Equilibrium of Particles in 3-D

Eqs. and FBD's are as in the 2-D above, but add the third (z) direction. ⇒

$$\Sigma F_x = 0 \quad (1)$$

$$\Sigma F_y = 0 \quad (2)$$

$$\Sigma F_z = 0 \quad (3)$$

3 eqs. ⇒ ?? unknowns ??

Review & Practice

FBD, FBD, FBD, FBD

See next pages & examples.

EXAMPLE 3-5

physical (space) diagram

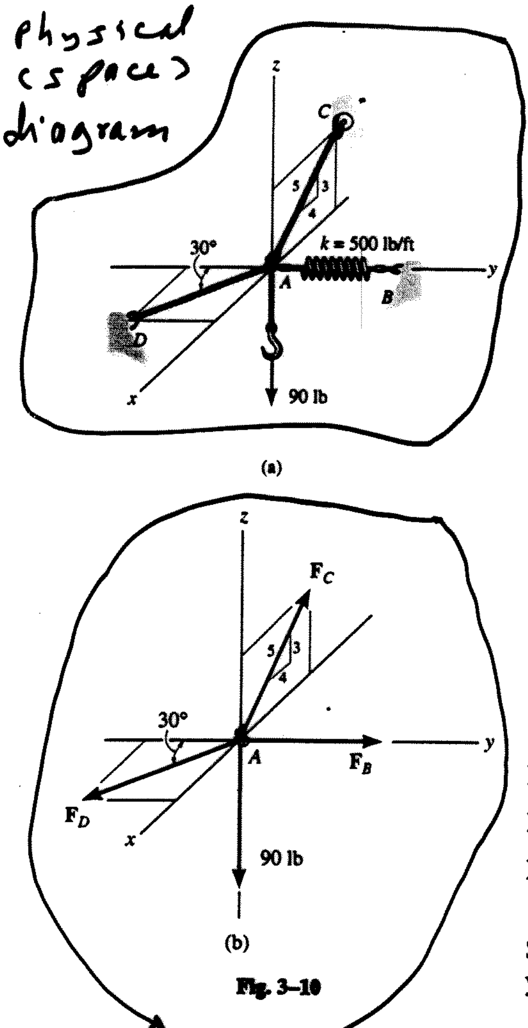


Fig. 3-10

FBD

3 eq. & 3 unknowns

A 90-lb load is suspended from the hook shown in Fig. 3-10*a*. The load is supported by two cables and a spring having a stiffness $k = 500$ lb/ft. Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.

Solution

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig. 3-10*b*.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be directly applied. Considering components directed along the positive axes as "positive," we have

$$\Sigma F_x = 0; \qquad F_D \sin 30° - \tfrac{4}{5}F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30° + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \tfrac{3}{5}F_C - 90 \text{ lb} = 0 \qquad (3)$$

Solving Eq. 3 for F_C, then Eq. 1 for F_D, and finally Eq. 2 for F_B, yields

$$F_C = 150 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 240 \text{ lb} \qquad \textit{Ans.}$$

$$F_B = 208 \text{ lb} \qquad \textit{Ans.}$$

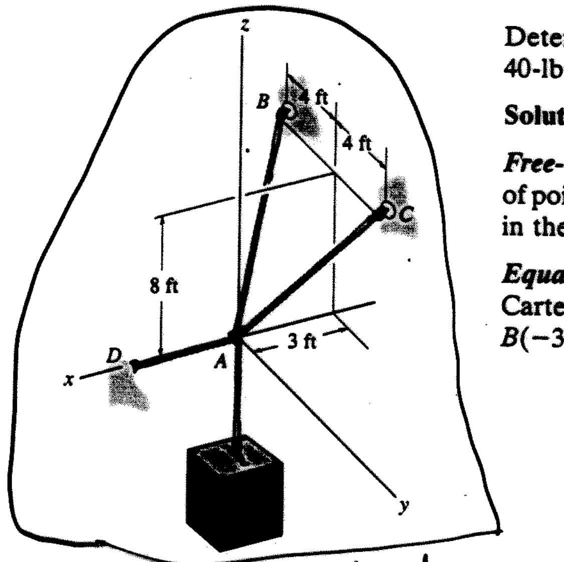
The stretch of the spring is therefore

$$F_B = ks_{AB}$$

$$208 \text{ lb} = 500 \text{ lb/ft}\,(s_{AB})$$

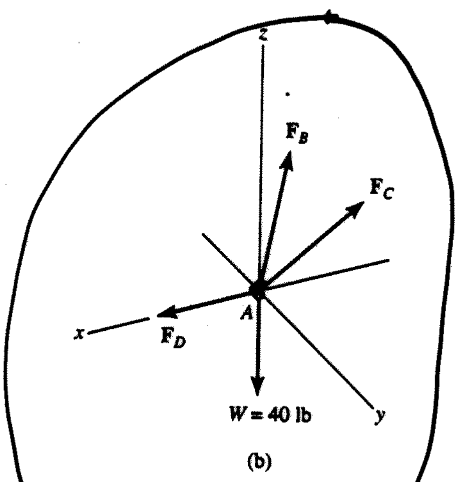
$$s_{AB} = 0.416 \text{ ft} \qquad \textit{Ans.}$$

EXAMPLE 3-7



space (physical) diagram

(a)



(b)

Fig. 3-12

FBD

Determine the force developed in each cable used to support the 40-lb crate shown in Fig. 3–12*a*.

Solution

Free-Body Diagram. As shown in Fig. 3–12*b*, the free-body diagram of point *A* is considered in order to "expose" the three unknown forces in the cables.

Equations of Equilibrium. First we will express each force in Cartesian vector form. Since the coordinates of points *B* and *C* are $B(-3\text{ ft}, -4\text{ ft}, 8\text{ ft})$ and $C(-3\text{ ft}, 4\text{ ft}, 8\text{ ft})$, we have

$$\mathbf{F}_B = F_B\left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}\right]$$
$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$\mathbf{F}_C = F_C\left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}\right]$$
$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$
$$\mathbf{F}_D = F_D\mathbf{i}$$
$$\mathbf{W} = \{-40\mathbf{k}\}\text{ lb}$$

Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$
$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k} - 0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero yields

$$\Sigma F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0 \qquad (1)$$
$$\Sigma F_y = 0; \qquad -0.424F_B + 0.424F_C = 0 \qquad (2)$$
$$\Sigma F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0 \qquad (3)$$

Equation 2 states that $F_B = F_C$. Thus, solving Eq. 3 for F_B and F_C and substituting the result into Eq. 1 to obtain F_D, we have

$$F_B = F_C = 23.6\text{ lb} \qquad \textit{Ans.}$$
$$F_D = 15.0\text{ lb} \qquad \textit{Ans.}$$

3 eq. & 3 unknowns