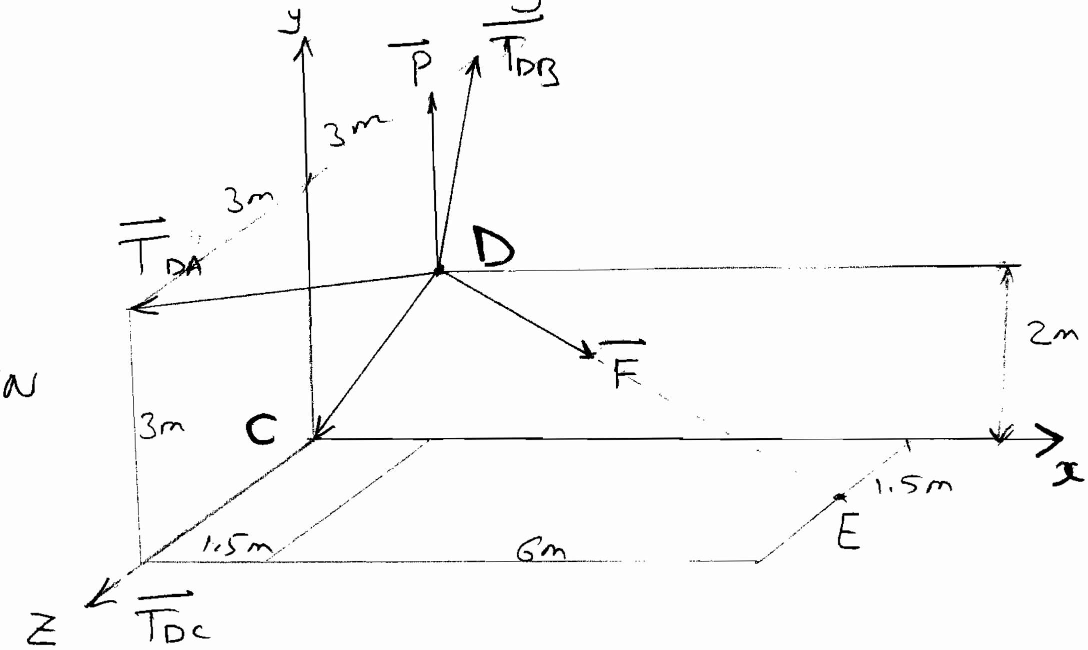
CE 201 - 4 & 5 (071)

H.W # 4. Key solution



Problem 1:

Given:

$F = 15.60$ KN

$T_{CD} = 0$

P in y-dir.

Fig. shown

Required:

The force P

Solution:-

The origin is at C

A (0, 3, 3)

B (0, 3, -3)

C (0, 0, 0)

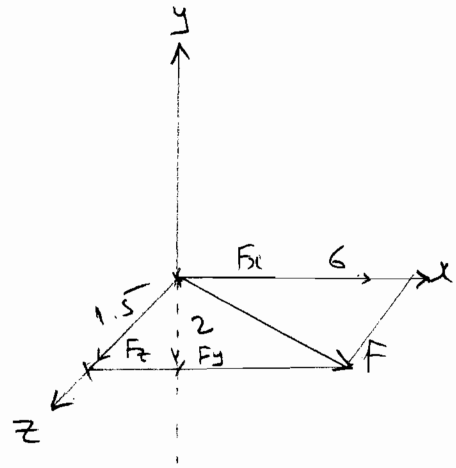
D (1.5, 2, 0)

E (6, 0, 1.5)

$$\vec{T}_{DA} = T_{DA}\,\frac{-1.5\vec{i} + \vec{j} + 3\vec{k}}{\sqrt{1.5^2 + 1 + 3^2}} = \frac{T_{DA}}{3.5}(-1.5\vec{i} + \vec{j} + 3\vec{k})$$

$$\vec{T}_{DB} = T_{DB}\,\frac{-1.5\vec{i} + \vec{j} - 3\vec{k}}{\sqrt{1.5^2 + 1^2 + 3^2}} = \frac{T_{DB}}{3.5}(-1.5\vec{i} + \vec{j} - 3\vec{k})$$

$$\vec{T}_{DC} = T_{DC}\,\frac{-1.5\vec{i} - 2\vec{j} + 0\vec{k}}{\sqrt{1.5^2 + 2^2}} = \frac{T_{DC}}{2.5}(-1.5\vec{i} - 2\vec{j})$$



finding components of $\vec{F}$

$$\left.\begin{aligned} 6/6.5 &= \cos\theta \Rightarrow \theta = 22.62 \\ -2/6.5 &= \cos\alpha \Rightarrow \alpha = 107.92 \\ 1.5/6.5 &= \cos\beta \Rightarrow \beta = 76.66 \end{aligned}\right\} \text{no need}$$

check $\cos\theta^2 + \cos\alpha^2 + \cos^2\beta = 1$ ∴ OK.

