CE 201 STATICS (Sections 4 & 6) [081]
H.W. # 3 Solutions.

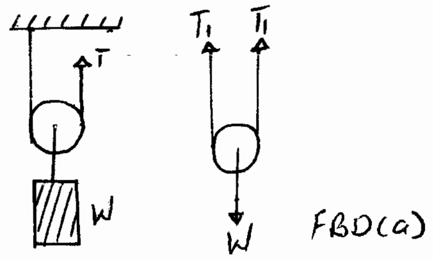
Problem 1

Given: Pulley systems containing one, two and three pulleys

Required: The force T, required to support the weights, W in each case

Solution:

(a)



(One pulley)

In FBD(a),

$T_1 = T$ (why?)

$+\uparrow \sum F_y = 0 \Rightarrow$

$T + T - W = 0$

$\Rightarrow \boxed{T = \frac{1}{2} W}$

b)

In FBD (b)①,

$T_1 = T$ (why?)

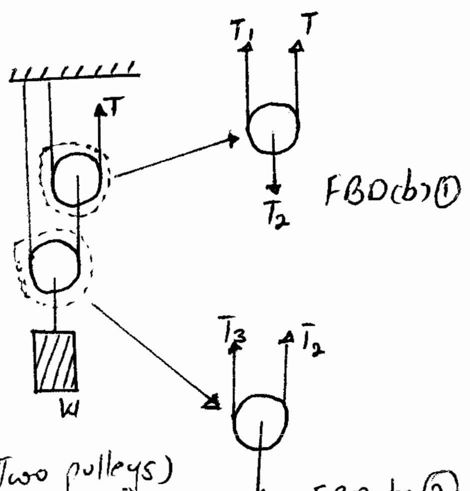
$+\uparrow \sum F_y = 0 \Rightarrow$

$T + T - T_2 = 0$

$\Rightarrow T_2 = 2T$

In FBD (b)②,

$T_3 = T_2 = 2T$ (why?)



(Two pulleys)

$+\uparrow \sum F_y = 0$

$T_2 + T_3 - W = 0 \Rightarrow$

$2T + 2T - W = 0$

$\Rightarrow \boxed{T = \frac{1}{4} W}$

(C)

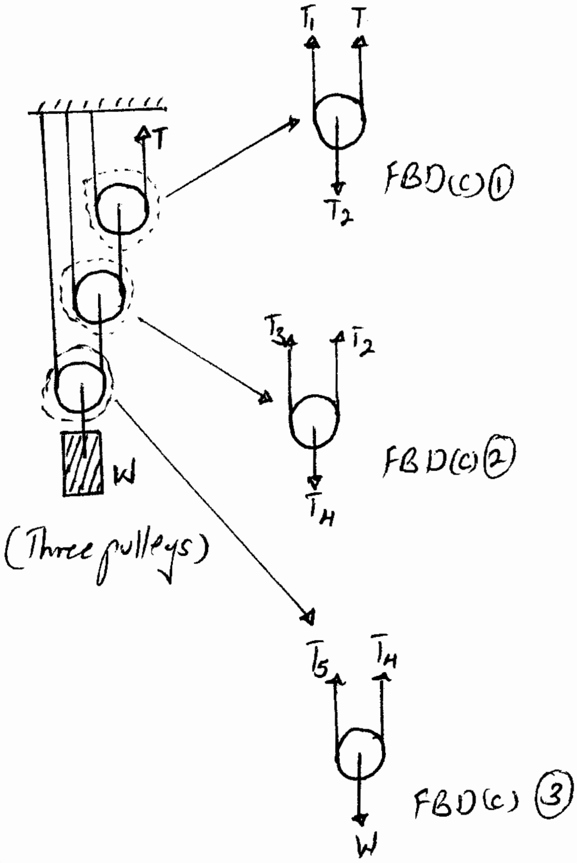
In FBD (c) 1,

$T_1 = T$ (why?)

$+\uparrow \sum F_y = 0 \Rightarrow$

$T + T - T_2 = 0$

$\Rightarrow T_2 = 2T$



In FBD (c) 2,

$T_2 = T_3 = 2T$ (why?)

