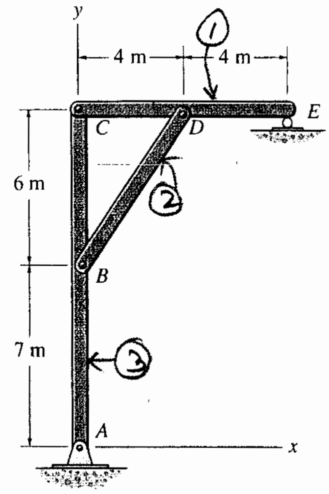
CE 201-2 (041)
H. W. # 13

Prob. 9-52 (P.-466)

Given:
mass per unit length = 6 kg/m.
Frame is shown in Figure.



Req.d:
I. Locate the Center of gravity $(\bar{x}, \bar{y})$
II. Reactions at the pin A and roller E.

Sol.n:

I). Centroid $(\bar{x}, \bar{y})$

$L_{②} = \sqrt{6^2 + 4^2} = \sqrt{52} = 7.2111\,m$

Segment	$L_i\,(m)$	$\bar{x}_i\,(m)$	$\bar{y}_i\,(m)$	$\bar{x}_i L_i\,(m^2)$	$\bar{y}_i L_i\,(m^2)$
1	8	4	13	32	104
2	7.2111	2	10	14.4222	72.111
3	13	0	6.5		84.5
Σ	28.2111			46.4222	260.611

$$\bar{x} = \frac{\Sigma \bar{x}_i L_i}{\Sigma L_i} = \frac{46.4222}{28.2111} \Rightarrow \boxed{\bar{x} = 1.646\,m}$$

$$\bar{y} = \frac{\Sigma \bar{y}_i L_i}{\Sigma L_i} = \frac{260.611}{28.2111} \Rightarrow \boxed{\bar{y} = 9.238\,m}$$

II). Reactions

FBD as shown

$W = (L)(\rho)(g)$

$= 28.2111 * 6 * 9.81 = 1660.51\,N$

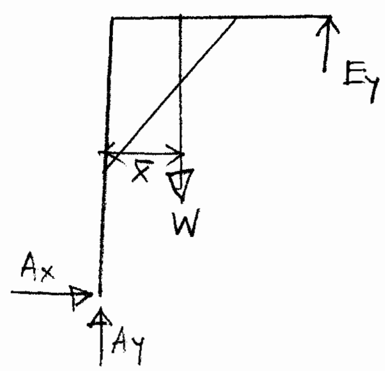
$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$\circlearrowleft + \;\Sigma M_A = 0 \Rightarrow 8 E_y - 1660.31 * 1.646 = 0$

$\therefore \boxed{E_y = 314.55\,N}$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y + 341.55 - 1660.51 = 0$

$\boxed{A_y = 1318.95\,N}$

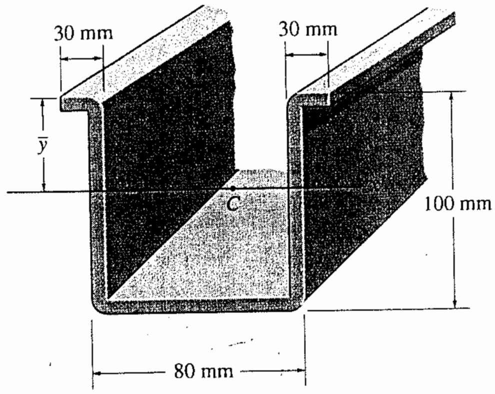


FBD

Prob: 9-55 (P.-466)

Given:
cross-section shown in Fig.
Thickness = 10 mm.