$$F_x = F\cos\theta = 15.60\,\overbrace{\cos 22.62}^{6/6.5} = 14.4 \text{ KN}$$

$$F_y = F\cos\alpha = 15.60\,\underbrace{\cos 107.92}_{-2/6.5} = -4.8 \text{ KN}$$

$$F_z = F\cos\beta = 15.60\,\underbrace{\cos 76.66}_{1.5/6.5} = 3.6 \text{ KN}$$

$$\overset{\rightarrow}{\Sigma F_x = 0} \Rightarrow \left(\frac{-1.5}{3.5}\right)T_{DB} - \left(\frac{1.5}{2.5}\right)T_{DC} - \left(\frac{1.5}{3.5}\right)T_{DA} + 14.4 = 0 \quad \text{— 1}$$

$$\uparrow \Sigma F_y = 0 \Rightarrow \frac{T_{DB}}{3.5} - \left(\frac{2}{2.5}\right)T_{DC} + \frac{T_{DA}}{3.5} + P + (-4.8) = 0 \quad \text{— 2}$$

$$\swarrow \Sigma F_z = 0 \Rightarrow \left(\frac{-3}{3.5}\right)T_{DB} + (3/3.5)\,T_{DA} + 3.6 = 0 \quad \text{— 3}$$

Solving 1 and 2

$$-\frac{3}{\cancel{3.5}} T_{DB} + \frac{3}{3.5} T_{DA} + 3.6 = 0 \quad \text{—— } (3)$$

$$\frac{3}{\cancel{3.5}} T_{DB} + \frac{3}{3.5} T_{DA} + 0 - 28.8 = 0$$

$$\frac{6}{3.5} T_{DA} - 25.2 = 0$$

$$T_{DA} = 14.7 \text{ KN}$$

$$T_{DB} = 18.9 \text{ KN}.$$

from 2

$$T_{DB}/3.5 + T_{DA}/3.5 + P + (-4.8) = 0$$

$$18.9/3.5 + 14.7/3.5 + P - 4.8 = 0$$

$$P = -4.8 \text{ KN}.$$

$$\therefore \boxed{P = 4.8 \text{ KN Downward}}$$

Problem 2

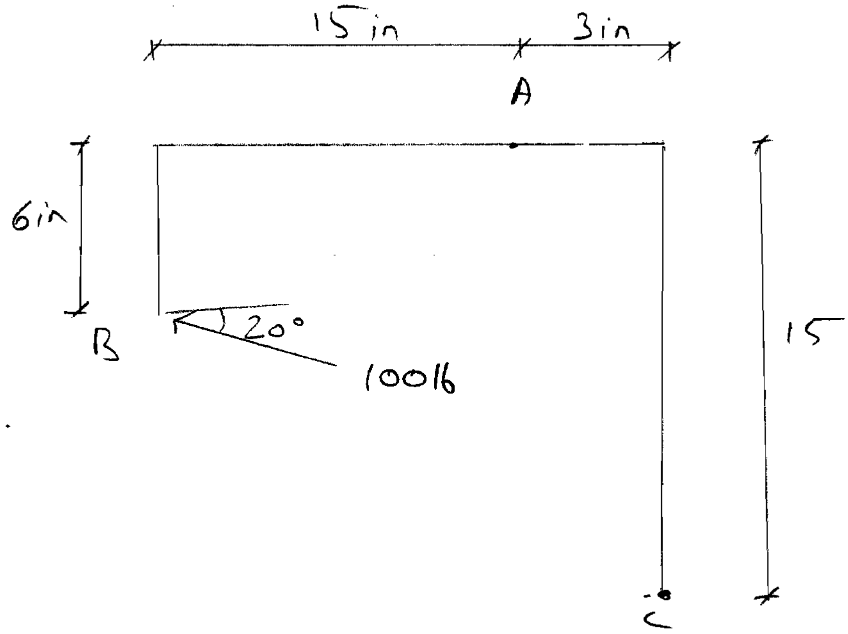
Given:

Force = 100 lb

Fig P2.