$+\uparrow \sum F_y = 0 \Rightarrow$

$T_2 + T_3 - T_4 = 0 \Rightarrow$

$2T + 2T - T_4 = 0 \Rightarrow$

$T_4 = 4T$

In FBD (c) 3,

$T_4 = T_5 = 4T$ (why?!)

$+\uparrow \sum F_y = 0 \Rightarrow$

$T_4 + T_5 - W = 0 \Rightarrow$

$4T + 4T - W = 0 \Rightarrow$

$$\boxed{T = \frac{1}{8} W}$$

What conclusion can you make from the answers in part (a), (b) and (c)?

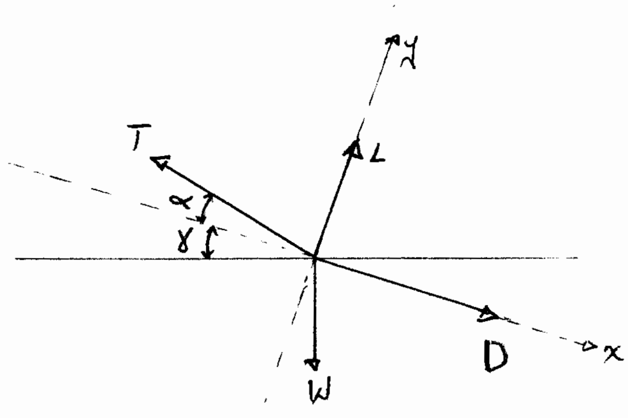
Problem 2

Given: The free body diagram as shown of an airplane flying in the vertical plane.

$\gamma = 6°$, $D = 125\,kN$, $L = 680\,kN$, and mass of airplane is 72,000 kg.

Required: Values of T and α necessary to maintain steady flight.

Solution



FBD

from the FBD,

$\overset{+}{\searrow} \Sigma F_x = 0 \Rightarrow$

$D - T\cos\alpha + W\sin\gamma = 0$ (1)

$\overset{+}{\nearrow} \Sigma F_y = 0 \Rightarrow$

$T\sin\alpha + L - W\cos\gamma = 0$ (2)

$W = mg = (72000)(9.81) = 706.32\,kN$

from equation (2), $\sin\alpha = \dfrac{W\cos\gamma - L}{T}$ (3)

from equation (1), $\cos\alpha = \dfrac{D + W\sin\gamma}{T}$ (4)

Divide equation (3) by (4) $\Rightarrow$

$$\tan\alpha = \frac{W\cos\gamma - L}{W\sin\gamma + D}$$

$$= \frac{706.32\cos 6° - 680}{706.32\sin 6° + 125}$$

$$= 0.11291$$

$\Rightarrow \boxed{\alpha = 6.442°}$

$\Rightarrow$ from equation (1), $\boxed{T = 200.1\,kN}$

Note that the thrust necessary for steady flight is about 28% of the airplane's weight.

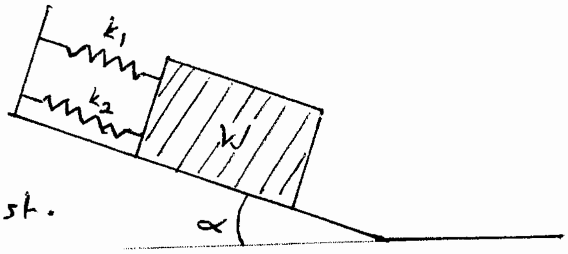
Problem 3

Given: As in the figure shown.

Required: To show that the magnitudes of the forces exerted by the two springs are:

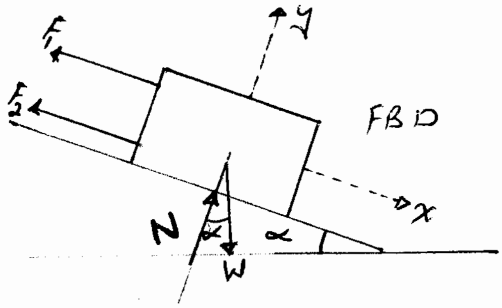
$F_1 = W\sin\alpha/(1+k_2/k_1)$,

$F_2 = W\sin\alpha/(1+k_1/k_2)$.



