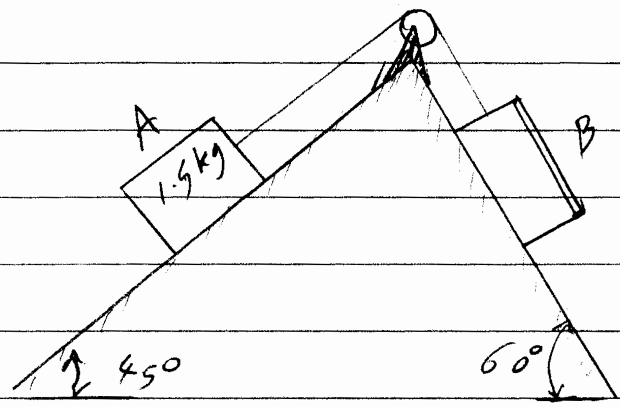
∴ $F_C < F_{C_{max}}$

∴ package C will not move.

Note: If C alone tries to move, but A & B did not move, what can you conclude ⇒

Two options
① ??!!
② ??!!

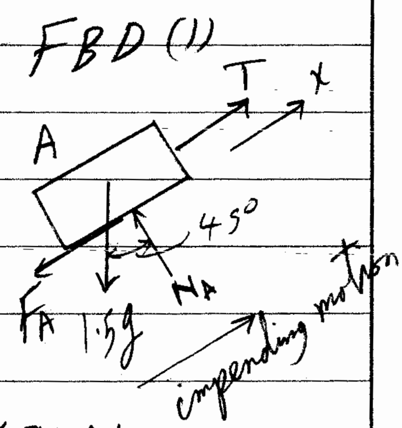
Problem # 4



Given: The system shown with $\mu_s = 0.2$

Required: The largest mass of block B without causing motion of the system.

Solution: From FBD (1)



$\Sigma F_y = 0 \Rightarrow$

$-1.5 g \cos 45 + N_A = \Rightarrow N_A = 1.5 g \cos 45$

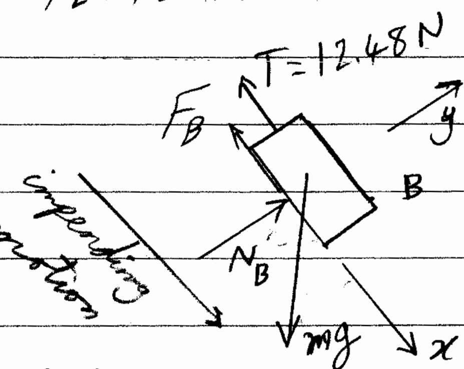
$\therefore F_A = \mu_s \times 1.5 g \cos 45 = 2.081 \text{ N}$

$\Sigma F_x = 0 \Rightarrow$

$T - F_A - 1.5 g \sin 45 = 0$

$\Rightarrow T = 2.081 + 1.5 g \sin 45 = 12.48 \text{ N}$

From FBD (2)



$\Sigma F_y = 0$

$-mg \cos 60 + N_B = 0$

$\Rightarrow N_B = mg \cos 60$

$\Rightarrow F_B = 0.2 mg \cos 60$

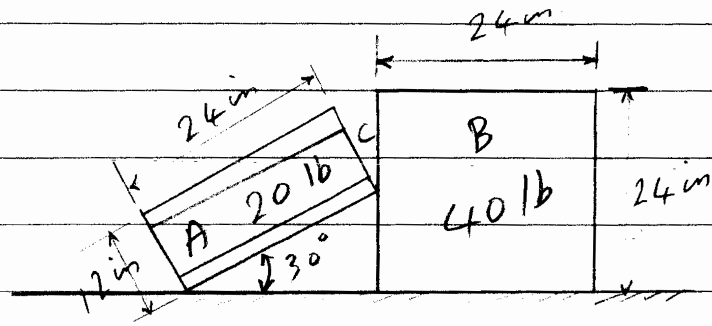
$\Sigma F_x = 0 \Rightarrow -T - F_B + mg \sin 60 = 0$

$\Rightarrow -12.48 - 0.2 mg \cos 60 + mg \sin 60 = 0$

$\Rightarrow \boxed{m = 1.66} \text{ kg}$ #

Note: why can't A move down and B up ??!!

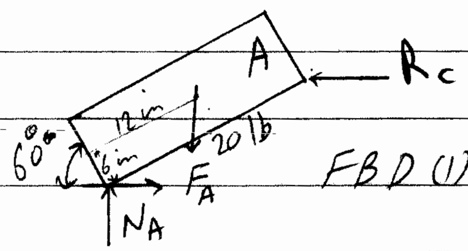
Problem #9



Given: The system shown with $\mu_s = 0.7$ between box A and floor and $\mu_s = 0.4$ between box B and floor.

Required: determine whether the boxes are in equilibrium.

Solution *From FBD(1)



$\circlearrowleft + \sum M_A = 0 \Rightarrow$

$R_c \times 24 \sin 30 +$

$-20(12\cos 30 - 6\cos 60) = 0$

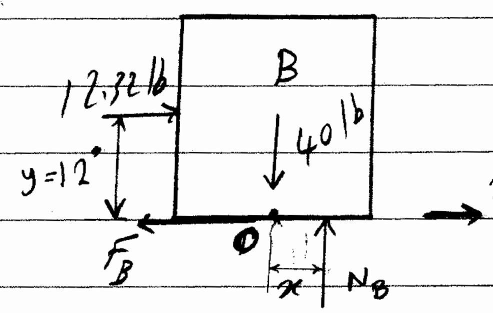
$\Rightarrow R_c = 12.32$ lb

$\sum F_y = 0 \Rightarrow -20 + N_A = 0 \Rightarrow N_A = 20$ lb

$\Rightarrow F_{A_{max}} = \mu_s N_A = 0.7 \times 20 = 14$ lb

$\sum F_x = 0 \Rightarrow F_A - R_c = 0 \Rightarrow F_A = 12.32 \text{ lb} < F_{A_{max}}$

$\therefore$ box A is in equilibrium if box B is in equilibrium.



*From FBD (2)

$\sum F_y = 0 \Rightarrow -40 + N_B = 0$

$\Rightarrow N_B = 40$ lb

$\Rightarrow F_{B_{max}} = \mu_s N_B = 16$ lb

$\sum F_x = 0 \Rightarrow -F_B + 12.32 = 0 \Rightarrow$ impending motion

$F_B = 12.32 < F_{B_{max}}$

$\therefore$ box B will not slide.

#

Continue problem # 5

Now let us check tipping for box B.

From FBD (2)

$y = 24 \sin 30 = 12$ in

$\circlearrowleft + \sum M_o = 0 \Rightarrow$

$40x - 12.32 * (12) = 0$

$\Rightarrow x = 3.7$ in < 12 in

$\therefore$ box B will not tip.

$\therefore$ box B is in equilibrium

$\therefore$ both boxs are in equilibrium.