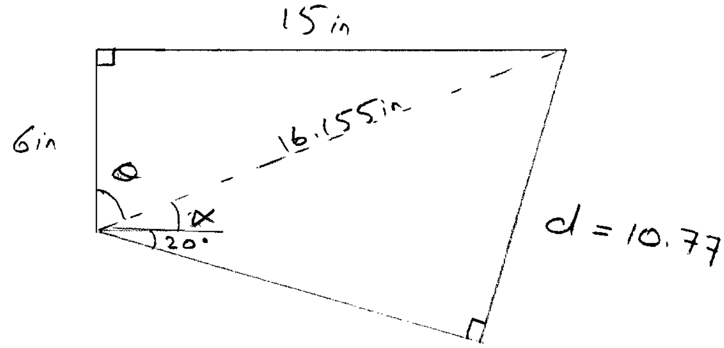
Required:-

Moment about A.



Solution:-

a)



$\theta = \sin^{-1}(15/16.155)$

$\theta = 68.20°$

$\alpha = 90 - 68.20 = 21.80°$

$d = \sin(21.80 + 20) \cdot 16.155$

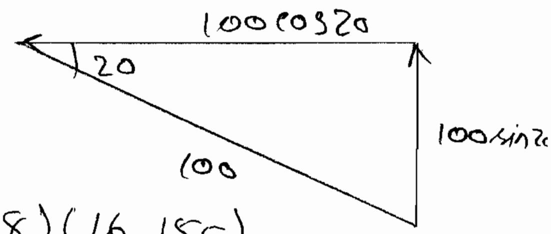
$d = 10.77 \text{ in}$

$M_A = F \cdot d = (100)(10.77)$

$\boxed{M_o = 1077 \text{ lb.in} \circlearrowright}$

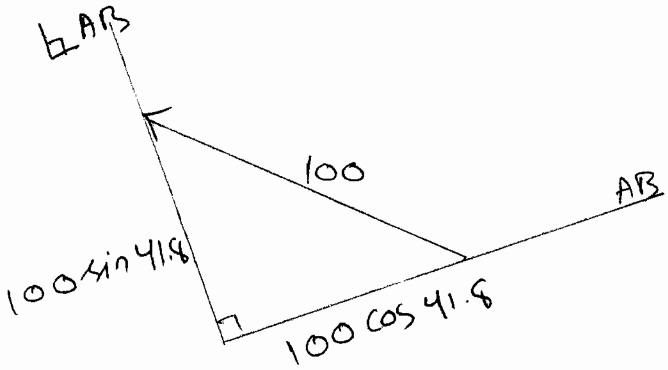
b) $M_A = (100\cos 20)(6) + (100\sin 20)(15)$

$\boxed{M_A = 1077 \text{ lb.in} \circlearrowright}$



c) $M_A = 100(\cos 41.8)0 + (100\sin 41.8)(16.155)$

$\boxed{M_A = 1077 \quad \text{lb.in} \circlearrowright}$



Problem 3:-

Given:-

$P = 420$ lb

Fig P_3

Required:-

Moment at O

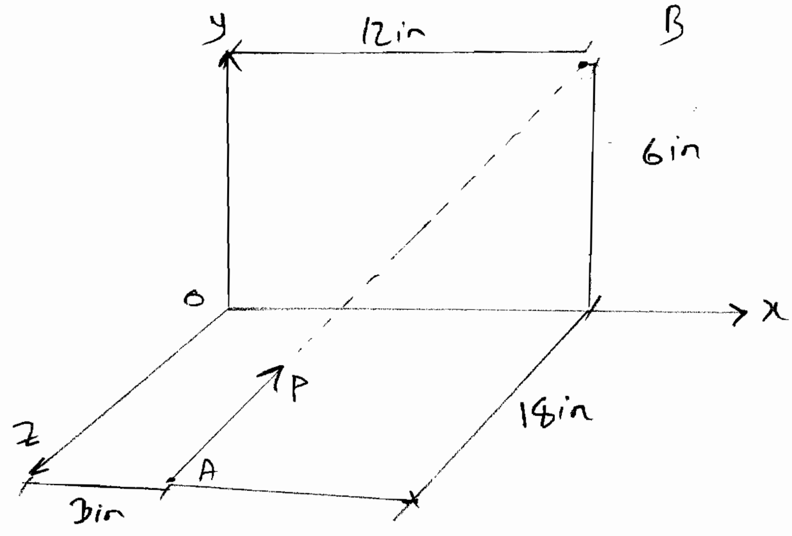


Fig. P3

Solution:-

a) i) $P = 420$ lb

$$\vec{M_o} = \vec{r_{OA}} \times \vec{P}$$

$$\vec{P} = (420)\,\frac{\overrightarrow{(12-3)i} + \overrightarrow{(6-0)j} + \overrightarrow{(-18)k}}{\sqrt{9^2 + 6^2 + 18^2}}$$

