

Problem 1

Given: Fig P1

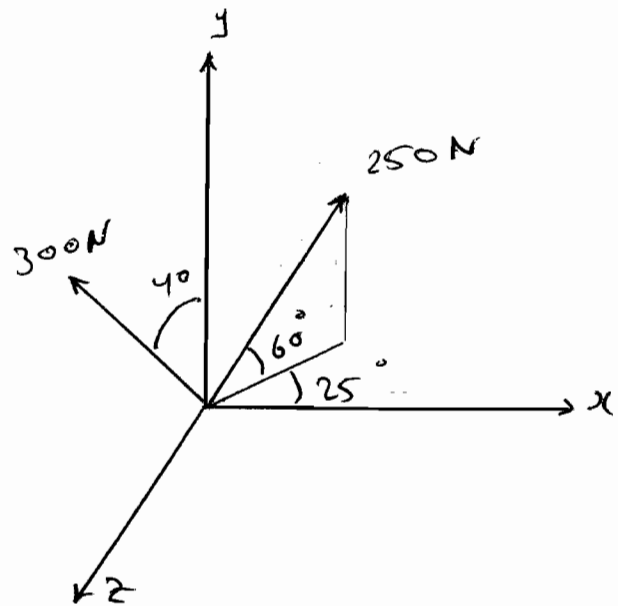
Required: value and resultant of the two forces

Solution:

$$\cos \theta_x = \frac{F_{1x}}{F_1}$$

$$\cos \theta_y = \frac{F_{1y}}{F_1}$$

$$\cos \theta_z = \frac{F_{1z}}{F_1}$$



$$F_{1x} = (300 \sin 40) \sin 20 = 65.95 \text{ N}$$

$$F_{1y} = 300 \cos 40 = 229.8 \text{ N}$$

$$F_{1z} = (300 \sin 40) \cos 20 = 181.2 \text{ N}$$

$$\therefore \vec{F}_1 = \{ 65.95 \vec{i} + 229.8 \vec{j} + 181.2 \vec{k} \}$$

$$F_{2x} = (250 \cos 60) \cos 25 = 113.3 \text{ N}$$

$$F_{2y} = 250 \sin 60 = 216.5 \text{ N}$$

$$F_{2z} = (250 \cos 60) \sin 25 = -52.83 \text{ N}$$

$$\therefore \vec{F}_2 = \{ 113.3 \vec{i} + 216.5 \vec{j} - 52.83 \vec{k} \}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

$$\vec{R} = \left[(65.95 + 113.3)\vec{i} + (229.8 + 216.5)\vec{j} + (181.2 - 52.83)\vec{k} \right]$$

$$\vec{R} = (179.25)\vec{i} + (446.3)\vec{j} + (128.37)\vec{k}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$R = \sqrt{(179.25)^2 + (446.3)^2 + (128.37)^2}$$

$$R = 497.8 \text{ N}$$

$$\vec{U}_R = \frac{R_x}{R}\vec{i} + \frac{R_y}{R}\vec{j} + \frac{R_z}{R}\vec{k}$$

$$= \frac{179.25}{497.8}\vec{i} + \frac{446.3}{497.8}\vec{j} + \frac{128.37}{497.8}\vec{k}$$

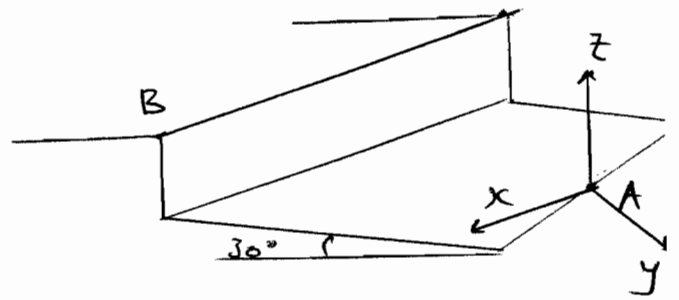
$$\theta_x = \cos^{-1}(0.360) \Rightarrow \theta_x = 68.89^\circ$$

$$\theta_y = \cos^{-1}(0.8965) \Rightarrow \theta_y = 26.29^\circ$$

$$\theta_z = \cos^{-1}(0.2579) \Rightarrow \theta_z = 75.06^\circ$$

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Problem 2 :-



Given:-

Forces at fig P2, $T_{AB} = 10.0 \text{ KN}$, $T_{AC} = 7.5 \text{ KN}$

Required:-

Magnitude and direction of resultant force on A.

