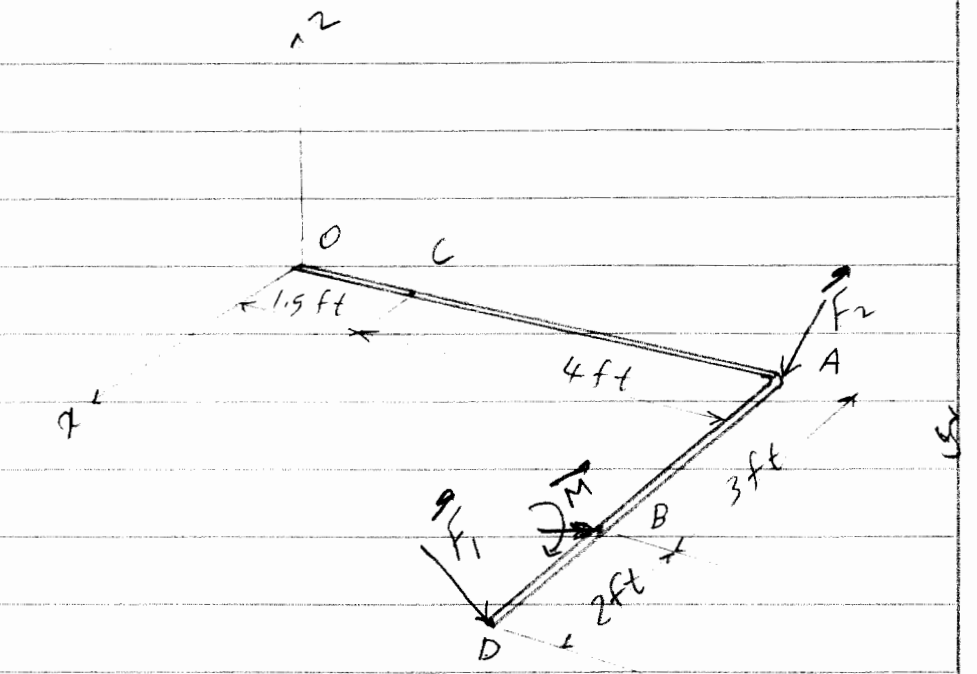
CE 201-9 [illegible]

Solution of HW # 4

Problem # 1



Given: The system shown in the figure with

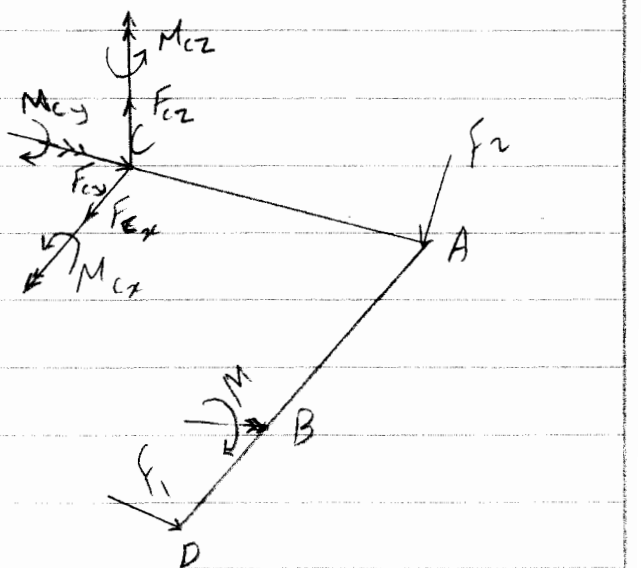
$\vec{F}_1 = -80\vec{i} + 200\vec{j} - 300\vec{k}$ lb

$\vec{F}_2 = 250\vec{i} - 150\vec{j} - 200\vec{k}$ lb

$\vec{M} = 200\vec{i} - 300\vec{j} + 400\vec{k}$ lb.ft

Required: the x, y, z components of internal loading at C.

