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Examples

Equilibrium of a Particle
(2-D : Coplanar Forces)Example 1:

Given:

The forces on the connection shown

$$A = 5000 \text{ N} \quad , \quad B = 2500 \text{ N}$$

The connection is in equilibrium.

Req'd:

The forces C and D

Soln.:

Equilibrium \Rightarrow From the FBD

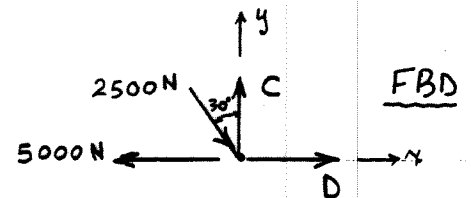
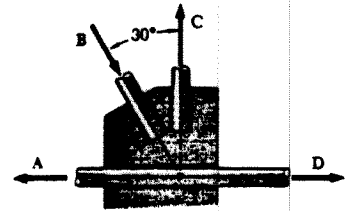
$$\rightarrow \Sigma F_x = 0 \Rightarrow -5000 + D + 2500 \sin 30^\circ = 0 \Rightarrow$$

$$D = 3750 \text{ N}$$

$$\uparrow \Sigma F_y = 0 \Rightarrow C - 2500 \cos 30^\circ = 0 \Rightarrow$$

$$C = 2165 \text{ N}$$

* Note: 2 equations & 2 unknowns

Example 2:

Given:

The electrically charged balls shown

$$m = 0.2 \text{ g} \quad \text{each}$$

Req'd:

The resultant horizontal force of repulsion F

Soln.:

From the FBD, it is easier to start with $\Sigma F_y = 0$. (Why?)

$$\Sigma F_y = 0 \Rightarrow$$

$$T \sin \theta - mg = 0$$

$$T (129.9/150) - 0.2 (10^{-3}) (9.81) = 0$$

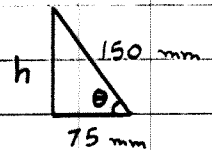
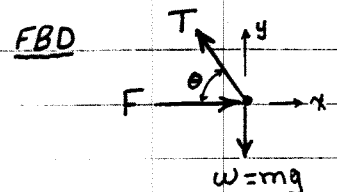
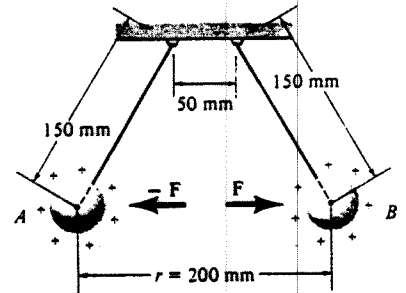
$$\Rightarrow T = 2.266 (10^{-3}) \text{ N} = 2.266 \text{ mN}$$

$$\Sigma F_x = 0 \Rightarrow$$

$$F - T \cos \theta = 0$$

$$F - 2.266 (75/150) = 0$$

$$\Rightarrow F = 1.13 \text{ mN}$$



$$h = \sqrt{(150)^2 - (75)^2} = 129.9 \text{ mm}$$

Example 3:

Given:

The system shown which is in equilibrium

Req'd.:

The mass at A and the angle θ

Sol'n.:

From the FBD shown, D is in equilibrium

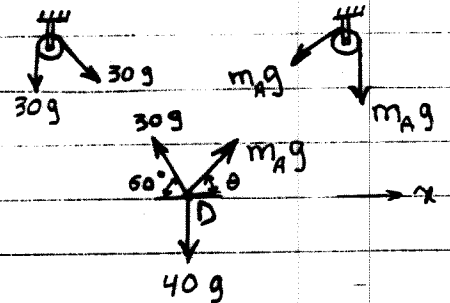
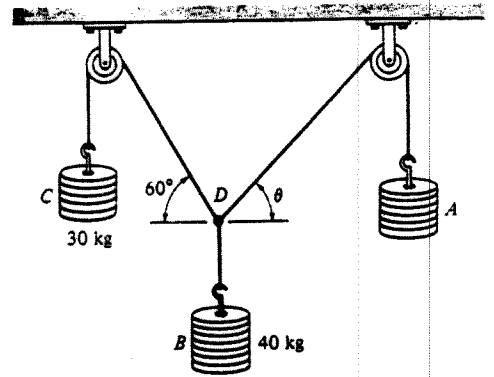
$$\Rightarrow \sum F_x = 0 \Rightarrow m_A g \cos \theta - 30g \cos 60^\circ = 0$$

$$\sum F_y = 0 \Rightarrow m_A g \sin \theta + 30g \sin 60^\circ - 40g = 0$$

$$\Rightarrow m_A = 15 / \cos \theta$$

$$\Rightarrow 15 \tan \theta + 30 \sin 60^\circ - 40 = 0$$

$$\Rightarrow \theta = 43.1^\circ \quad \Rightarrow m_A = \frac{15}{\cos 43.1^\circ} \Rightarrow m_A = 20.5 \text{ kg}$$



Notes: ① $w = mg$; ② pulleys are assumed frictionless (smooth).

Example 4:

Given:

The springs shown in the figure which are in their equilibrium position

Req'd.:

The stretch in each spring

Sol'n.: ((First, guess the answers!))

From the FBD, $\sum F_x = 0 \Rightarrow$

$$F_{AB} \left(\frac{4}{5}\right) - F_{AC} \left(\frac{3}{\sqrt{18}}\right) = 0 \Rightarrow F_{AC} = 1.1314 F_{AB}$$

$\sum F_y = 0 \Rightarrow$

$$-2g + F_{AB} \left(\frac{3}{5}\right) + F_{AC} \left(\frac{3}{\sqrt{18}}\right) = 0$$

$$\frac{3}{5} F_{AB} + 1.1314 \left(\frac{3}{\sqrt{18}}\right) F_{AB} = 2g \Rightarrow F_{AB} = 14.01 \text{ N}$$

$$\Rightarrow F_{AC} = 1.1314 (14.01) = 15.86 \text{ N}$$

$$F_{\text{spring}} = k s \Rightarrow s = F/k \Rightarrow s_{AB} = \frac{14.01}{30} \Rightarrow s_{AB} = 0.467 \text{ m}$$

$$s_{AC} = \frac{15.86}{20} \Rightarrow s_{AC} = 0.793 \text{ m}$$

$$s_{AD} = \frac{2(9.81)}{40} \Rightarrow s_{AD} = 0.491 \text{ m}$$

$$F_{AD} = 2(9) \text{ N}$$

Answers as expected?!