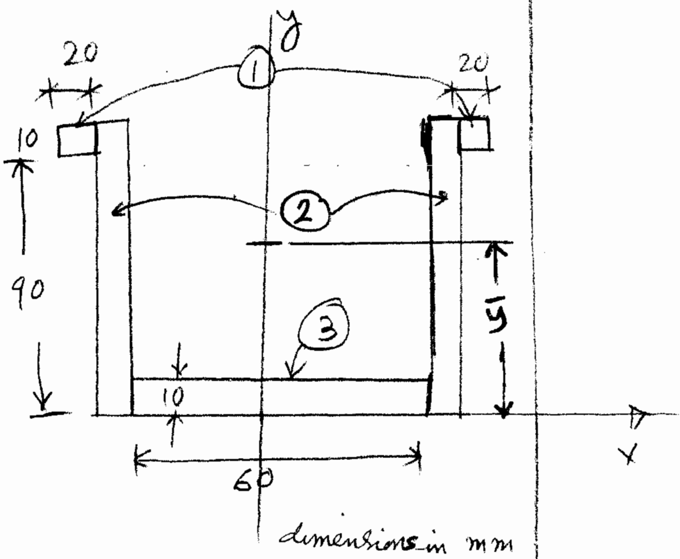
Req.d:
The location of the centroid $\bar{Y}$ of the area.



Prob. 9–55

Sol.n:

Note the cross-section is symmetry about Y-axis



Segment	Area$_i$ (mm^2)	$\bar{y}_i$ (mm)	$\bar{y}_i A_i$ (mm^3)
1	400	95	38,000
2	2,000	50	100,000
3	600	5	3,000
Σ	3,000		141,000

$$\bar{Y} = \frac{\Sigma \bar{Y}_i A_i}{\Sigma A_i} = \frac{141,000}{3,000}$$

$\therefore \boxed{\bar{Y} = 47 \text{ mm}}$

$\bar{Y}$ = 47 mm measured from bottom

(or $\bar{Y}$ = 53 mm " " top)

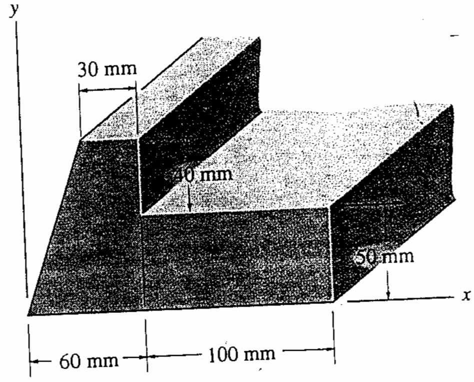
Prob. 9.65 (P. -469)

Given:

The member's cross-sectional area shown in figure.

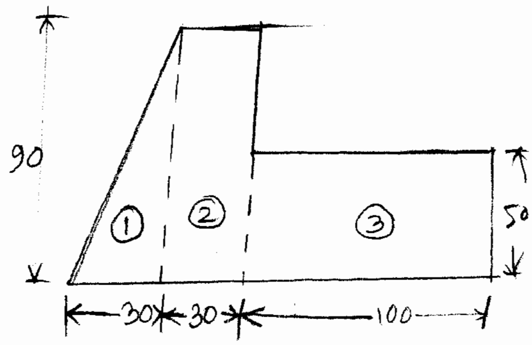
Req.d:

Location of the centroid $(\bar{x}, \bar{y})$.



Sol.n:

All dimensions in mm



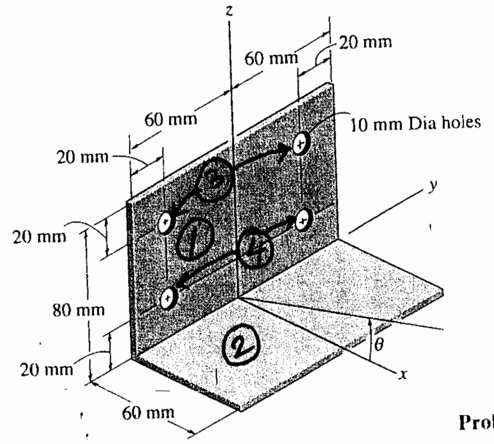
Segment	A_i (mm^2)	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	$\bar{x}_i A_i$ (mm^3)	$\bar{y}_i A_i$ (mm^3)
1	1350	20	30	27,000	40,500
2	2700	45	45	121,500	121,500
3	5000	110	25	550,000	125,000
Σ	9050			698,500	287,000

$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = \frac{698,500}{9,050} \Rightarrow \boxed{\bar{x} = 77.182 \text{ mm}}$$

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{287,000}{9,050} \Rightarrow \boxed{\bar{y} = 31.713 \text{ mm}}$$

Prob. 9-73 (P.-470)

Given: The sheet metal bracket shown in Figure.