Solution:



FBD

Continuous problem # 1

From the F.B.D

$$\sum \vec{F} = \vec{0}$$

$$\vec{F_1} + \vec{F_2} + \vec{F_c} = 0$$

$$\Rightarrow (-80\vec{i} + 200\vec{j} - 300\vec{k}) + (250\vec{i} - 150\vec{j} - 200\vec{k}) + \vec{F_c} = 0$$

$$\Rightarrow \boxed{\vec{F_c} = -170\vec{i} - 50\vec{j} + 500\vec{k}}\ lb$$

$$\therefore F_{cx} = -170\ lb \quad \& \quad F_{cy} = -50\ lb \quad \& \quad F_{cz} = 500\ lb$$

$$\sum \vec{M} = \vec{0}$$

$$(\vec{r}_{CA} \times \vec{F_2}) + (\vec{r}_{CD} \times \vec{F_1}) + \vec{M} + \vec{M_c} = 0$$

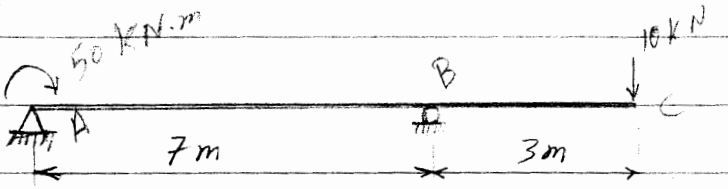
$$\Rightarrow \vec{M_c} + 200\vec{i} - 300\vec{j} + 400\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & 0 \\ 250 & -150 & -200 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 4 & 0 \\ -80 & +200 & -300 \end{vmatrix} = 0$$

$$M_c + 200\vec{i} - 300\vec{j} + 400\vec{k} - 800\vec{i} - 1000\vec{k} - 1200\vec{i} + 1500\vec{j} + 1320\vec{k} = 0$$

$$\Rightarrow \boxed{\vec{M_c} = 1800\vec{i} - 1200\vec{j} - 720\vec{k}}\ lb\text{-}ft \quad \#$$

Solution of HW #

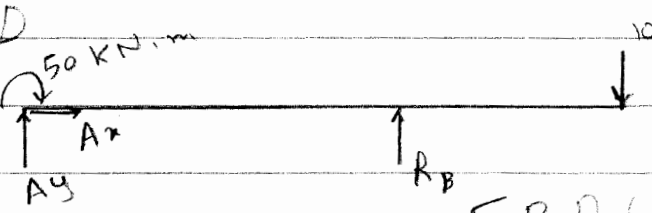
Problem # 2



Given: The beam shown in the figure.

Required: SFD & BMD

Solution First we find the reactions



FBD (1)

From FBD(1)

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma M_A = 0 \quad +\circlearrowleft \quad -50 + R_B \times 7 - 10 \times 10 = 0$

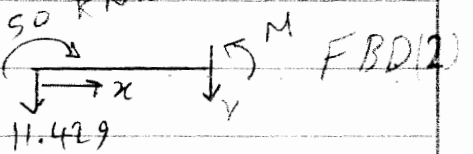
$\Rightarrow R_B = 21.429 \text{ kN}$

$\Sigma F_y = 0 \Rightarrow A_y + 21.429 - 10 = 0$

$\Rightarrow A_y = -11.429 \text{ kN}$

Two section are needed

AB ($0 \le x \le 7$)



FBD(2)

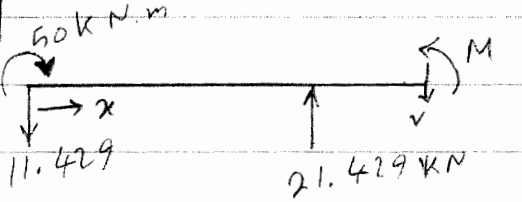
From FBD(2)

$\Sigma F_y = 0 \Rightarrow -11.429 - V = 0 \Rightarrow \boxed{V = -11.429 \text{ kN}}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow -50 + 11.429 \times x + M = 0$

