

H.W. # 12 Solutions.

Problem 1.

Given: The semicircular part of a homogeneous slender bar as shown in figure below.

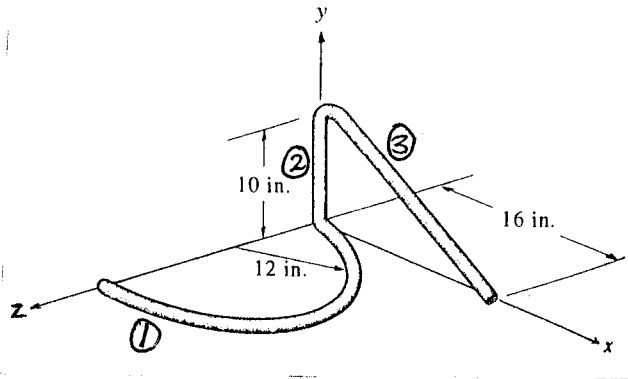
Required: The centre of mass of the bar.

Solution.

The bar is divided into 3 segments.

$$\bar{x} = \frac{\sum_{i=1}^3 L_i \bar{x}_i}{\sum_{i=1}^3 L_i}$$

$$\bar{y} = \frac{\sum_{i=1}^3 L_i \bar{y}_i}{\sum_{i=1}^3 L_i}, \quad \bar{z} = \frac{\sum_{i=1}^3 L_i \bar{z}_i}{\sum_{i=1}^3 L_i}$$



Seg #	L_i (in)	x_i (in)	y_i (in)	\bar{z}_i (in)	$\bar{x}_i L_i$ (in ²)	$\bar{y}_i L_i$ (in ²)	$\bar{z}_i L_i$ (in ²)
①	$\pi r = 12\pi$ = 37.6991	$\frac{2r}{\pi} = \frac{24}{\pi}$ = 7.63944	0	12	288	0	452.309
②	10	0	5	0	0	50	0
③	$\sqrt{16^2 + 10^2}$ = 18.8680	8	5	0	150.994	94.3398	0
Σ	66.5671				438.944	144.3398	452.389

$$\bar{x} = \frac{438.944}{66.5671} \Rightarrow \boxed{\bar{x} = 6.594 \text{ in}}$$

$$\bar{y} = \frac{144.3398}{66.5671} \Rightarrow \boxed{\bar{y} = 2.168 \text{ in}}$$

$$\bar{z} = \frac{452.389}{66.5671} \Rightarrow \boxed{\bar{z} = 6.796 \text{ in}}$$

Are the answers reasonable?!

Problem 2

$\frac{2}{5}$

Given: A homogeneous wire ABCD as shown in Figure P2 of question sheet.

Required: The length, L for which portion AB of the wire is horizontal.

Solution:

→ We draw the wire when portion AB is horizontal →

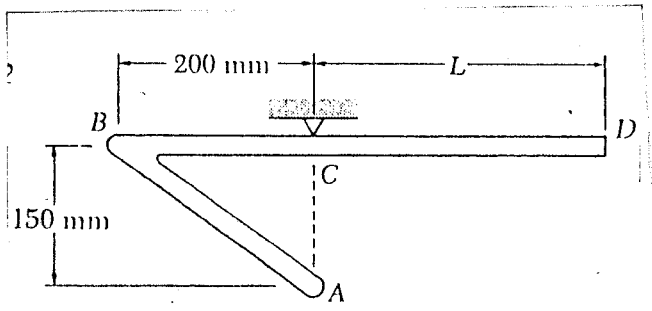
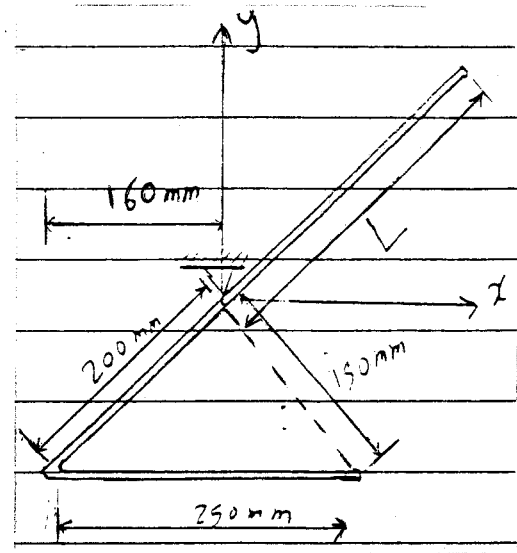


Figure P2.