$$\vec{P} = (180\vec{i} + 120\vec{j} - 360\vec{k})\ \text{lb}$$

$$\vec{M_o} = \vec{r_{OA}} \times \vec{P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 18 \\ 180 & 120 & -360 \end{vmatrix}$$

$$= (0 - 18 \times 120)\vec{i} - (-1080 - 3240)\vec{j} + (360 - 0)\vec{k}$$

$$\boxed{\vec{M_o} = (-2160\vec{i} + 4320\vec{j} + 360\vec{k})\ \text{lb.in.}}$$

a)ii)

$$\vec{M_O} = \vec{r_{OB}} \times \vec{P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 12 & 6 & 0 \\ 180 & 120 & -360 \end{vmatrix}$$

$$= (-2160 - 0)\vec{i} - (-4320 - 0)\vec{j} + (1440 - 1080)\vec{k}$$

$$\boxed{\vec{M_O} = (-2160\vec{i} + 4320\vec{j} + 360\vec{k}) \text{ lb.in}}$$

b) $M_O = P \cdot d$

$M = \sqrt{(2160)^2 + (4320)^2 + (360)^2} = 4843.3 \text{ lb.in}$

$P = 420 \text{ lb}$

$\therefore d = M_O / P = \dfrac{4843.3}{420}$

$$\boxed{d = 11.53 \text{ in}}$$

Problem (4)

Given :-

$Q = 450$ KN

Fig P.4

Required

Moment at O & D.

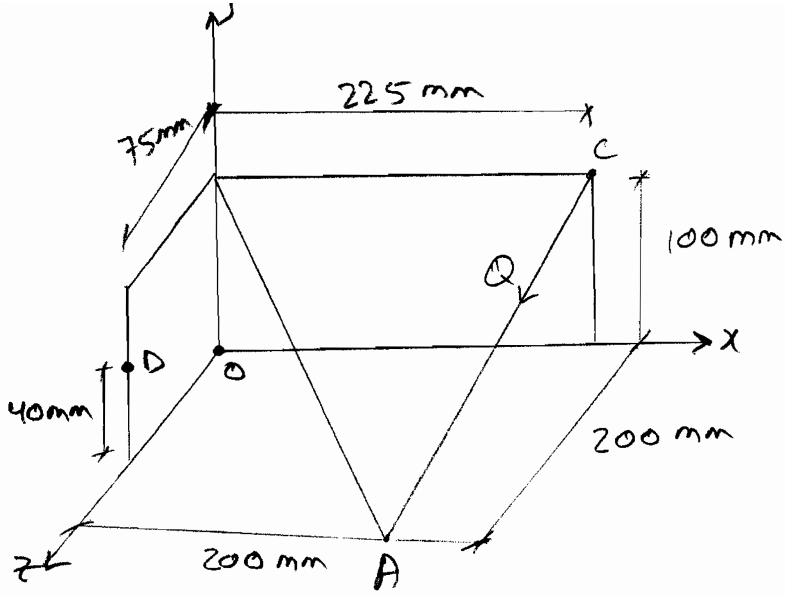


Fig P4

Solution :-

a) $O\ (0, 0, 0)$

$A\ (200, 0, 200)$

$C\ (225, 100, 0)$

$$\overline{M_O} = \overrightarrow{r_{OA}} \times \overline{Q}$$

$$\overline{r_{OA}} = (200\,i + 200\,\vec{k}), \quad \vec{r}_{OC} = (225\,\vec{i} + 100\,\vec{j} + 0\,\vec{k})$$

$$\vec{Q} = 450 \frac{(200-225)\vec{i} - 100\,j + 200\,k}{\sqrt{25^2 + 100^2 + 200^2}}$$

$$\vec{Q} = (-50\,\vec{i} - 200\,\vec{j} + 400\,\vec{k})\ N$$

$$\vec{M_O} = \overline{r_{OA}} \times \vec{Q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 200 & 0 & 200 \\ -50 & -200 & 400 \end{vmatrix}$$