$\Rightarrow \boxed{M = 50 - 11.429x}$

BC ($7 \le x \le 10$)



From FBD(3)

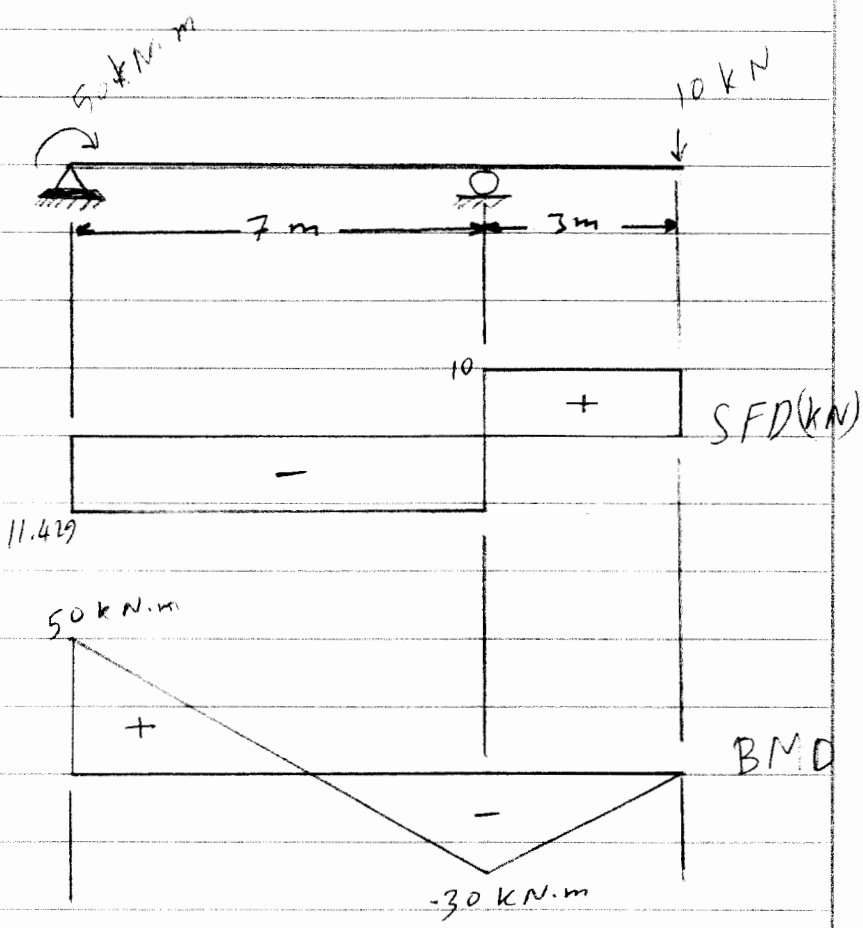
$\Sigma F_y = 0 \Rightarrow -11.429 + 21.429 - V = 0 \Rightarrow$

$\boxed{V = 10 \text{ kN}}$

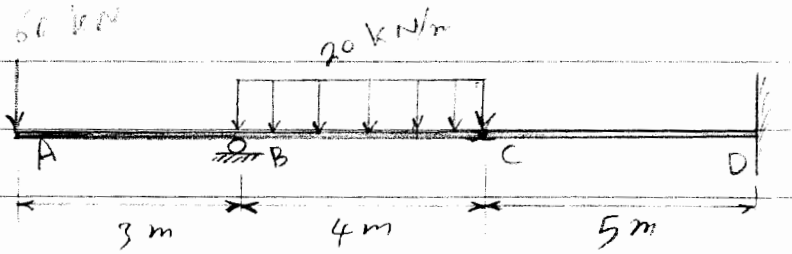
$+\circlearrowleft \Sigma M = 0 \Rightarrow -50 + 11.429 \times x - 21.429(x-7) + M = 0$

