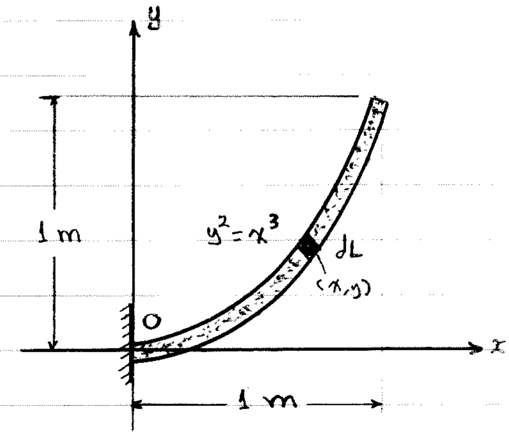
#23

Examples
Centroid
Integration Method

Example 1:



Given:

The bent homogeneous rod shown

$\rho = 0.5$ kg/m

Req.d.:

a) the center of mass $\bar{x}$

b) the reactions at the fixed support O

Soln.:

a) $\bar{x} = \dfrac{\int \tilde{x}\, dL}{\int dL}$ $\Rightarrow$ $dL^2 = dx^2 + dy^2$ $\Rightarrow$ $dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + (\frac{dy}{dx})^2}\ dx$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(x^{\frac{3}{2}}) = \frac{3}{2} x^{\frac{1}{2}}$ $\Rightarrow$ $\int dL = \int_0^1 \sqrt{1 + (\frac{3}{2} x^{\frac{1}{2}})^2}\, dx = \int_0^1 \sqrt{1 + \frac{9}{4} x}\ dx = \frac{4}{9}(\frac{2}{3})(1 + \frac{9}{4}x)^{\frac{3}{2}} \Big|_0^1$

$= 1.4397$ m

$\int \tilde{x}\, dL = \int_0^1 x \sqrt{1 + \frac{9}{4} x}\ dx$

let $u = 1 + \frac{9}{4} x$

$\Rightarrow dx = \frac{4}{9} du$; $x = \frac{4}{9}(u - 1)$

$= (\frac{4}{9})^2 \Big[\frac{2}{5}(1 + \frac{9}{4} x)^{\frac{5}{2}} - \frac{2}{3}(1 + \frac{9}{4} x)^{\frac{3}{2}} \Big]_0^1$

$= 0.78566\ m^2$

$\Rightarrow \bar{x} = 0.78566 / 1.4397$ $\Rightarrow$ $\boxed{\bar{x} = 0.546 \text{ m}}$

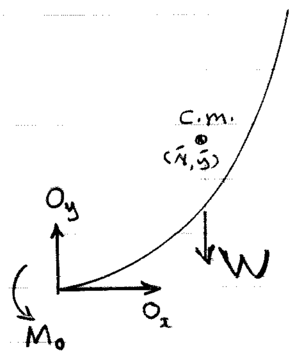
b) $\Sigma F_x = 0$ $\Rightarrow$ $\boxed{O_x = 0}$

$+\uparrow \Sigma F_y = 0 \Rightarrow O_y - W = 0$

$W = mg = L \rho g = 1.4397(0.5)9.81$

$= 7.061$ N

$\Rightarrow O_y - 7.061 = 0 \Rightarrow$ $\boxed{O_y = 7.06 \text{ N} \uparrow}$



$\circlearrowleft + \Sigma M_o = 0 \Rightarrow$

$M_o - W\bar{x} = 0 \Rightarrow M_o = 7.061(0.546)$

$\Rightarrow$ $\boxed{M_o = 3.85 \text{ N.m} \circlearrowleft}$

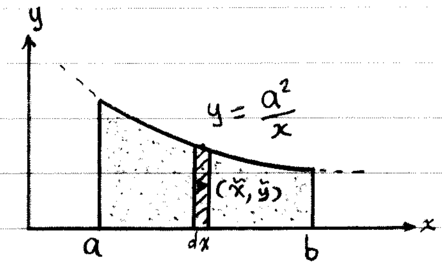
Example 2 :

Given :

The area shown

Reqd. :

The centroid



Soln. :

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} \quad ; \quad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$\tilde{x} = x$