$$\vec{M_O} = (40\,\vec{i} - 90\,\vec{j} - 40\,\vec{k})\ N.m$$

b) $\vec{M}_D = \vec{r}_{DA} \times \vec{Q}$

$$\vec{r}_{DA} = (200\vec{i} - 40\vec{j} + 125\vec{k})$$

$$\vec{M}_D = \vec{r}_{DA} \times \vec{Q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 200 & -40 & 125 \\ -50 & -200 & 400 \end{vmatrix}$$

$$\vec{M}_D = (-16000 + 25000)\vec{i} - (80000 + 6250) + (-40000 - 2000)$$

$$= +(9000\vec{i} - 86250\vec{j} - 42000\vec{k})\ N.mm$$

$$\boxed{\vec{M}_D = (90\vec{i} - 86.250\vec{j} - 42\vec{k})\ N.m}$$

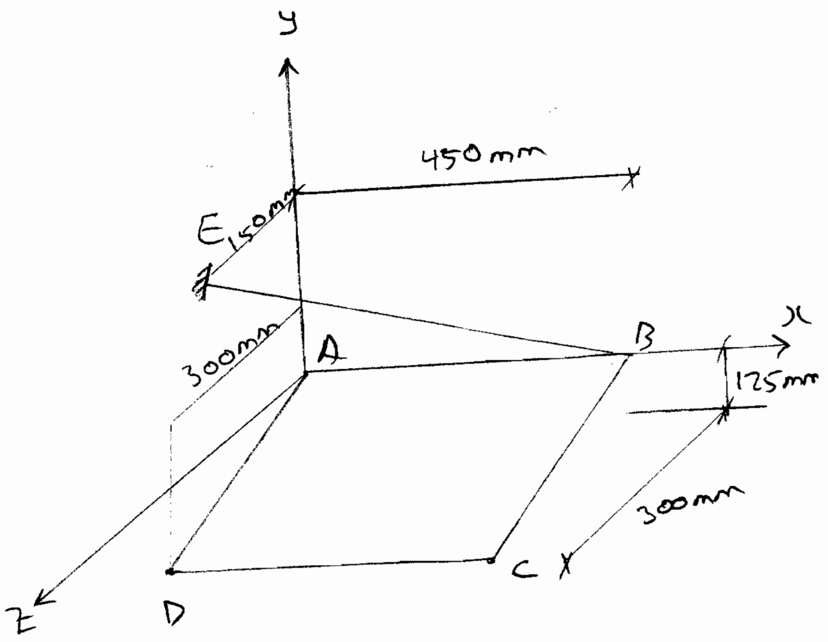
Note: You can use $\vec{r}_{DC}$ instead of $\vec{r}_{DA}$. Try it!

Problem 5 :-

Given :-

$T = 546\ N$

Fig P5



Required:

Moment about AD

Solution :-

A (0, 0, 0)

B (450, 0, 0)

C (450, −125, 300)

D (0, −125, 300)

E (0, 225, 150)

$$M_{AD} = \vec{U}_{AD} \cdot (\vec{r}_{AB} \times \vec{T})$$

$$\vec{U}_{AD} = \frac{-125\vec{j} + 300\vec{k}}{\sqrt{125^2 + 300^2}} = \frac{-125}{325}\vec{j} + \frac{300}{325}\vec{k}$$

$$\left.\begin{aligned} \vec{r}_{AB} &= (450\vec{i}) \\ \vec{r}_{AE} &= 225\vec{j} + 150\vec{k} \end{aligned}\right\} \text{may use one of them}$$

$$\vec{T} = 546 \; \frac{-450\vec{i} + 225\vec{j} + 150\vec{K}}{\sqrt{450^2 + 225^2 + 150^2}}$$

$$\vec{T} = (-468\vec{i} + 234\vec{j} + 156\vec{k})\ N$$

$$M_{AD} = \vec{U}_{AD} \cdot (\vec{r}_{AB} \times \vec{T}) = \begin{vmatrix} 0 & \frac{-125}{325} & \frac{300}{325} \\ 450 & 0 & 0 \\ -468 & 234 & 156 \end{vmatrix}$$

$$M_{AD} = 0 + \frac{125}{325}(70200) + \frac{300}{325}(105300)$$

$$= 27000 + 97200 = 124200 \ \text{N.mm}$$

$$\boxed{M_{AD} = 124.2 \ \text{N.m.}}$$