$\Rightarrow \boxed{M = -100 + 10x}$

Continuous problem 7 :



Problem # 3

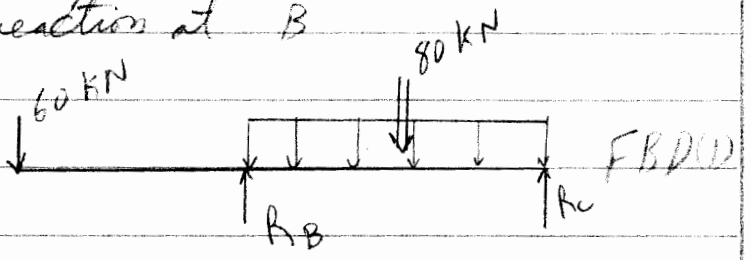


Given: The beam shown in the figure

Required: SFD & BMD

Solution: First we find reaction at B

From FBD(1)

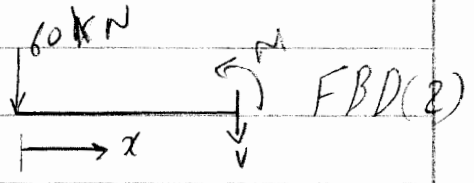


$+\circlearrowleft \Sigma M_C = 0 \Rightarrow 60 \times 7 - R_B \times 4 + 80 \times 2 = 0$

$\Rightarrow R_B = 145\ kN$

Note: we don't need to find reaction at D!

AB ($0 \le x \le 3$) from FBD(2)



$\Sigma F_y = 0 \Rightarrow$

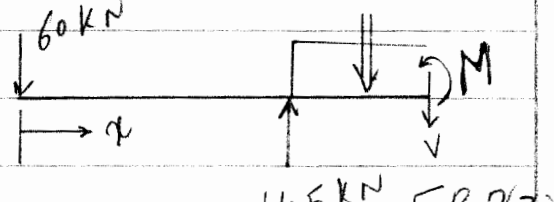
$-60 - V = 0 \Rightarrow \boxed{V = -60\ kN}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow$

$60 \times x + M = 0 \Rightarrow \boxed{M = -60x}\ kN.m$

BC ($3 \le x \le 7$)

From FBD(3)



$\Sigma F_y = 0$

$-60 + 145 - 20(x-3) - V = 0$

$\Rightarrow \boxed{V = 85 - 20(x-3)}\ kN \Rightarrow \boxed{V = 145 - 20x}\ kN$

$+\circlearrowleft \Sigma M = 0 \Rightarrow$

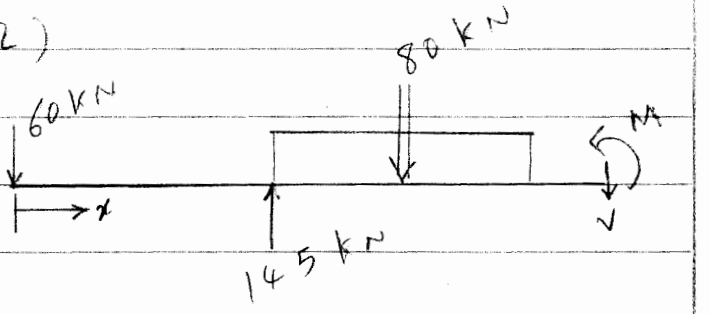
$60 \times x - 145(x-3) + 20(x-3)\frac{(x-3)}{2} + M = 0$

$\Rightarrow \boxed{M = 145x - 10x^2 - 525}\ kN.m$

Continue problem #3

CD ($7 \le x \le 12$)

From FBD (4)