$\tilde{y} = \frac{1}{2} y$

$dA = y\, dx$ (vertical element)

$$A = \int dA = \int_a^b y\, dx = \int_a^b a^2 \frac{dx}{x} = a^2(\ln b - \ln a) = a^2 \ln b/a$$

$$\int \tilde{x}\, dA = \int_a^b x(y\,dx) = \int_a^b x\left(\frac{a^2}{x}\right) dx = \int_a^b a^2 dx = a^2(b-a)$$

$$\int \tilde{y}\, dA = \int \frac{y}{2}(y\,dx) = \frac{a^4}{2}\int_a^b \frac{1}{x^2}\,dx = \frac{a^4}{2}(-1\,x^{-1})\Big|_a^b$$

$$= \frac{a^4}{2}\left[-\left(\frac{1}{b} - \frac{1}{a}\right)\right] = \frac{b-a}{ab}\left(\frac{a^4}{2}\right)$$

$$\Rightarrow \bar{x} = \frac{a^2(b-a)}{a^2 \ln b/a} \Rightarrow \boxed{\bar{x} = \frac{b-a}{\ln b/a}}$$

$$\bar{y} = \frac{a^4(b-a)}{2ab\,(a^2 \ln b/a)} \Rightarrow \boxed{\bar{y} = \frac{a(b-a)}{2b \ln b/a}}$$

Try to solve this problem using "horizontal" element.
Do you see any problem or difficulties in this ?!
What method is easier and shorter ?

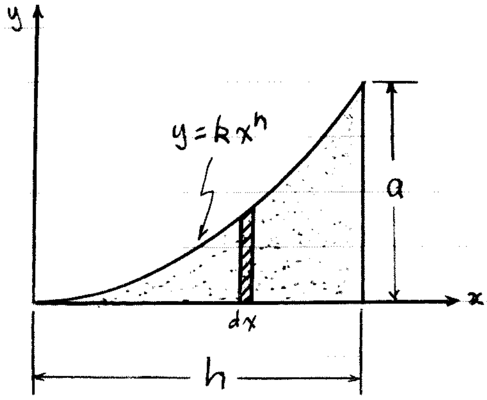
Example 3:

Given:

The figure shown

Reqd.:

The centroid of the volume obtained by rotating the area shown about the x-axis



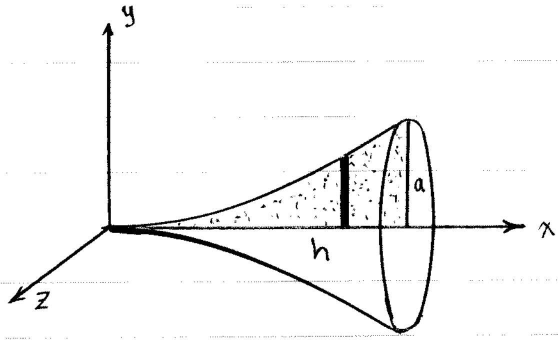
Soln.:

Take a strip as shown in the figure

From B.C.,

$k = a/h^n$

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV}$$



$$dV = \pi y^2 dx = \pi k^2 x^{2n} dx$$

$$\Rightarrow V = \int_0^h \pi k^2 x^{2n} dx$$

$$= \frac{\pi k^2}{2n+1} h^{2n+1}$$

$$\tilde{x} = x$$

$$\Rightarrow \int \tilde{x}\, dV = \int \pi k^2 x^{2n+1} dx$$

$$= \frac{\pi k^2}{2n+2} h^{2n+2}$$

$$\Rightarrow \bar{x} = \frac{\frac{\pi k^2}{2n+2} h^{2n+2}}{\frac{\pi k^2}{2n+1} h^{2n+1}} \Rightarrow \boxed{\bar{x} = \frac{2n+1}{2(n+1)} h}$$

Note that this expression can be used to derive some of the formulas given in the book (back cover), such as cones $(n=1)$

$\Rightarrow \bar{x} = \frac{3}{4} h$ and paraboloids of revolution $(n = \frac{1}{2}) \Rightarrow$

$\bar{x} = \frac{2}{3} h$, ... etc.