Solution:-

$$A(0, 0, 0)$$

$$B(12, -18 \cos 30^\circ, 6 + 18 \sin 30^\circ)$$

$$C(-15, -18 \cos 30^\circ, 9.60 + 18 \sin 30^\circ)$$

\therefore coordinates:-

$$A(0, 0, 0)$$

$$B(12, -15.6, 15)$$

$$C(-15, -15.6, 18.6)$$

$$\vec{AB} = 12\vec{i} - 15.6\vec{j} + 15.0\vec{k} \text{ m} \implies AB = 24.74 \text{ m}$$

$$\vec{AC} = -15\vec{i} - 15.6\vec{j} + 18.6\vec{k} \text{ m} \implies AC = 28.5 \text{ m}$$

$$\vec{F}_{AB} = F_{AB} \vec{U}_{AB} = F_{AB} \frac{\vec{AB}}{AB}$$

$$F_{AB} = \frac{10}{24.75} (12\vec{i} - 15.6\vec{j} + 15\vec{k})$$

$$\vec{F}_{AB} = 4.85\vec{i} - 6.30\vec{j} + 6.06\vec{k} \quad (\text{KN})$$

$$\vec{F}_{AC} = F_{AC} \vec{U}_{AC} = F_{AC} \frac{\vec{AC}}{AC}$$

$$F_{AC} = \frac{7.5}{28.50} (-15\vec{i} - 15.6\vec{j} + 18.6\vec{k})$$

$$\vec{F}_{AC} = -3.95\vec{i} - 4.10\vec{j} + 4.89\vec{k} \quad (\text{KN})$$

$$\therefore \vec{R} = \sum \vec{F} = \vec{F}_{AB} + \vec{F}_{AC}$$

$$= (4.85 - 3.95)\vec{i} + (-6.30 - 4.10)\vec{j} + (6.06 + 4.89)\vec{k}$$

$$\vec{R} = 0.90\vec{i} - 10.4\vec{j} + 10.95\vec{k}$$

$$R = \sqrt{(0.90)^2 + (-10.4)^2 + (10.95)^2} \Rightarrow$$

$$R = 15.13 \text{ KN}$$

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$$\cos \theta_x = \frac{R_x}{R} = \frac{0.9}{15.13} \Rightarrow \theta_x = 86.0^\circ$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{-10.4}{15.13} \Rightarrow \theta_y = 133^\circ$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{10.95}{15.13} \Rightarrow \theta_z = 43.6^\circ$$

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Problem 3

Given:-

fig P₃

Required:-

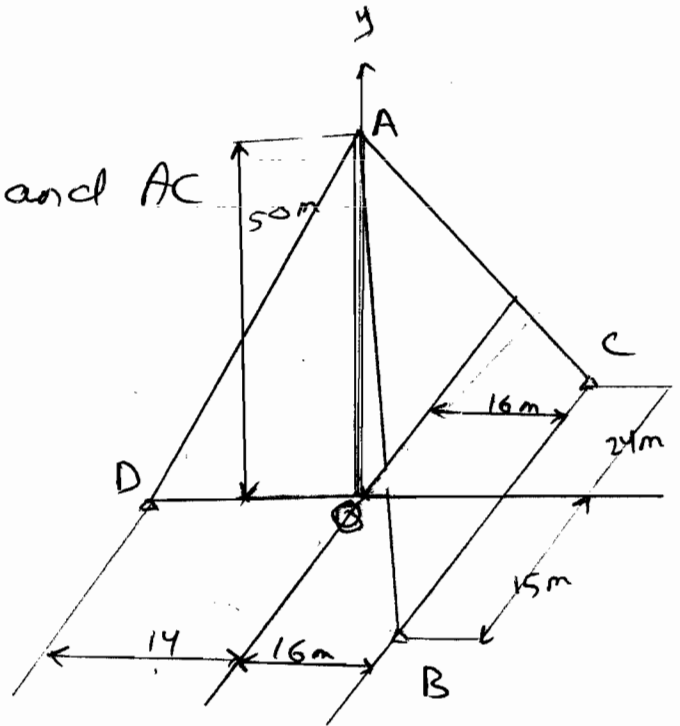
Angle formed between AB and AC

Solution:-

$$A(0, 50, 0)$$

$$B(16, 0, 15)$$

$$C(16, 0, -24)$$



$$\vec{r}_{AB} = (16-0)\vec{i} + (0-50)\vec{j} + (15-0)\vec{k}$$

$$\vec{r}_{AB} = 16\vec{i} - 50\vec{j} + 15\vec{k}$$

$$r_{AB} = \sqrt{16^2 + (-50)^2 + 15^2} \implies r_{AB} = 54.59\text{m}$$

$$\vec{r}_{AC} = (16-0)\vec{i} + (0-50)\vec{j} + (-24-0)\vec{k}$$

$$\vec{r}_{AC} = 16\vec{i} - 50\vec{j} - 24\vec{k}$$

$$r_{AC} = \sqrt{16^2 + (-50)^2 + (-24)^2} \implies r_{AC} = 57.72\text{m}$$

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$$\cos \theta = \frac{\vec{r}_{AB} \cdot \vec{r}_{AC}}{r_{AB} r_{AC}}$$

$$\cos \theta = \frac{(16)(16) + (-50)(-50) + (15)(-24)}{(54.59)(57.72)}$$

$$\theta = 40.5^\circ$$

Problem (4)

Given:- fig P4, $T_{BC} = 280 \text{ lb}$

Required:

a. Angle between BC and BA

b. Projection on BA exerted by cable BC at point B.