For the wire to be in equilibrium in this position, \bar{x} should equal to zero (Why??!!)

$$\bar{x} = \frac{\sum_{i=1}^3 \bar{x}_i L_i}{\sum_{i=1}^3 L_i} = 0 \Rightarrow \sum_{i=1}^3 \bar{x}_i L_i = 0$$

$$\sum_{i=1}^3 \bar{x}_i L_i = 250[-(160-125)] + 200 \times \left(\frac{-160}{80}\right) + L \left(\frac{1}{2} \frac{200}{250}\right) = 0$$

$$\Rightarrow \boxed{L = 248.75 \text{ mm}}$$

Note: Try to solve the problem for the cases;

- ① Portion BD is horizontal.
- ② Portion BA is vertical.

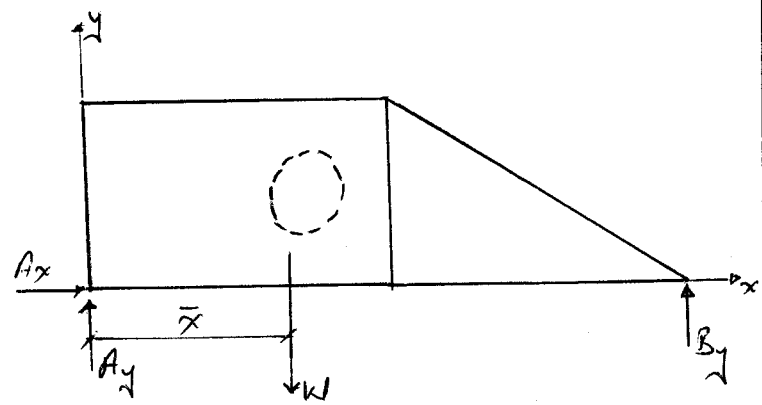
Problem 3

Given: The homogeneous plate shown in Figure P3 of question paper.

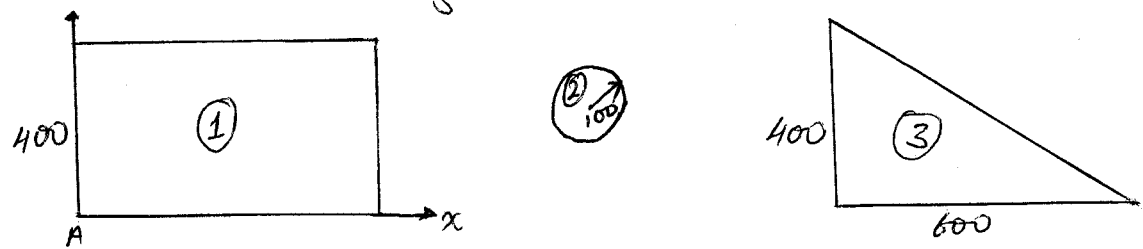
Required: The reactions

Solution:

In order to find the reactions, we need to locate the centroid (\bar{x}) first. Why?!



The area is divided into 3 segments



Seg #	A_i (mm ²)	\bar{x}_i (mm)	$\bar{x}_i A_i$ (mm ³)
1	$400 \times 800 = 320,000$	400	$128,000,000$ $\approx 1.28(10)^8$
2	$-\frac{\pi(100)^2}{2} = -31,415.93$	600	-18849556
3	$\frac{600 \times 400}{2} = 120,000$	$800 + \frac{600}{3} = 1,000$	$1.2(10)^8$
Σ	408,584		$2.29150(10)^8$

$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = \frac{2.2915(10)^8}{408,584} = 560.84 \text{ mm (reasonable?!)}$$

Note that \bar{y} is not calculated because it is not needed.

In the FBD: $\rightarrow \Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$\uparrow \Sigma M_A = 0 \Rightarrow -500(560.84) + 1400B_y = 0 \Rightarrow \boxed{B_y = 200.3 \text{ N}}$

$\uparrow \Sigma F_y = 0 \Rightarrow A_y + 200.3 - 500 = 0 \Rightarrow \boxed{A_y = 299.7 \text{ N}}$

Problem 4

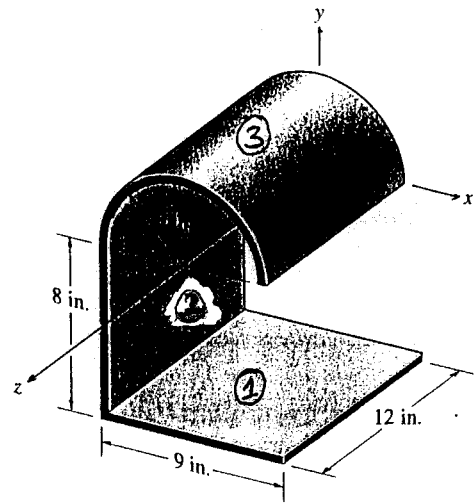
Given: The figure P4 as shown in the question sheet.

Required: The centroid of the sheet.

Solution:

The object is divided into 3 parts.

Note that the "thickness" is assumed very small and thus ignored.