Solution:

The FBD is drawn first.



$\searrow^{+}\ \Sigma F_x = 0 \Rightarrow$

$W\sin\alpha - F_1 - F_2 = 0$ (1)

For the two springs,

$S_1 \ (\text{or } \Delta L_1) = S_2 \ (\text{or } \Delta L_2) = S$

Both have the same stretch s, (Why?!)

$\Rightarrow F_1 = k_1 S$ — (2)

$F_2 = k_2 S$ (3)

Dividing (2) by (3); $\dfrac{F_1}{F_2} = \dfrac{k_1}{k_2}$

Dividing (1) by F_2,

$$\frac{W\sin\alpha}{F_2} - \frac{F_1}{F_2} - 1 = 0$$

$$\frac{W\sin\alpha}{F_2} - \frac{k_1}{k_2} - 1 = 0$$

$$\frac{W\sin\alpha}{F_2} = 1 + \frac{k_1}{k_2}, \qquad \boxed{F_2 = \frac{W\sin\alpha}{1 + k_1/k_2}}$$

From equation ①

$$F_1 = W \sin\alpha - F_2$$

$$= W \sin\alpha - \frac{W \sin\alpha}{1 + k_1/k_2}$$

$$= W \sin\alpha \left(1 - \frac{1}{1 + k_1/k_2}\right)$$

$$= W \sin\alpha \left[\frac{1 + k_1/k_2 - 1}{1 + k_1/k_2}\right]$$

$$= W \sin\alpha \left[\frac{k_1/k_2}{1 + k_1/k_2}\right]$$

$$= W \sin\alpha \left[\frac{k_1/k_2}{1 + k_1/k_2}\right] \frac{k_2/k_1}{k_2/k_1}$$

$$\boxed{F_1 = \frac{W \sin\alpha}{1 + \frac{k_2}{k_1}}}$$

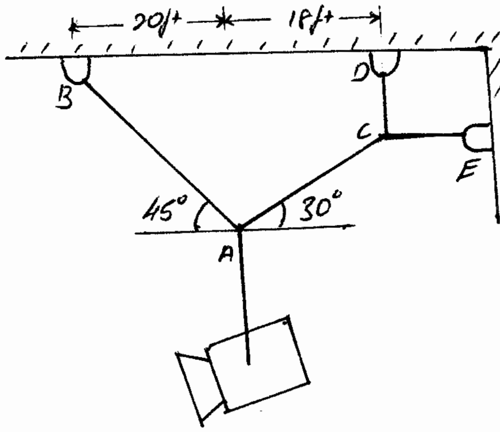
<u>Note</u>: We can find F_1 as we found F_2, by dividing equation ① by F_1; it could be easier than the method above.

Problem 4

Given: A system of cables suspending a 1000-lb bank of lights.

Required: a) Tension in cables AB, CD and CE.

b) Tension in cable AB when cable CE is removed.



Solution

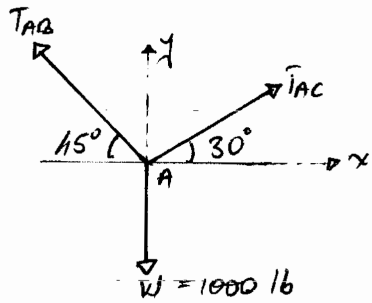
(a) First FBD ① is drawn. (Why?!)