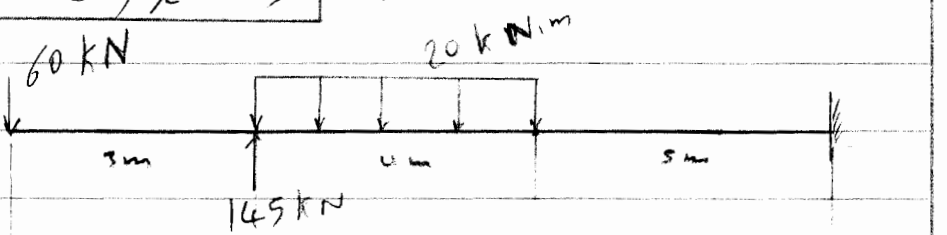
$\Sigma F_y = 0 \Rightarrow$

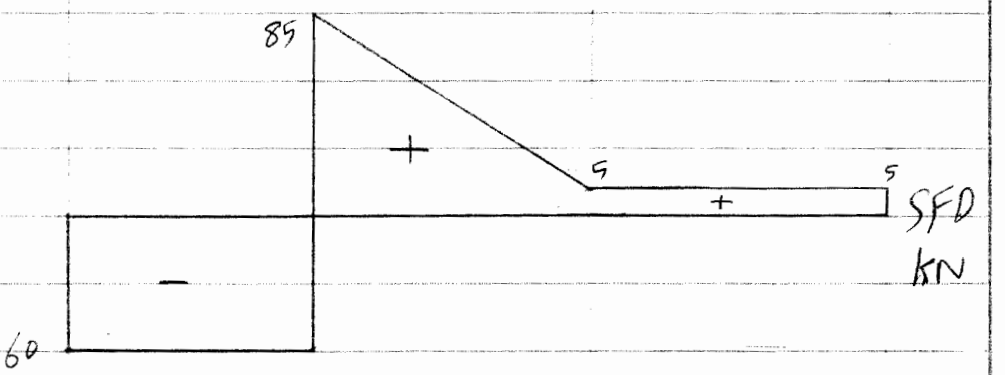
$-60 + 145 - 80 - V = 0 \Rightarrow \boxed{V = 5}$ kN