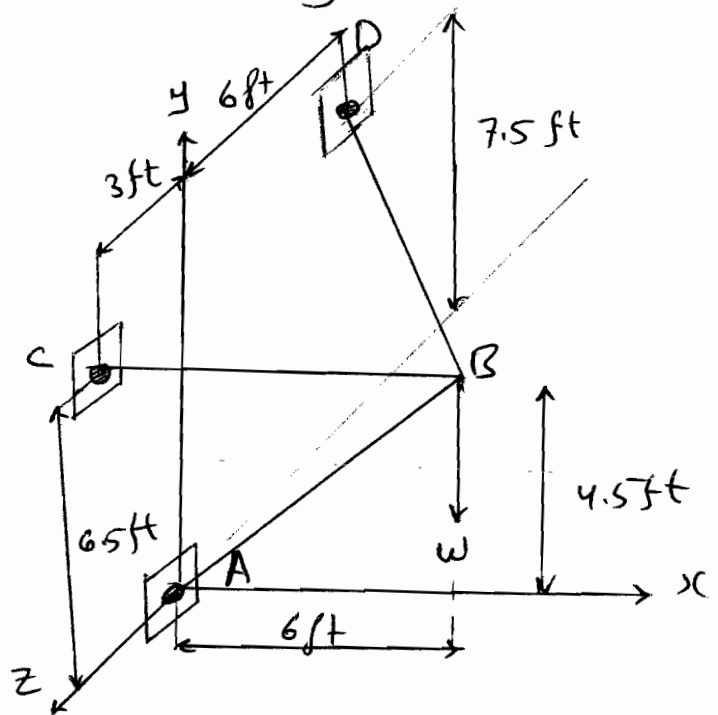
Solution:-

a.

$$A(0, 0, 0)$$

$$B(6, 4.5, 0)$$

$$C(0, 6.5, 3)$$



$$\vec{r}_{BC} = (0 - 6)\vec{i} + (6.5 - 4.5)\vec{j} + (3 - 0)\vec{k}$$

$$\vec{r}_{BC} = -6\vec{i} + 2\vec{j} + 3\vec{k}$$

$$r_{BC} = \sqrt{-6^2 + 2^2 + 3^2} \Rightarrow r_{BC} = 7.0 \text{ ft}$$

$$\vec{r}_{BA} = (0 - 6)\vec{i} + (0 - 4.5)\vec{j} + 0\vec{k}$$

$$\vec{r}_{BA} = -6\vec{i} - 4.5\vec{j}$$

$$r_{BA} = \sqrt{(-6)^2 + (-4.5)^2} \Rightarrow r_{BA} = 7.5 \text{ ft}$$

$$\begin{aligned} \cos \theta &= \frac{\vec{r}_{BA} \cdot \vec{r}_{BC}}{r_{BA} r_{BC}} \\ &= \frac{(-6)(-6) + (-4.5)(2) + (0)(3)}{7.5 \times 7} = 0.514 \end{aligned}$$

$$\theta = 59.05^\circ$$

b-(i) (if Angle θ was not found)

$$(\vec{F}_{BC})_{BA} = F_{BC} \vec{U}_{BC}$$

$$\vec{F}_{BC} = \frac{280}{7} (-6\vec{i} + 2\vec{j} + 3\vec{k})$$

$$\vec{F}_{BC} = -240\vec{i} + 80\vec{j} + 120\vec{k}$$

$$(\vec{F}_{BC})_{BA} = \vec{F}_{BC} \cdot \vec{U}_{BA}$$

$$= (-240\vec{i} + 80\vec{j} + 120\vec{k}) \cdot \left(-\frac{6}{7.5}\vec{i} - \frac{4.5}{7.5}\vec{j}\right)$$

$$= 192 - 48 = 144 \text{ lb}$$

$$\therefore \boxed{F_{BA} = 144 \text{ lb}}$$

(ii) if angle θ was found.

$$\begin{aligned}(F_{BC})_{BA} &= F_{BC} \cos \theta \\ &= 280 \cos 59.05 \\ &= 144.0 \text{ N}\end{aligned}$$

$$\boxed{(F_{BC})_{BA} = 144. \text{ N}}$$