$\xrightarrow{+} \sum F_x = 0 \Rightarrow$

$T_{AC} \cos 30° - T_{AB} \cos 45° = 0$ ①

$+\uparrow \sum F_y = 0 \Rightarrow$

$T_{AB} \sin 45° + T_{AC} \sin 30° - W = 0$ ②



FBD ①

from ①, $T_{AC} = \dfrac{\cos 45°}{\cos 30°} T_{AB}$ ③

Substituting ③ into ②

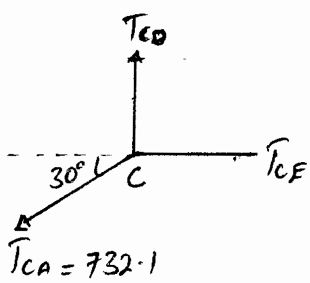
$T_{AB} \sin 45° + \dfrac{\cos 45°}{\cos 30°} \sin 30° \, T_{AB} - W = 0$

$\Rightarrow \boxed{T_{AB} = 896.6 \text{ lb}}$

From ③, $\boxed{T_{AC} = 732.1 \text{ lb}}$

Now, FBD ② is drawn.

Note the direction of T_{AC} (Why?!)



FBD ②

$\xrightarrow{+} \sum F_x = 0 \Rightarrow$

$T_{CE} - 732.1 \cos 30° = 0 \quad \boxed{T_{CE} = 634.0 \text{ lb}}$

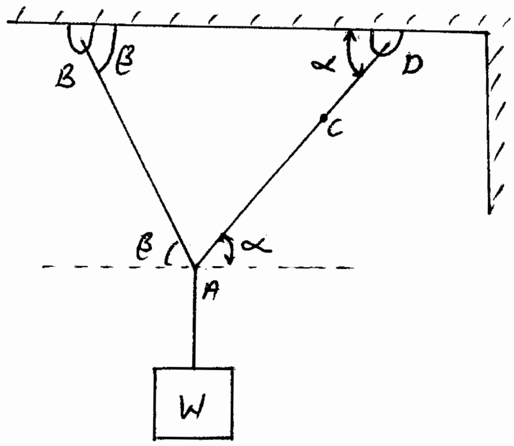
$+\uparrow \sum F_y = 0 \Rightarrow$

$T_{CD} - 732.1 \sin 30° \Rightarrow \boxed{T_{CD} = 366.0 \text{ lb}}$

(b)

After removing the cable CE, then the system will look like the figure shown. (Why?)

From the original figure (given in the problem),



$AB \cos 45^\circ = 20 \Rightarrow AB = 28.2843\ ft$

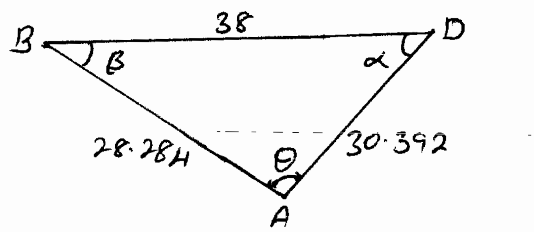
$AC \cos 30^\circ = 18 \Rightarrow AC = 20.7846\ ft$

$CD = AB \sin 45^\circ - AC \sin 30^\circ$

$\quad = 20 \tan 45 - 18 \tan 30^\circ$

$CD = 9.6077\ ft$

Thus, $AD = AC + CD = 30.392\ ft$



Using Cosine Law

$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2 - 2(AD)(BD) \cos\alpha$

$\Rightarrow \cos\alpha = 0.67871$

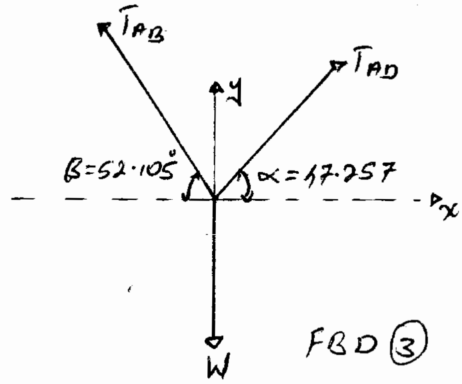
$\Rightarrow \alpha = 47.257^\circ$

$\overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2 - 2(AB)(BD) \cos\beta$

$\Rightarrow \cos\beta = 0.61422$

$\Rightarrow \beta = 52.105^\circ$

Now, FBD ③ is drawn.