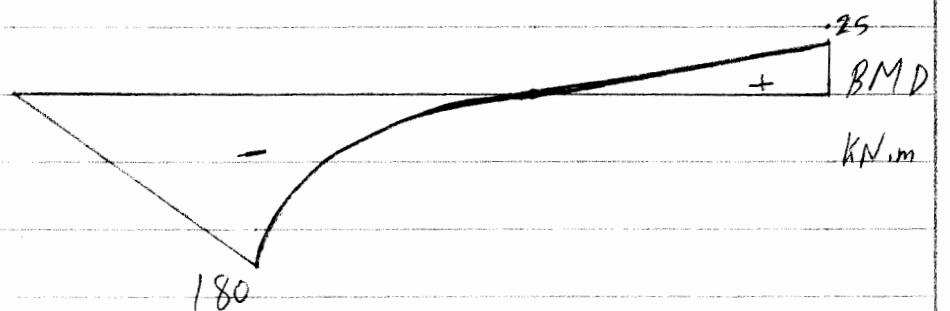
$\circlearrowleft \Sigma M = 0$

$60x - 145(x-3) + 80(x-5) + M = 0$

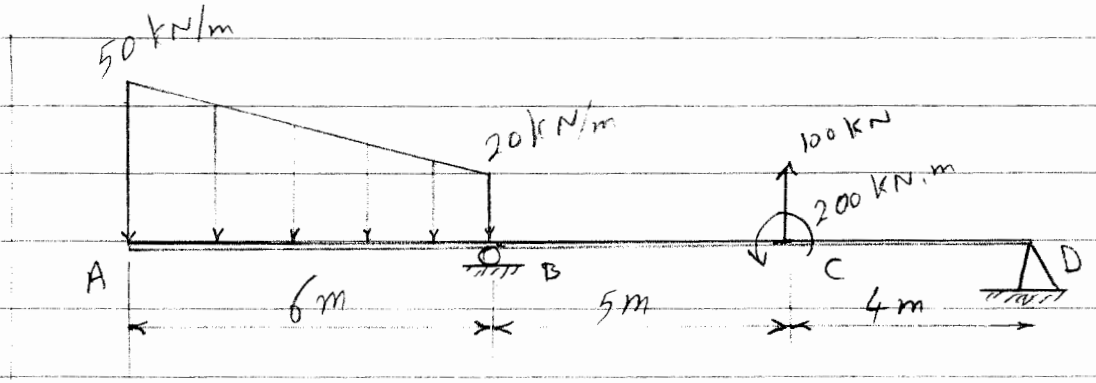
$\Rightarrow \boxed{M = 5x - 35}$ kN.m







Problem # 4

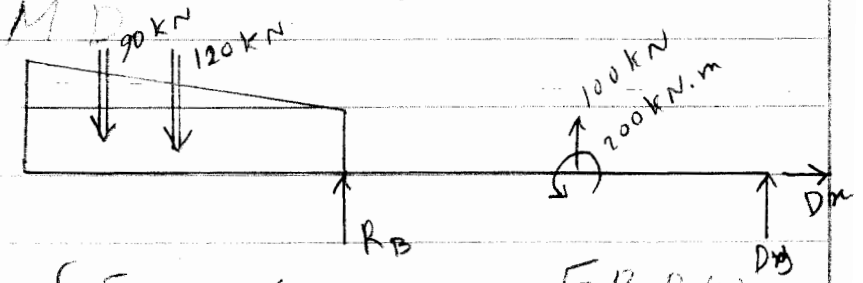


Give: The beam shown in the figure.

Required: SFD & BMD

Solution: First find the reaction

From FBD(1)



FBD(1)

$F_1 = 20 \times 6 = 120$ kN & $F_2 = 30 \times \frac{6}{2} = 90$ kN

$\Sigma F_x = 0 \Rightarrow D_x = 0$

$+\circlearrowleft \Sigma M_D = 0 \quad 90 \times 13 + 120 \times 12 - R_B \times 9 - 100 \times 4 + 200 = 0$

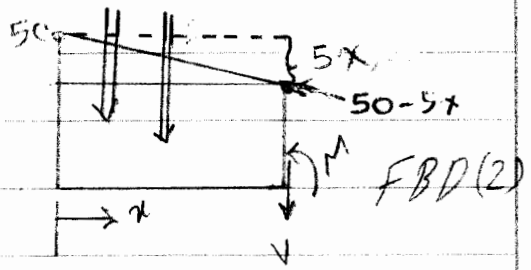
$\Rightarrow R_B = 267.778$ kN

$\Sigma F_y = 0 \Rightarrow -90 - 120 + 267.778 + 100 + D_y = 0$

$\Rightarrow D_y = -157.778$ kN

AB ($0 \le x \le 6$)

From FBD(2)



FBD(2)

[[See details next page]] ⊗

((or another method))

$\Sigma F_y = 0 \Rightarrow -(50 - 5x)x - 5x\frac{x}{2} - V = 0$

$\Rightarrow \boxed{V = -50x + 2.5x^2}$ kN

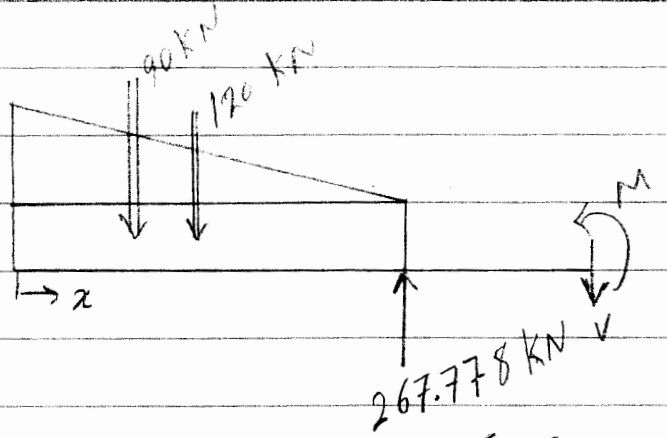
$+\circlearrowleft \Sigma M = 0 \Rightarrow (50 - 5x)x\frac{x}{2} + \frac{5}{2}x^2 \times \frac{2}{3}x + M = 0$