Prob. 9–73

Req.d:

1) location of center of gravity
2) The maximum angle of tilt θ

Sol.n:

1/ Note that segment ① is the whole vertical sheet including the four holes.
Also note that we can deal with areas directly or with volumes after multiplying all areas by the constant thickness t. ⇒ same answers.

Segment	A_i (mm²)	$\bar{x}_i$ (mm)	$\bar{z}_i$ (mm)	$\bar{x}A_i$ (mm³)	$\bar{z}_iA_i$ (mm³)
①	120 * 80 = 9,600	0	40	0	384,000
②	120 * 60 = 7,200	30	0	216,000	0
③*	$-2\left[\frac{\pi}{4}(10)^2\right] = -157.0796$	0	60	0	−9424.778
④*	$-2\left[\frac{\pi}{4}(10)^2\right] = -157.0796$	0	20	0	−3141.593
Σ	16,485.841			216,000	371,433.629

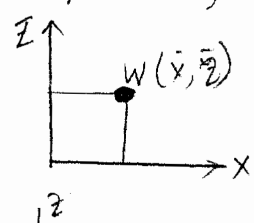
* why we take two holes together?! Can we always do it?!

$$\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i} = \frac{216,000}{16,485.841} \Rightarrow \boxed{\bar{x} = 13.10 \text{ mm}}$$

$$\bar{z} = \frac{\Sigma \bar{z}_i A_i}{\Sigma A_i} = \frac{371,433.629}{16,485.841} \Rightarrow \boxed{\bar{z} = 22.53 \text{ mm}}$$

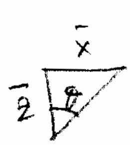
2/ Maximum angle "θ"

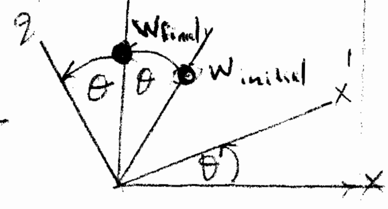
Since the bracket is resting on the horizontal x-y plane, then "θ" can be increased until the reaction is just on the y-axis. To prevent rotation about y-axis at this stage, the weight and the reaction must coincide.



Thus $\tan\theta = \frac{\bar{x}}{\bar{z}} = \frac{13.10}{22.53}$

$\therefore \boxed{\theta = 30.18°}$





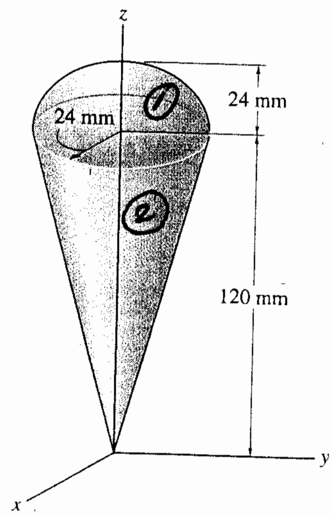
Prob.-79 (P. 472)

Given:

The cone is shown in Figure.

Req.d:

location of the centroid $\bar{z}$ of the top made from a hemisphere and a cone.



Sol. n:

Segment	V_i (mm^3)	$\bar{z}_i$ (mm)	$\bar{z}_i V_i$ (mm^4)
1 (hemi-sphere)	$\frac{2}{3}\pi(24)^3 = 28,952.918$	$120+\frac{3}{8}(24) = 129$	3,73,4926.409
2 (cone)	$\frac{1}{3}\pi(24)^2(120) = 72,382.295$	$\frac{3}{4}(120) = 90$	6,514,406.526
	101,335.213		10,249,332.94

$$\bar{z} = \frac{\Sigma \bar{z}_i V_i}{\Sigma V_i} = \frac{10,249,332.94}{101,335.213}$$

$$\Rightarrow \boxed{\bar{z} = 101.14 \text{ mm}}$$