FBD ③

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow$

$T_{AD} \cos\alpha - T_{AB} \cos\beta = 0$

$\Rightarrow T_{AD} = \dfrac{\cos\beta}{\cos\alpha} T_{AB}$ (i)

$+\uparrow \sum F_y = 0 \Rightarrow$

$T_{AB} \sin\beta + T_{AD} \sin\alpha - W = 0$ (ii)

Substituting (i) into (ii)

$T_{AB} \sin\beta + \dfrac{\cos\beta}{\cos\alpha} \sin\alpha\, T_{AB} - W = 0$

$\boxed{T_{AB} = 687.9\ lb}$

Problem 5

Given: Cable AB attached to the top of the vertical 3-m post, and its tension is 50kN

Required: Tensions in cable AO, AC, and AD

Solution

① Coordinates:

A (6, 2, 0), B (12, 3, 0), C (0, 8, 5), D (0, 4, -5), O (0, 0, 0)

② Position Vectors

$\vec{AB} = (B) - (A) = 6\vec{i} + 1\vec{j} + 0\vec{k} \Rightarrow AB = \sqrt{37}$ m

$\vec{AC} = (C) - (A) = -6\vec{i} + 6\vec{j} + 5\vec{k} \Rightarrow AC = \sqrt{97}$ m

$\vec{AD} = (D) - (A) = -6\vec{i} + 2\vec{j} - 5\vec{k} \Rightarrow AD = \sqrt{65}$ m

$AO = (O) - (A) = -6\vec{i} - 2\vec{j} + 0\vec{k} \Rightarrow AO = \sqrt{40}$ m

③ Force Vectors

$$\vec{T}_{AB} = \frac{T_{AB}}{AB}(\vec{AB}) = 49.320\vec{i} + 8.2199\vec{j} + 0\vec{k}$$

(Note that T_{AB} is given as 50kN)

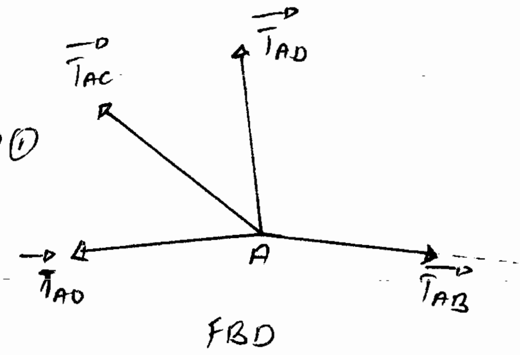
$$\vec{T}_{AC} = \frac{T_{AC}}{AC}(\vec{AC}) = T_{AC}(-0.60921\vec{i} + 0.60921\vec{j} + 0.50767\vec{k})$$

$$\vec{T}_{AD} = \frac{T_{AD}}{AD}(\vec{AD}) = T_{AD}(-0.74421\vec{i} + 0.24807\vec{j} - 0.62017\vec{k})$$

$$\vec{T}_{AO} = \frac{T_{AO}}{AO}(\vec{AO}) = T_{AO}(-0.94868i - 0.31623j + 0\vec{k})$$

(4) The FBD is drawn as shown below.

$\Sigma F_x = 0 \Rightarrow$



FBD

$$49.320 - 0.60921 T_{AC} - 0.74421 T_{AD} - 0.94868 T_{AO} = 0 \quad (1)$$

9/9

$\Sigma F_y = 0 \Rightarrow$

$$8.2199 + 0.60921 T_{AC} + 0.24807 T_{AD} - 0.31623 T_{AO} = 0 \quad (2)$$

$\Sigma F_z = 0 \Rightarrow$

$$0.50767 T_{AC} - 0.62017 T_{AD} = 0 \quad (3)$$

Solving equations (1), (2) and (3)

From equation (3)

$$T_{AD} = 0.81860 T_{AC} \quad \text{—(i)}$$

Substituting Equation (i) into equation (2)

$$T_O = 25.9934 + 2.56864 T_{AC} \quad \text{—(ii)}$$

Substituting equation (i) and (ii) into (1)

$$\boxed{T_{AC} = 6.747 \text{ kN}}$$

$$\boxed{T_{AD} = 5.523 \text{ kN}}$$

$$\boxed{T_{AO} = 43.323 \text{ kN}}$$