$\Rightarrow \boxed{M = -25x^2 + 0.833333x^3}$ kN.m

Continue problem # 4

BC ($6 \le x \le 11$)

From FBD (3)



FBD(3)

$\Sigma F_y = 0 \Rightarrow$

$-90 - 120 + 267.778 - V = 0$

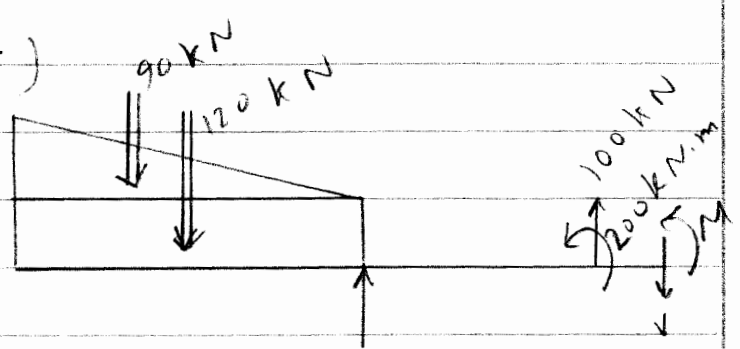
$\Rightarrow \boxed{V = 57.778 \text{ kN}}$

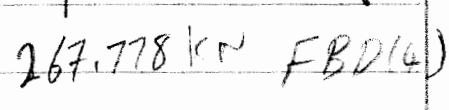
$\circlearrowleft \Sigma M = 0 \Rightarrow 120(x-3) + 90(x-2) - 267.778(x-6) + M = 0$

$\Rightarrow \boxed{M = 57.778x - 1066.68 \text{ kN}}$

CD ($11 \le x \le 15$)

From FBD(4)





FBD(4)

$\Sigma F_y = 0 \rightarrow -90 - 120 + 267.778 + 100 - V = 0$

$\boxed{V = 157.778 \text{ kN}}$

$\circlearrowleft \Sigma M = 0 \Rightarrow$

$90(x-2) + 120(x-3) - 267.778(x-6) - 100(x-11) + 200 + M = 0$

$\Rightarrow \boxed{M = 157.778x - 2366.668 \text{ kN}}$

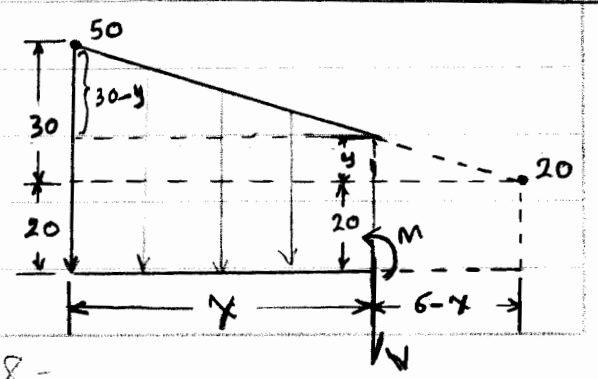
⊛ Details $\frac{y}{30} = \frac{6-x}{6} \Rightarrow y = 5(6-x)$

$+\uparrow \Sigma F_y = 0 = -V - 20x - y(x) - (30-y)\frac{x}{2}$

$\Rightarrow V = -50x + \frac{5}{2}x^2$

$+\circlearrowleft \Sigma M = 0 = M + 20x\left(\frac{x}{2}\right) + y(x)\left(\frac{x}{2}\right) + (30-y)\frac{x}{2}\left(\frac{2x}{3}\right)$

$\Rightarrow M = -25x^2 + \frac{5}{6}x^3$



Continue problem # 4