

Cross Product

(Vector ")

$$\vec{C} = \vec{A} \times \vec{B}$$

The result of a cross (vector) product of two vectors \vec{A} and \vec{B} is a vector \vec{C} with the following properties:

1) Magnitude:

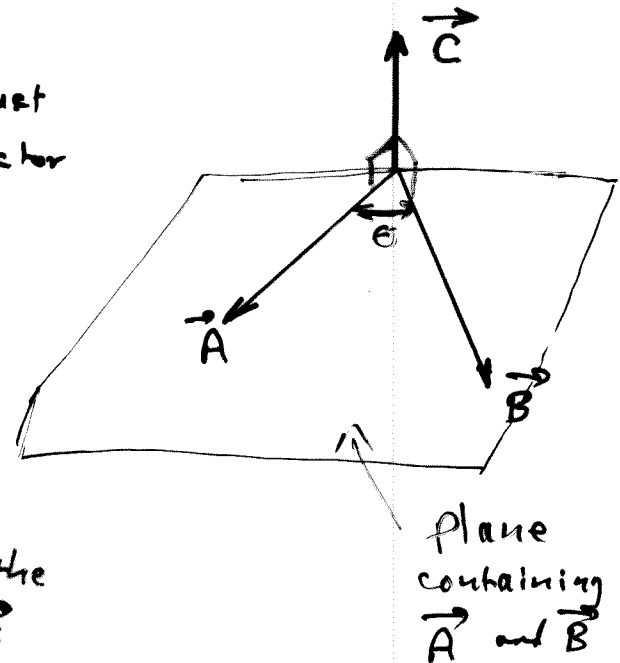
$$C = AB \sin \theta$$

2) Line of action:

\perp (perpendicular) to the plane containing \vec{A} and \vec{B}

3) Sense:

right hand rule (RHR)



Laws of Operations:

$$1) \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \\ = -\vec{B} \times \vec{A} \quad \text{« RHR »}$$

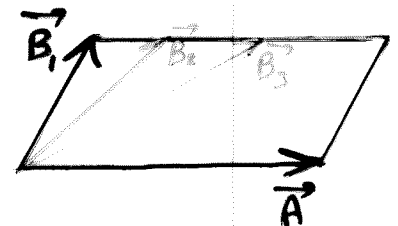
$$2) n(\vec{A} \times \vec{B}) = (n\vec{A}) \times \vec{B} = \vec{A} \times (n\vec{B}) = (\vec{A} \times \vec{B})n$$

n is a scalar

$$3) \vec{A} \times (\vec{B} + \vec{D}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{D})$$

$$4) \vec{A} \times \vec{B}_1 \stackrel{?}{=} \vec{A} \times \vec{B}_2 \stackrel{?}{=} \vec{A} \times \vec{B}_3$$

yes! yes!



$$5) (\vec{A} \times \vec{B}) \times \vec{D} \stackrel{?}{=} \vec{A} \times (\vec{B} \times \vec{D})$$

no!

* Cartesian Vector / Rectangular Coordinate

$$\vec{i} \times \vec{j} = \vec{k}$$

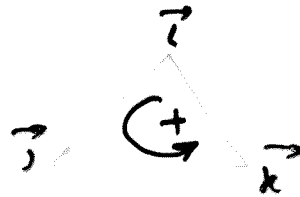
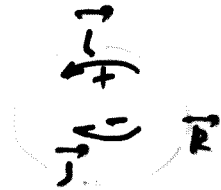
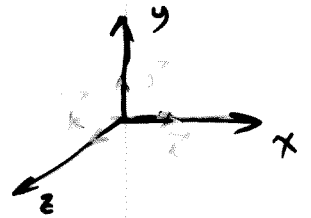
$$\vec{j} \times \vec{i} = -\vec{k}$$

$$\vec{j} \times \vec{k} = \vec{i}$$

$$\vec{k} \times \vec{j} = -\vec{i}$$

$$\vec{k} \times \vec{i} = \vec{j}$$

$$\vec{i} \times \vec{k} = -\vec{j}$$



$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k}) \\ &= A_x B_x (\vec{i} \times \vec{i}) + A_x B_y (\vec{i} \times \vec{j}) + \dots \\ &= 0 + A_x B_y \vec{k} + \dots \end{aligned}$$

⇒

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad (\text{determinant})$$