Seg #	$A_i (\text{in}^2)$	$\bar{x}_i (\text{in})$	$\bar{y}_i (\text{in})$	$\bar{z}_i (\text{in})$	$\bar{x}_i A_i (\text{in}^3)$	$\bar{y}_i A_i (\text{in}^3)$	$\bar{z}_i A_i (\text{in}^3)$
1	$9 \times 12 = 108$	0.5	-8	6	54	-864	648
2	$8 \times 12 = 96$	-4	-4	6	-384	-384	576
3	$\pi r h = \pi(4)(12) = 48\pi \approx 150.796$	0	$\frac{2r}{\pi} = \frac{2(4)}{\pi} = \frac{8}{\pi} \approx 2.54648$	6	0	384	904.779
Σ	354.796				-330	-864	2,128.779

$$\bar{x} = \frac{\sum_{i=1}^3 \bar{x}_i A_i}{\sum_{i=1}^3 A_i} = \frac{-330}{354.796} \Rightarrow \boxed{\bar{x} = -0.9301 \text{ in}}$$

$$\bar{y} = \frac{\sum_{i=1}^3 \bar{y}_i A_i}{\sum_{i=1}^3 A_i} = \frac{-864}{354.796} \Rightarrow \boxed{\bar{y} = -2.435 \text{ in}}$$

$$\bar{z} = \frac{\sum_{i=1}^3 \bar{z}_i A_i}{\sum_{i=1}^3 A_i} = \frac{2128.779}{354.796} \Rightarrow \boxed{\bar{z} = 6 \text{ in.}}$$

Are the answers reasonable?!

* Note that we can get $\bar{z} = 6$ directly due to symmetry.

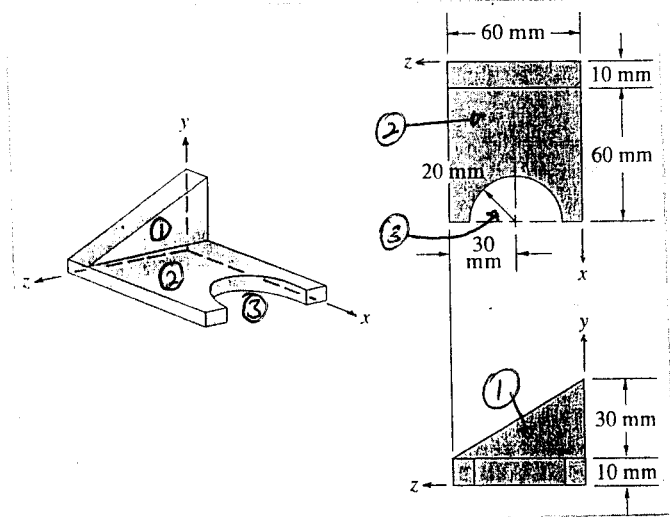
Problem 5

Given: A homogeneous object as shown in Figure P5 of question sheet.

Required: The center of mass

Solution.

The object is divided into 3 parts.



PART	$V_i (\text{mm}^3)$	$\bar{x}_i (\text{mm})$	$\bar{y}_i (\text{mm})$	$\bar{z}_i (\text{mm})$	$\bar{x}_i V_i (\text{mm}^4)$	$\bar{y}_i V_i (\text{mm}^4)$	$\bar{z}_i V_i (\text{mm}^4)$
①	$\frac{1}{2}(60)(30)(10) = 9,000$	$\frac{10}{2} = 5$	$10 + \frac{80}{3} = 20$	$\frac{60}{3} = 20$	45,000	180,000	180,000
②	$60(60+10)(10) = 42,000$	$\frac{70}{2} = 35$	$\frac{10}{2} = 5$	$\frac{60}{2} = 30$	1,470,000	210,000	1,260,000
③	$\frac{-\bar{x}_r^2}{2} = \frac{-\pi(30)^2}{2} = -6,283.19$	$\frac{10+60-100}{3\pi} = 61.5117$	$\frac{10}{2} = 5$	$\frac{60}{2} = 30$	-386,489.6	-31,415.9	-188,495.6
Σ	44,716.8				1,128,510	358,584	1,251,504

$$\bar{x} = \frac{\sum_{i=1}^3 \bar{x}_i V_i}{\sum_{i=1}^3 V_i} = \frac{1,128,510}{44,716.8} \Rightarrow \bar{x} = 25.24 \text{ mm}$$

$$\bar{y} = \frac{\sum_{i=1}^3 \bar{y}_i V_i}{\sum_{i=1}^3 V_i} = \frac{358,584}{44,716.8} \Rightarrow \bar{y} = 8.019 \text{ mm}$$

$$\bar{z} = \frac{\sum_{i=1}^3 \bar{z}_i V_i}{\sum_{i=1}^3 V_i} = \frac{1,251,504}{44,716.8} \Rightarrow \bar{z} = 27.99 \text{ mm}$$

Are the answers reasonable?!